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Scientific Research

Reactive Power Reserve Improvement Using Power Systems Inherent Structural Characteristics

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ABSTRACT

This paper considers the use of the inherent structural characteristics of power system networks for improving the reactive power reserve margins for both topologically weak and strong networks. The inherent structural characteristics of the network are derived from the Schur complement of the partitioned Y-admittance matrix using circuit theory representations. Results show that topologically strong networks, operating close to the upper voltage limit could be made to increase their loadability margin by locating reactive power compensators close to generator sources, whereas topologically weak (ill conditioned) networks could be made to operate within the feasible operating limits by locating reactive power compensators on buses farther from generator sources.

Keywords: Power System Networks Inherent Characteristics; Reactive Power Reserve Margin; Loadability Margin; Schur Complement

1. Introduction

Transmission network plays a critical role in power systems operations. Its role in ensuring reliable operations of power systems was acknowledged after post-mortem analysis of major blackouts in many advanced countries [1]. However, the concept of using transmission networks inherent structural characteristics to resolve power systems operational issues has not been fully considered. Transmission networks traditionally serve the purpose of transporting power from generating stations to load centres. The amount of power that could be supplied from generating stations to load centres and the routes for the power flows depend on the transmission network structural interconnections [2]. The structural interconnections between the power system nodes define the power system inherent structural characteristics [3]. These characteristics are governed by the value of line impedances and how they are interconnected. Line impedances consist of resistive and reactive components. The reactive components account for the reactive power presence on the network, which majorly affect the network operating voltages and the amount of transferable active power a transmission network can support [4]. Transmission networks with excess reactive power are in general topologically strong networks and have network bus voltages that are very high beyond the nominal limit. On the other hand, topologically weak (ill conditioned) networks, which are in deficit of reactive power, have low voltages below the nominal limit [5]. Therefore, there is the need to balance the amount of reactive power in a network against the desired voltage operating limits [6]. However, due to scarce resources, the difficulty of securing rights of way and environmental issues, power system networks are forced to operate within tight technical constraints. The effect in recent times is the total blackouts caused by voltage collapsed experienced by many matured power system networks around the world [1].

So far, the approaches used to address reactive power reserve mainly range from the linear programming technique to nonlinear programming techniques [7-13]. However, the challenge for these optimisation techniques is the nonlinear, non-convex nature of the problem formulation [14]. Due to the non-convex nature of the problem, many optimisation techniques could easily be trapped in local minima [11]. Secondly, ill conditioned networks could lead to suboptimal solutions because of the need to locate fictitious reactive power compensators first, to achieve convergence of load flow before the loadability of the network can be properly addressed [15]. Thirdly, the large sizes of practical networks could be a challenge when the nonlinearity of the problem formulation is fully considered, since large solution variables would need to be generated and may present memory storage issues [13]. Finally, different buses on the network affect the network operations differently because of the nonlinearity of the network parameters [16]. Because of these challenges, there is therefore, the need to reconsider the fundamental circuit theory properties of network, in order to identify its inherent structural characteristics that could be used to achieve better reactive power management.

This paper focuses on the inherent structural characteristics of power system networks as a solution guide to the issue of reactive power reserve management. For the remainder of the paper, section II presents a brief overview of reactive power reserve management tools, section III discusses the inherent structural characteristics of networks and section IV presents a case study and result discussion. Finally, section V concludes the paper and highlights the major findings.

2. Techniques for Assessing Reactive Power Reserve Margin

The purpose of adequate reactive power in a network is to ensure operation of the network at both normal and stressed conditions. The stressed condition consists of lost of major lines, transformers, generators or a situation where load gradually increase until the network cannot support such load demand corresponding to the nose curve point D in **Figure 1**, referred to as voltage collapse point [15]. A power system with operating voltage at point A is ill conditioned, because, it is operating below the nominal voltage limit. This is caused by the unavailability of sufficient reactive power in the network, hence, load flow may not converge for such networks [17]. It is necessary to move the operating point to between points B and C for which load flow will converge [15]. **Figure 1** shows that as the network voltages move more towards point C, the amount of extra power demand it can support increases (*i.e.* increased loadability margin) until beyond point C where it is infeasible to operate the network. The relationship between voltages and network loadability is nonlinear. The amount of load demand the network can support before voltage collapse is referred to as its maximum loadability margin. The loadability margin is a function of reactive power reserve in the network [18].

The techniques used for assessing the maximum loadability of networks are continuous power flow (CPF) technique and optimal power flow-direct method (OPF-DM) or mathematical optimisation techniques [14,18,19]. The difference between the two techniques is that in the latter, to ensure adequate reactive power margin, system security variables to be maintained within limits must be defined, hence constituting an indirect approach to security assessment of the network [19].

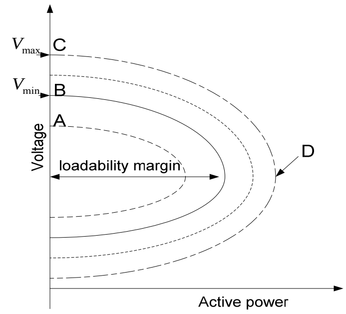

Figure 1. Power-voltage curve.

The CPF technique uses a modified power flow whose loading margin can be expressed as

$$\lambda = \lambda_c - \lambda_0 \tag{1}$$

where λ_c is maximum loading at the critical point and λ_0 is the current or base loading margin. As λ changes with load increase, it relationship with variation in generation and load pattern are

$$P_G = P_{G0} + \left(\lambda + K_G\right) P_S \tag{2}$$

$$P_L = P_{L0} + \lambda P_D \tag{3}$$

$$Q_L = Q_{L0} + \lambda K_L P_D \tag{4}$$

where P_{G0} is the generation base level, P_{L0} and Q_{L0} are the base level of active and reactive respectively, K_G represents distributed slack bus and K_L represents loads with constant power factor. P_S and P_D represent generation and load directions respectively [19].

In the case of optimisation techniques, the maximum loadability can be expressed as [14]

$$\max_{\delta, V_L, V_G, Q_G, \lambda} \lambda \tag{5}$$

Subject to

$$P_{Gi} - \lambda P_{Li} - \hat{G}_P\left(\delta, V_{L,} V_{G,} G_{ij,} B_{ij}\right) = 0 \tag{6}$$

$$Q_{Gi} - \lambda Q_{Li} - \hat{G}_q\left(\delta, V_{L,} V_{G,} G_{ij,} B_{ij}\right) = 0 \tag{7}$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \tag{8}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \tag{9}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{10}$$

where $i = 1 \cdots n$, P_{Gi} is the active power generation at bus i, Q_{Gi} is the reactive power generation at bus *i*, V_G is the generator bus voltage magnitude, V_L is the load bus voltage magnitude, δ is the bus voltage phase angles, P_{Li} is the active power demand at bus *i*, Q_{Li} is the reactive power demand at bus *i*, G_{ij} is the conductance of line ij and B_{ij} is the susceptance of line ij [19]. Equations (6) and (7) are the power balance equations of the network, while Equations (8) - (10) are the inequalities that must be satisfied for the network to operate between points B and C of **Figure 1**. For topologically weak (ill conditioned) networks, convergence of load flow may not hold since the network is operating around point A [17]. Other approaches besides those presented in this section are necessary to identify suitable locations for reactive compensators for such networks [20]. On the other hand, topological strong networks have voltages that are between points B and C of **Figure 1**; however, if the loadability of the network is to be increased then suitable locations for reactive power compensators are required. In order to satisfy these objectives, the inherent structural characteristics of network which may serve as a guide in selecting suitable reaction power compensators locations is presented in the next section.

3. Inherent Structural Characteristics of Networks

The fundamental circuit theorem law applicable to power system networks can be written as

$$I = YV \tag{11}$$

where I is current, Y is network admittance and V is voltage.

Suppose that the Y-admittance matrix is partitioned as

$$Y = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \tag{12}$$

where Y_{GG} is the generator-generator coupling in the system admittance matrix with dimension $G \times G$, Y_{GL} is the generator-load coupling in the system admittance matrix with dimension $G \times L$, Y_{LG} is the load-generator coupling in the system admittance matrix with dimension $L \times G$ and Y_{LL} is the load-load coupling in the system admittance matrix with dimension $L \times L$. G and L are the number of generator and load buses in the network respectively.

We can express (11) as

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \tag{13}$$

where I_G is injected generator bus currents, I_L is injected load bus currents, V_G is generator bus complex voltages and V_L is load bus complex voltages.

Since Y_{GG}, the leading submatrix in (12) is non-singular, the Schur complement [21,22] of Y_{GG} in Y is

$$YY_{LL} - Y_{LG} Y_{GG}^{-1} Y_{GL} \tag{14}$$

The determinant of the Y-admittance matrix based on Schur complement formula [21] is

$$\det Y = \det Y_{GG} \det \left(Y_{LL} - Y_{LG} Y_{GG}^{-1} Y_{GL} \right) \tag{15}$$

In compact form (15) is expressed as

$$\det Y = \det Y_{GG} \det C_{LL} \tag{16}$$

where $C_{LL} = Y_{LL} - Y_{LG} Y_{GG}^{-1} Y_{GL}$ and represents the equivalent admittance of the network with all influences associated to generators eliminated.

The importance of matrix C_{LL} in relation to power system network is clearer from the algebraic manipulation of (13) which gives

$$V_L = C_{LL}^{-1} \left\{ I_L - W_{LG} I_G \right\} \tag{17}$$

where $W_{LG} = Y_{LG} Y_{GG}^{-1}$

The right hand side of (17) shows that matrix C_{LL} is inversely related to the network bus voltages. Combining this fact with the determinant relationship of this matrix with the entire network structure presented in equation (16), it shows that matrix C_{LL} holds essential information about the network structure. In order to identify these inherent structural characteristics contained in matrix C_{LL}, eigenvalue decomposition technique [23] is applied as

$$C_{LL} = MRM^* = \sum_{i=1}^{n} m_i \mu_i m_i^* \tag{18}$$

where M is a orthonormal matrix with eigenvectors m_i, while μ_i are the eigenvalues.

Since the inverse of matrix C_{LL} exist due to the non-singularity of Y_{GG}, the generalized inverse of matrix C_{LL} is

$$C_{LL}^{-1} = MR^{-1}M^* = \sum_{i=1}^{n} \frac{m_i m_i^*}{\mu_i} \tag{19}$$

Substituting Equation (19) into Equation (17) gives

$$V_L = \sum_{i=1}^{n} \frac{v_i m_i^T}{\mu_i} \left\{ I_L - W_{LG} I_G \right\} \tag{20}$$

The buses associated with the smallest eigenvalues in matrix C_{LL} would have the most effect on the network bus voltages as mathematically expressed in Equation (20), due to the reciprocal relationship between eigenvalues and the load voltages. From power system perspective, the smallest eigenvalue $\mu_n = 0$ based on a predefined precision level, will occur when the network buses are electrically far from one another, because of the shortage of adequate reactive element within the network structure. The corresponding left eigenvectors (matrix M) in this case will have column vectors with constant values, indicating non-participation between the network buses. This indicates a topologically weak (ill conditioned) network [5]. As previously discussed, topologically weak network have low voltages [5]. Buses associated with the smallest eigenvalues indicate where reactive power support are required [20]. Hence, to improve the overall voltage profile these buses are suitable locations for reactive power compensators [24].

On the hand, when the smallest eigenvalue is $\mu_n > 0$ based on a predefined precision level, there is adequate reactive element presence in the network structure. The degree of sufficiency of the reactive elements dependent on the participation between the network buses observable from the eigenvectors of matrix *M*. Networks that exhibit such characteristics are topologically strong networks [5]. As already mentioned, such networks have adequate voltages. In order to improve the loadability margin for such networks, generators should be prevented from reaching their reactive power limits by adding reactive power compensators close to the generators. Buses associated with the largest eigenvalues are suitable locations for achieving this objective, since they are the ones closest to the generator buses. The next section illustrates this concept with a case study.

4. Case Study and Discussion of Results

The test network is a 40 bus Southwest networks shown in **Figure 2**. The voltage profile of this test network without any reactive power compensator is shown in **Figure 3**. The purpose of adding reactive power compensators is mainly for increasing the loadability margin of this test network [14].

The smallest eigenvalue for this network is 0.0045 (in absolute value) from the application of equation (19). A set of five suitable locations for installing reactive power compensators associated with the largest eigenvalues for improving the loadability margin of the test network are presented in **Table 1**.

In order to ascertain the effectiveness of these locations on reactive power reserve margin, comparison with locations obtained using multi start-Benders decomposition technique published in [14] for the same network is used in this paper. Continuous power flow (CPF) reactive power assessment technique implemented in Power System Analysis Toolbox (PSAT) with generation and load directions set was used to determine the maximum loadability margin of the test network. The maximum loadability for both approaches are shown in **Table 2** using a Static Var Compensator (SVC) of $\pm 0.04\,pu$.

The power-voltage curves for the lowest voltage of each approach are shown in **Figure 4** for the maximum loadability corresponding to installation of five SVCs.

The proposed approach improves the network loadability margin better compared to the multi start-Benders decomposition technique as shown in **Table 2** and **Figure 4** respectively. This is because the propose approach seeks to locate reactive power compensators close to generators, in order to prevent the generators from

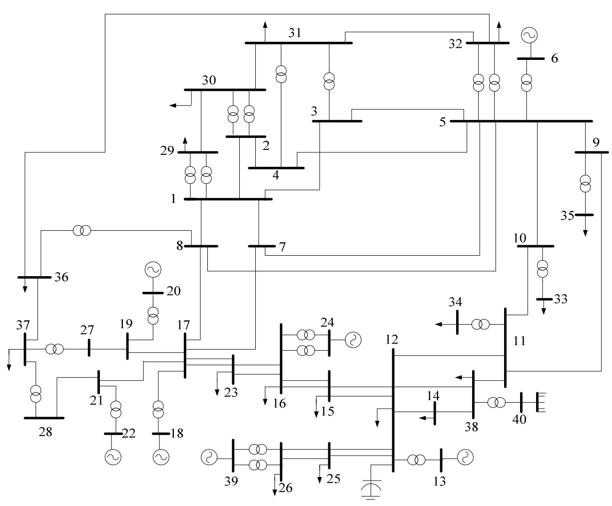

Figure 2. Southwest England 40 bus network.

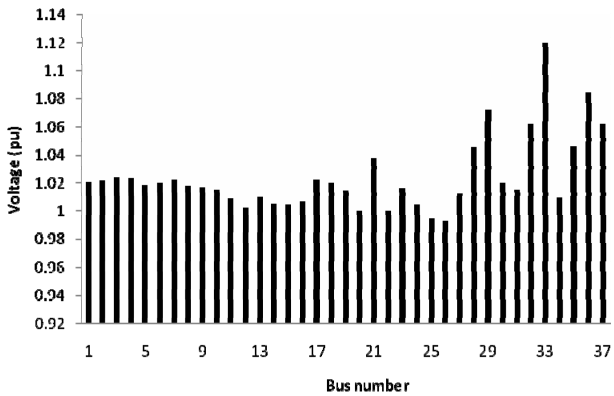

Figure 3. Voltage profile of Southwest 40 bus network.

Table 1. Suitable locations for reactive power compensators.

S/N	Largest eigenvalues	Bus number
1	16.4309	9
2	9.9136	11
3	9.3788	10
4	8.2647	12
5	7.3916	2

Table 2. Comparison of maximum loadability.

Number of SVCs	Proposed Approach		Multi start-Benders decomposition	
	Bus number	λ (p.u)	Bus number	λ (p.u)
1	9	1.1434	29	1.1237
2	9,11	1.1939	29,30	1.174
3	9,11,10	1.2445	29,30,32	1.1978
4	9,11,10,12	1.2934	29,30,32,31	1.2045
5	9,11,10,12,2	1.3405	29,30,32,31,28	1.2119

Base (No SVC) Maximum loadability (λ) = 1.0909 p.u.

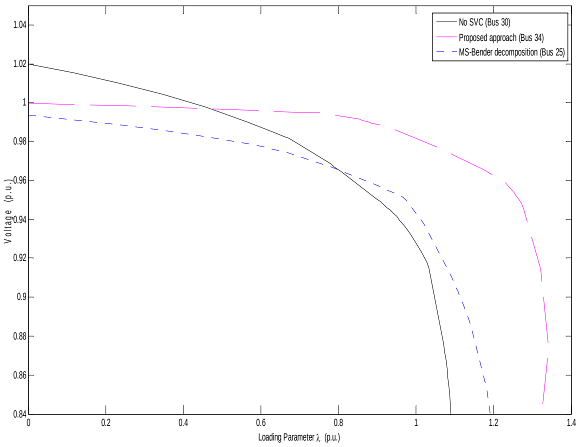

Figure 4. Power-voltage curve of the test network.

reaching their reactive power limits. This allows the generators to be free to supply more active power as the load demand increases in topologically strong networks. On the other hand, for topologically weak networks, the compensators should be located on nodes farthest from the generators, (i.e. on buses associated with the smallest eigenvalues) [20,24] to ensure that the networks would be within the acceptable voltage limits.

5. Conclusions

This paper has demonstrated that the network inherent characteristics derivable from the Schur complement of the partitioned Y-admittance matrix could be used to identify suitable locations for improving reactive power reserve margins in power system networks. For the case of topologically weak (ill conditioned) networks, buses associated with the smallest eigenvalues are suitable locations for installing reactive power compensators to ensure feasibility of the network operating voltages. On the other hand, topologically strong networks, operating well within the desired voltage limits could increase their loadability margin by installing reactive power compensators on buses associated with the largest eigenvalues.

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