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Capacity Choice in a Private Duopoly: A Unilateral Externality Case*

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ABSTRACT

This paper studies capacity choice in a quantity-setting and price-setting private duopoly with differentiated goods wherein either of two firms has a price-raising effect on the price level of the product of the opponent firm. In both quantity-setting and price-setting competition, whether the price-raising effect of the product of one firm on the price level of the other firm's product is strong or weak strictly depends on the differences between the quantities and capacity levels of both firms. More precisely, in the quantity-setting competition, when the price-raising effect is sufficiently strong, both firms choose under-capacity, whereas when such an effect is sufficiently weak, both firms choose over-capacity. Furthermore, in the price-setting competition, when the price-raising effect is sufficiently strong, both firms choose over-capacity, whereas when such an effect is sufficiently weak, both firms choose under-capacity. Therefore, the presence of the price-raising effect as the unilateral externality strikingly changes the difference between each firm's quantity and capacity level in the contexts of both the quantity-setting competition and the price competition in a private duopoly with differentiated goods.

Keywords: Private Duopoly; Externality; Capacity Choice

1. Introduction

This paper considers the capacity choices of two profit-maximizer firms in a private duopoly with unilateral externality. More precisely, in the contexts of both quantity-setting competition and price-setting competition wherein one firm has a price-raising effect on the price level of the product of the opponent firm as the unilateral externality, we investigate the difference between the quantity and capacity level of each firm. We introduce the price-raising effect as the unilateral externality into each firm's demand function à la Choi and Lu [1]. The purpose of this paper is to check the robustness of the result on the difference between each firm's quantity and capacity level obtained in the standard private duopoly with differentiated goods against the introduction of the price-raising effect of the one firm's product on the price level of the opponent firm's product in the fashion of Choi and Lu [1].

In the model of this paper, following the model employed in Choi and Lu [1], we consider the situation where both firms produce a homogeneous good if there does not exist the price-raising effect of the product of the one firm as the unilateral externality[1]. Similar to Choi and Lu [1], since only the quantity of the product of the one firm (say firm 1) has the price-raising effect on the price level of the opponent firm's product, we refer to this effect as the unilateral externality. When the price-raising effect of firm 1's product is sufficiently strong, its product becomes a complement to the product of the firm without the price-raising effect (say firm 0). Hence, when the price-raising of firm 1's product as the unilateral externality is sufficiently strong, the strategic relations between the quantities/price levels and between the capacity levels as the strategic variables of both firms are changed in the context of both the quantity-setting competition and the price-setting competition. Cabral and Majure [2] found theoretical and empirical evidence on the asymmetry of the strategic relation in the Portuguese banking industry. More precisely, they indicated that for some banks, the number of branches of rival banks is a

[1]As indicated in Choi and Lu [1], although we can investigate the model such that the products of both firms can be differentiated even if there is the price-raising effect of the one firm as the unilateral externality, such an assumption does not change the qualitative results obtained in this paper.

strategic complement, whereas for other banks, it is a strategic substitute[2]. By considering a standard oligopolistic market without unilateral externality, Bulow *et al*. [3] and Bulow *et al*. [4] found that the dominant firm in an industry can consider the products of fringe firms as strategic complements, whereas the fringe firms can consider the output level of the dominant firm as strategic substitutes. In addition, Tombak [5] considered a two-stage game in which one firm regards its rival's second-stage strategic variable as a strategic complement whereas the other firm regards its rival's second-stage strategic variable as a strategic substitute[3].

In contrast to the works in this field including Choi and Lu [1], in this paper, we elaborate on the influences of the price-raising effect that the dominant firm has on not only the strategic variables in the market but also the additional strategies (the capacity levels) of the dominant firm and the fringe firm, that is, whether the over-capacity or under-capacity is achieved in both firms[4].

In this paper, we show that when the price-raising effect of the one firm's product on the price level of the other firm's product is sufficiently strong, in the quantity-setting competition, both firms choose under-capacity, while in the price-setting competition, both firms choose over-capacity. These results sharply contrast with those obtained in the standard quantity-setting and price-setting competitions with differentiated goods and without the price-raising effect[5]. As described above, the strength of the price-raising effect of firm 1's product changes the strategic relation between the strategic variables of both firms, *i.e.*, their quantities/price levels and capacity levels. More precisely, when the price-raising effect of the product of firm 1 is sufficiently strong (weak), in the quantity-setting competition, the strategic

relation of firm 0's quantity with firm 1's quantity is a strategic complement (substitute) in firm 0's reaction function in the quantity-setting stage[6]. On the other hand, in the price-setting competition, when the price-raising effect of the product of firm 1 is sufficiently strong (weak), the strategic relation of firm 0's price level with firm 1's price level is a strategic substitute (complement) in firm 0's reaction function in the price-setting stage[7]. Furthermore, in the contexts of both quantity-setting competition and price-setting compe-tition, the influence of firm 1's capacity level on firm 0's quantity/price level is changed when the price-raising effect is sufficiently strong. Thus, when the price-raising effect is sufficiently strong, the changes of the strategic relations between the strategic variables yield changes of the signs of the differences between the quantities and capacity levels of both firms in the context of both quantity-setting compe-tition and the price-setting competition.

The remainder of this paper is organized as follows. In Section 2, we formulate a private duopolistic model with the prise-raising effect of the one firm's product as the unilateral externality and with the capacity choices of both firms that will be investigated throughout this paper. In Section 3, from the viewpoints of quantity-setting competition and the price-setting competition, we consider the differences between the quantities and capacity levels of both firms using the model built in Section 2. Section 4 concludes with several remarks. The market outcomes including each firm's equilibrium price level and profit are relegated to the appendix.

2. Model

Following Choi and Lu [1], we formulate an asymmetric duopolistic model with unilateral externality wherein both firms determine not only their quantities/price levels but also their capacity levels. The products are perfect substitutes if the unilateral externality does not exist. When the unilateral externality is introduced, the quantity of firm 1's products can have an price-raising effect on the price level of firm 0's products. On the basis of Choi and Lu [1], we call the effect such that the output level of firm 1 can raise firm 0's price level the "unilateral externality". The inverse demand functions of firms 0 and 1 are given as follows:

$$p_0 = a - q_0 + (\theta - 1) q_1, \quad p_1 = a - (q_0 + q_1), \text{ respectively,}$$

where p_i and q_i denote firm i's price level and output level, respectively, and parameter θ measures the

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[2]The findings obtained in Cabral and Majure [2] are explained by the following two facts: i) the geographic differences (urban or rural areas) of the expansion patterns of incumbents (public banks) and entrants (private banks) and ii) the degree of customer honesty (in general, rural customers are more honest than urban customers).

[3]More precisely, Tombak [5] investigated the situation where only the dominant firm decides the investment for R&D in the first stage.

[4]By taking into account the managerial delegation in the fashion of Fershtman and Judd [6], Sklivas [7], and Vickers [8], although Choi and Lu [1] investigated the multi-stage game where the delegation parameters of both the firms as well as their quantities/price levels are determined, their main focus was to derive the result on the equilibrium timing of setting the quantities of both firms obtained in the endogenous timing game (the consequent order of the game). We consider how the setting of the strategic variable for the firm without the price-raising effect except for its strategic variable in the market is influenced by the setting of the strategic variables for the firm with the price-raising effect as the unilateral externality.

[5]In the appendix, we give the results on the differences in the quantities and capacity levels between both firms in the standard quantity-setting and price-setting competitions with differentiated goods and without the price-raising effect. More precisely, in a standard quantity-setting competition, both firms always choose over-capacity irrespective of the degree of product differentiation, whereas in a standard price-setting competition, both firms choose under-capacity irrespective of the degree of product differentiation.

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[6]In firm 1's reaction function in the quantity-setting competition, the strategic relation of firm 1's quantity with firm 0's quantity is a strategic substitute irrespective of the strength of the price-raising effect of firm 1.

[7]In firm 1's reaction function in the price-setting competition, the strategic relation of firm 1's price level with firm 0's price level is a strategic complement irrespective of the strength of price-raising effect of firm 1.

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degree of the price-raising effect as the unilateral externality, $(i = 0,1)$. Note that a denotes the demand parameter and $\theta \in (0,1) \cup (1,2)$[8]. $\theta \in (1,2)$ indicates that the price-raising effect of firm 1's output level is relatively strong whereas $\theta \in (0,1)$ indicates that the price-raising effect of firm 1's output level is relatively weak. Futhermore, $\theta \in (1,2)$ implies that firm 1's product becomes a complement to firm 0's product whereas $\theta \in (0,1)$ implies that firm 1's product becomes a substitute for firm 0's product.

Both firms adopt identical technologies represented by the cost function $C_i(q_i, x_i)$, where x_i is the capacity level of firm i, $(i = 0,1)$. Following Vives [9], Ogawa [10], Bárcena-Ruiz and Garzón [11], Tomaru *et al.* [12], Nakamura and Saito [13], Nakamura and Saito [14], and Nakamura [15], we assume that the cost function is given by $C_i(q_i, x_i) = mq_i + (q_i - x_i)^2$, $(i = 0,1)$. This cost function implies that if each firm's output level equals its capacity level, $q_i = x_i$, the long-run average cost is minimized. The profit of firm i is given by $\Pi_i = p_i q_i - C_i(q_i, x_i)$ $(i = 0,1)$.

We investigate the game with the following orders of each firm's moves: In the first stage, firms 0 and 1 simultaneously set their capacity levels. In the second stage, after both firms observe their capacity levels, they engage in either quantity-setting competition or price-setting competition with each other.

3. Equilibrium Analysis

3.1. Quantity Competition

We first solve the quantity-setting game by backward induction from the second stage to obtain the subgame perfect Nash equilibrium. In the second stage, firm i maximizes its profit Π_i with respect to q_i, $(i = 0,1)$. The best response functions of firms 0 and 1 in the second stage are given as follows:

$$q_0(q_1; x_0, x_1) = (a - m - q_1 + 2x_0 + q_1\theta)/4, \quad (1)$$

$$q_1(q_0; x_1) = (a - m - q_0 + 2x_1)/4. \quad (2)$$

From Equations (1) and (2), when $\theta \in (0,1)$, q_0 is decreasing in q_1, whereas when $\theta \in (1,2)$, q_0 is increasing in q_1. For any $\theta \in (0,1) \cup (1,2)$, q_1 is decreasing in q_0. Thus, the strategic relation of the quantity of firm 0 with the quantity of firm 1 strictly depends on the value of θ; that is, when $\theta \in (0,1)$, the quantity of firm 0 is a strategic substitute for the quantity of firm 1, whereas when $\theta \in (1,2)$, the quantity of firm 0 is a strategic complement to the quantity of firm 1. On the other hand, the strategic relation of the quantity of firm 1 is a strategic substitute for the quantity of firm 0

irrespective of $\theta \in (0,1) \cup (1,2)$.

Furthermore, we obtain the following equilibrium quantities of firms 0 and 1 as the functions of their capacity levels in the Nash equilibrium in the quantity-setting stage:

$$q_0 = \frac{8x_0 - 2x_1 + 2x_1\theta + a(3+\theta) - m(3+\theta)}{15+\theta}, \quad (3)$$

$$q_1 = \frac{3(a - m) - 2x_0 + 8x_1}{15+\theta}. \quad (4)$$

In the first stage, firms 0 and 1 realize that the choices of their capacity levels influence their quantity determined in the second stage. Provided Equations (3) and (4), respectively, firms 0 and 1 simultaneously and independently set their capacity levels with respect to their profits. Thus, by solving the first-order conditions of the profits of firms 0 and 1, we obtain

$$x_0(x_1) = \frac{16\left[2x_1(\theta-1) + a(3+\theta) - m(3+\theta)\right]}{97 + 30\theta + \theta^2}, \quad (5)$$

$$x_1(x_0) = \frac{16(3a - 3m - 2x_0)}{97 + 30\theta + \theta^2}, \quad (6)$$

yielding

$$x_0^q = \frac{16(a-m)(13 + 18\theta + \theta^2)}{559 + 419\theta + 45\theta^2 + \theta^3},$$

$$x_1^q = \frac{16(a-m)(13 + 3\theta)}{559 + 419\theta + 45\theta^2 + \theta^3}.$$

Note that superscript q is used to represent the subgame perfect Nash equilibrium market outcomes in the quantity-setting competition. Then, the equilibrium quantities of firms 0 and 1 are given as follows:

$$q_0^q = \frac{(a-m)(15+\theta)(13 + 18\theta + \theta^2)}{559 + 419\theta + 45\theta^2 + \theta^3},$$

$$q_1^q = \frac{(a-m)(15+\theta)(13 + 3\theta)}{559 + 419\theta + 45\theta^2 + \theta^3}.$$

Therefore, from easy calculations, we obtain the following equilibrium result on the differences between the quantities and capacity levels of firms 0 and 1:

$$q_0^q - x_0^q = \frac{(a-m)(\theta-1)(13 + 18\theta + \theta^2)}{559 + 419\theta + 45\theta^2 + \theta^3},$$

$$q_1^q - x_1^q = \frac{(a-m)(\theta-1)(13 + 3\theta)}{559 + 419\theta + 45\theta^2 + \theta^3}.$$

Proposition 1 *In the quantity-setting competition, when the price-raising effect of firm 1's product on the price level of firm 0's product is sufficiently strong, i.e., $\theta \in (1,2)$, both firms 0 and 1 choose under-capacity. In contrast, in the quantity-setting competition, when such a price-raising effect of firm 1's product is sufficiently*

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[8]Similar to Choi and Lu [1], we omit the case of $\theta = 1$ since firm 0's price level is independent of firm 1's output level; this is a trivial case.

weak, i.e., $\theta \in (0,1)$, both firms 0 *and* 1 *choose over-capacity.*

Proposition 1 indicates that whether either of the two firms has a price-raising effect on the price level of the product of its opponent firm strictly determines the differences between the quantities and capacity levels of both firms. More precisely, when the price-raising effect of the one firm's product on the price level of the product of its opponent firm is sufficiently strong, both firms choose under-capacity, whereas when such an effect is sufficiently weak, both choose over-capacity. This result is strikingly different from that obtained in the standard quantity-setting competition with differentiated goods[9]. In the standard private duopolistic competition without the price-raising effect, since firm i pays attention to its market share in order to increase its profit, it attempts to decrease the quantity of its opponent firm j in order to increase its market share by increasing its capacity level when the relation between their output levels is substitutable, $(i, j = 0,1; i \neq j)$. In addition, in the case wherein the relation between the products of both firms is complementary, firm i has an incentive to increase its market share by decreasing the quantity of its opponent firm j by decreasing its own capacity, and thus, this behavior of decreasing firm i's capacity also decreases firm j's capacity since their capacity levels are strategic complements in the first stage, $(i, j = 0,1; i \neq j)$. Although the capacity levels of both firms tend to be low, their quantities are lower than their capacity levels since their capacity levels are positively associated with their own quantities. Thus, irrespective of whether the relation of the goods produced by both firms are substitutable or complementary, in a standard quantity-setting duopoly without the price-raising effect, they choose over-capacity.

In the quantity-setting competition wherein the one firm has the price-raising effect of its product on the price level of the product of the other firm, if such an effect is sufficiently weak, i.e., $\theta \in (0,1)$, the intuition behind the result that both firms 0 and 1 choose over-capacity is similar to that given in the standard quantity-setting competition with substitutable goods and without the price-raising effect[10]. From Equations (3) and (4), when $\theta \in (0,1)$, both firms 0 and 1 have incentive to decrease the quantities of their respective opponent firms through increasing their own capacity in order to expand their respective market share, implying that both firms

choose over-capacity. On the other hand, when the price-raising effect of firm 1's product is sufficiently strong, i.e., $\theta \in (1,2)$, we obtain the result that the differences between the quantities and capacity levels of firms 0 and 1 are positive, which is strikingly different from the results obtained for i) the quantity-setting competition without the price-raising effect and ii) the case wherein such a price-raising effect is sufficiently weak, i.e., $\theta \in (0,1)$. The intuition behind this result is as follows: from Equation (3), firm 1 which has the price-raising effect has an incentive to decrease firm 0's quantity by decreasing its own capacity, and consequently, firm 1 chooses under-capacity. On the other hand, from Equation (4), firm 0, which does not have the price-raising effect has an incentive to decrease firm 1's quantity by increasing its own capacity, similar to the case where $\theta \in (0,1)$. However, taking Equations (5) and (6) into account, firm 0's capacity tends not to become so high when $\theta \in (1,2)$. Therefore, since the quantity of firm 1 does not become so low because of the relatively low capacity of firm 0, from Equation (1), the quantity of firm 0 is higher relative to its capacity when $\theta \in (1,2)$. Consequently, firm 0 also chooses under-capacity. We emphasize that the strength of the price-raising effect and the change of the relation of the goods produced by firms 0 and 1 on the basis of such an effect determine the difference between the quantity and capacity of each firm in the quantity-setting competition with the price-raising effect as the unilateral effect.

3.2. Price Competition

We next solve the price-setting game by backward induction from the second stage to obtain the subgame perfect Nash equilibrium. In the second stage, firm i maximizes its profit with respect to p_i, $(i = 0,1)$. The best-response functions of firms 0 and 1 in the second stage are given as follows:

$$p_0 = \frac{p_1\left(2 - \theta - \theta^2\right) + \theta\left[m - 2x_0 + a(2 + \theta)\right]}{2(1 + \theta)}, \quad (7)$$

$$p_1 = \frac{2p_0 + m\theta + p_0\theta - 2x_1\theta}{2(1 + \theta)}. \quad (8)$$

From Equations (7) and (8), when $\theta \in (0,1)$, p_0 is increasing in p_1, whereas when $\theta \in (1,2)$, p_0 is decreasing in p_1. For any $\theta \in (0,1) \cup (1,2)$, p_1 is increasing in p_0. Thus, the strategic relation of the price level of firm 0 with the price level of firm 1 strictly depends on the value of θ; that is, when $\theta \in (0,1)$, the strategic relation of the price level of firm 0 is a strategic complement to the price level of firm 1, whereas when $\theta \in (1,2)$, the strategic relation of the price level of firm 0 is a strategic substitute for the price level of firm 1. On

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[9]This results on the differences between the quantities and capacity levels of firms 0 and 1, which are obtained in the standard quantity-setting competition with differentiated goods and without the price-raising effect, are given in the appendix. More concretely, in both cases wherein firms 0 and 1 produce substitutable goods or complementary goods, it was shown that they both always choose over-capacity.
[10]As described in the setting of this model, we recall that the fact that the price-raising effect is sufficiently weak implies that the products firms 0 and 1 are substitutable.

the other hand, the strategic relation of the price level of firm 1 is a strategic complement to the price level of firm 0 irrespective of $\theta \in (0,1) \cup (1,2)$.

$$p_0 = \frac{-4x_0 - 4x_1 - 4x_0\theta + 2x_1\theta + 2x_1\theta^2 + m(4 + \theta - \theta^2) + 2a(2 + 3\theta + \theta^2)}{8 + 7\theta + \theta^2}, \tag{9}$$

$$p_1 = \frac{a(2+\theta)^2 + m(4+3\theta) - 2[2x_1(1+\theta) + x_0(2+\theta)]}{8 + 7\theta + \theta^2}. \tag{10}$$

In the first stage, firms 0 and 1 realize that the choices of their capacity levels influence their price levels in the second stage. Provided Equations (9) and (10), respectively, firms 0 and 1 set their capacity levels with respect to their profits. Thus, by solving the first-order conditions of the profits of firms 0 and 1 in the first stage, we obtain

$$x_0(x_1) = \frac{4(3 + 4\theta + \theta)[x_1(\theta - 1) + a(1+\theta) - m(1+\theta)]}{28 + 52\theta + 37\theta^2 + 10\theta^3 + \theta^4}, \tag{11}$$

$$x_1(x_0) = \frac{2(3 + 4\theta + \theta^2)[-2x_0 + a(2+\theta) - m(2+\theta)]}{28 + 52\theta + 37\theta^2 + 10\theta^3 + \theta^4}, \tag{12}$$

yielding

$$x_0^p = \frac{4(a-m)(1+\theta)^2(6 + 17\theta + 8\theta^2 + \theta^3)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6},$$

$$x_1^p = \frac{2(a-m)(12 + 34\theta + 43\theta^2 + 29\theta^3 + 9\theta^4 + \theta^5)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6}.$$

Note that superscript p represents the subgame perfect Nash equilibrium market outcomes in the price-setting competition. Then, the equilibrium quantities of firms 0 and 1 are given as follows:

$$q_0^p = \frac{2(a-m)(8 + 7\theta + \theta^2)(2 + 7\theta + 6\theta^2 + \theta^3)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6},$$

$$q_1^p = \frac{(a-m)(8 + 7\theta + \theta^2)(4 + 6\theta + 5\theta^2 + \theta^3)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6}$$

Therefore, from easy calculations, we obtain the following equilibrium result on the differences between the quantities and capacity levels of firms 0 and 1:

$$q_0^p - x_0^p$$

$$= \frac{2(a-m)(1-\theta)(1+\theta)(2+\theta)(2 + 5\theta + \theta^2)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6},$$

$$q_1^p - x_1^p$$

$$= \frac{(a-m)(1-\theta)(2+\theta)(4 + 6\theta + 5\theta^2 + \theta^3)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6}.$$

Proposition 2 *In the price-setting competition, when the price-raising effect of firm* 1*'s product on the price level of firm* 0*'s product is sufficiently strong, i.e.,* $\theta \in (1,2)$*, both firms* 0 *and* 1 *choose over-capacity. In contrast, when such a price-raising effect of firm* 1*'s product is sufficiently weak, i.e.,* $\theta \in (0,1)$*, both firms* 0 *and* 1 *choose under-capacity.*

Similar to the quantity-setting competition with the price-raising effect of the product of firm 1, Proposotion 2 indicates that the differences between the quantities and capacity levels of firms 0 and 1 strictly depend on the strength of the price-raising effect of firm 1's product on the price level of firm 0's product. More concretely, it is shown that when the price-raising effect of firm 1's product is sufficiently strong, the differences between the quantities and capacity levels of firms 0 and 1 are negative, whereas when such an effect is sufficiently weak, the differences between their quantities and capacity levels are positive[11].

In the price-setting competition with the price-raising effect, when such an effect is sufficiently weak, *i.e.*, $\theta \in (0,1)$ from Equations (9) and (10), both firms 0 and 1 attempt to decrease the quantity of their respective opponent firm by decreasing their capacity in order to expand their own market share. Thus, when $\theta \in (0,1)$, both firms 0 and 1 choose under-capacity, which is the same result as that obtained in the standard price-setting competition with differentiated goods and without the

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[11]These results on the differences between the quantities and capacity levels of firms 0 and 1, which are obtained in the standard price-setting competition with differentiated goods and without the price-raising effect, are given in the appendix. More concretely, in both cases wherein firms 0 and 1 produce substitutable goods or complementary goods with each other, it is shown that they always choose under-capacity together.

price-raising effect[12].

In contrast, the intuition behind the result that both firms 0 and 1 choose over-capacity in the price-setting competition when $\theta \in (1,2)$ is given as follows: from Equation (9), firm 1 which has the price-raising effect attempts to increase firm 0's price level by setting its capacity level high in order to decrease the quantity of firm 0, implying that it chooses over-capacity. Furthermore, similar to the case wherein $\theta \in (0,1)$, from Equation (10), firm 0 tends to decrease its own capacity level in order to decrease firm 1's quantity by increasing the price level of firm 1 when $\theta \in (1,2)$ as well. However, taking Equations (11) and (12) into account, firm 0's capacity level tends not to become low when $\theta \in (1,2)$. Therefore, since the price level of firm 1 does not become so high because of the relatively high capacity level of firm 0, from Equation (7), the price of firm 0 becomes relatively high, Consequently, since the quantity of firm 0 becomes low, firm 0 also chooses over-capacity when $\theta \in (1,2)$.

For both the quantity-setting competition with the price-raising effect and the price-setting competition with the price-raising effect, we find that the choices of the quantity/price level and the capacity level for the firm with the price-raising effect influences not only the selection of the quantity/price level for its opponent firm in the market, but also the selection of the capacity level for its opponent firm. These choices change the differences between the quantities and capacity levels of the two firm.

4. Conclusions

This paper investigated the differences between the output and capacity levels in a private duopoly composed of two profit-maximizer firms in both quantity-setting competition and price-setting competition, particularly when one firm has the price-raising effect on the price level of the product of its opponent firm as the unilateral exter-

nality. In both the standard quantity-setting competition and price-setting competition with differentiated goods and without the price-raising effect, the differences between the quantity and capacity levels of the two firms do not depend on the relation between their products, that is, whether they are substitutable or complementary. More precisely, in the standard quantity-setting competition with differentiated goods, both firms 0 and 1 always choose over-capacity, whereas in the standard price-setting competition with differentiated goods, they always choose under-capacity. In contrast, in the private duopoly with the price-raising effect of the product of the one firm, the differences between the quantities and capacity levels of both firms strictly depend on the strength of such an effect in not only the quantity-setting competition but also the price-setting competition. In particular, when the price-raising effect is sufficiently strong, both firms choose under-capacity in the quantity-setting competition whereas they both choose over-capacity in the price-setting competition. In the model of this paper, in the quantity-setting and price-setting competitions when the price-raising effect of the one firm is sufficiently strong, the product of the firm with such an effect is a complement to the product of the firm without such an effect, while the product of the firm without such an effect is always a substitute for the product of the firm with such an effect. Thus, the change of the strategic relation between the quantities/price levels and capacity levels of both firms along with the change of the relation of the product of the firm that has the price-raising effect as the unilateral externality with the product of the firm without such an effect (*i.e.*, substitutable or complementary) strikingly influences the differences between their quantities and capacity levels even if their strategic variables (q or p) are fixed in the market. The above findings comprise the most important contribution of this paper.

Finally, we mention an issue to be addressed in the future. Throughout this paper, we explored the differences between the quantity and capacity level of each firm by adopting the private duopolistic model with the unilateral externality à la Choi and Lu [1]. However, we did not consider the impact of the separation between ownership and management, which was investigated in Choi and Lu [1], on the difference between the quantities and capacity levels of the firms[13]. Future research must deal with the above problems.

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[12]In the standard price-setting duopoly with differentiated goods, when the goods produced by both firms are substitutable, since each firm's capacity is negatively associated with the opponent firm's price level, each attempts to decrease the quantity of its opponent firm by increasing its opponent firm's price level by decreasing its own capacity level in order to increase its own market share, and hence, each firm's quantity is relatively higher than its capacity level. Thus, both firms choose under-capacity. On the other hand, when the relation between the products of the firms is complementary, each firm has an incentive to increase its opponent firm's price level by increasing its capacity level in order to increase its market share, and thus, such capacity-increasing behavior also increases its opponent firm's capacity since the capacity levels of both firms are strategic complements in the first stage. Although the capacity levels of both firms tend to become high, their quantities are higher than their capacity levels since their capacity levels are negatively associated with their own price levels. Thus, whether the relation of the goods produced by both firms is substitutable or complementary, in a standard price-setting duopoly without the price-raising effect, they choose under-capacity.

[13]Several types of strategic delegation, along with the separation between ownership and management, are presented as the objective functions of managers on the basis of the manager's bonus. For instances, Jansen *et al.* [16] introduced the weighted sum of the firm's profit and its market share as the objective function of its manager, and Miller and Pazgal [17] and Miller and Pazgal [18] considered the weighted sum of the firm's profit and (the sum of) the profit(s) of its opponent firm(s).

REFERENCES

[1] K. Choi and Y. Lu, "A Note on Endogenous Timing with Strategic Delegation: Unilateral Externality Case," *Bulletin of Economic Research*, Vol. 64, No. 2, 1983, pp. 253-264. doi:10.1111/j.1467-8586.2011.00392.x

[2] L. Cabral and W. R. Majure, "An Empirical Analysis of Bank Branching: Portugal 1989-1991," In: D. Neven and L.-H. Roller, Eds., *The Empirical Analysis of Industrial Organization*, CEPR, London, 1994.

[3] J. I. Bulow, J. D. Geanakoplos and P. D. Klemperer, "Holding Idle Capacity to Deter Entry," *Economic Journal*, Vol. 95, No. 377, 1985, pp. 178-182. doi:10.2307/2233477

[4] J. I. Bulow, J. D. Geanakoplos and P. D. Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, Vol. 93, No. 3, 1985, pp. 488-511. doi:10.1086/261312

[5] M. M. Tombak, "Strategic Asymmetry," *Journal of Economic Behavior & Organization*, Vol. 61, No. 3, 2006, pp. 339-350. doi:10.1016/j.jebo.2004.11.014

[6] C. Fershtman and K. Judd, "Equilibrium Incentives in Oligopoly," *American Economic Review*, Vol. 77, No. 5, 1987, pp. 927-940.

[7] S. D. Sklivas, "The Strategic Choice of Management Incentives," *RAND Journal of Economics*, Vol. 18, No. 3, 1987, pp. 452-458. doi:10.2307/2555609

[8] J. Vickers, "Delegation and the Theory of the Firm," *Economic Journal*, Vol. 95, No. 380a, 1985, pp. 138-147. doi:10.2307/2232877

[9] X. Vives, "Commitment, Flexibility, and Market Outcomes," *International Journal of Industrial Organization*, Vol. 4, No. 2, 1986, pp. 217-229. doi:10.1016/0167-7187(86)90032-9

[10] H. Ogawa, "Capacity Choice in the Mixed Duopoly with Product Differentiation," *Economics Bulletin*, Vol. 12, No. 8, 2006, pp. 1-6.

[11] J. C. Bárcena-Ruiz and M. B. Garzón, "Capacity Choice in a Mixed Duopoly under Price Competition," *Economics Bulletin*, Vol. 12, No. 26, 2007, pp. 1-7.

[12] Y. Tomaru, Y. Nakamura and M. Saito, "Capacity Choice in a Mixed Duopoly with Managerial Delegation," *Economics Bulletin*, Vol. 29, No. 3, 2009, pp. 1904-1924.

[13] Y. Nakamura and M. Saito, "Capacity Choice in a Price-Setting Mixed Duopoly: The Relative Performance Approach," *Modern Economy*, Vol. 4, No. 4, 2013, pp. 273-280. doi:10.4236/me.2013.44031

[14] Y. Nakamura and M. Saito, "Capacity Choice in a Mixed Duopoly: The Relative Performance Approach," *Theoretical Economics Letters*, Vol. 3, No. 2, 2013, pp. 124-133. doi:10.4236/tel.2013.32020

[15] Y. Nakamura, "Capacity Choice in a Price-Setting Mixed Duopoly with Network Effects," *Modern Economy*, Vol. 4, No. 5, 2013, pp. 418-425. doi:10.4236/me.2013.45044

[16] T. Jansen, A. V. Lier and A. V. Witteloostruijn, "A Note on Strategic Delegation: The Market Share Case," *International Journal of Industrial Organization*, Vol. 25, No. 3, 2007, pp. 531-539. doi:10.1016/j.ijindorg.2006.04.017

[17] N. Miller and A. Pazgal, "The Equivalence of Price and Quantity Competition with Delegation," *RAND Journal of Economics*, Vol. 32, No. 2, 2001, pp. 284-301. doi:10.2307/2696410

[18] N. Miller and A. Pazgal, "Relative Performance as a Strategic Commitment Mechanism," *Managerial and Decision Economics*, Vol. 23, No. 2, 2002, pp. 51-68. doi:10.1002/mde.1045

Appendix

Equilibrium Outcomes in a Standard Quantity-Setting Competition with Differentiated Goods

In this appendix, we formulate a standard private quantity-setting competition with differentiated goods wherein firms 0 and 1 choose not only their output levels but also their capacity levels. It is assumed that the inverse demand functions of firms 0 and 1 are $p_0 = a - q_0 - bq_1$ and $p_1 = a - bq_0 - q_1$, respectively. Note that a and b denote the demand parameter and the degree of product differentiation, respectively. Moreover, similar to the setting on the cost functions of firms 0 and 1 in the main body of this paper, their cost functions are represented as $C_i(q_i; x_i) = mq_i + (q_i - x_i)^2$, $(i = 0,1)$. Then, in the standard quantity-setting competition without the price-raising effect, we obtain the following equilibrium quantities and capacity levels of firms 0 and 1:

$$q_i^* = \frac{(16 - b^2)(a - m)}{32 + 16b - 4b^2 - b^3},$$

$$x_i^* = \frac{16(a - m)}{32 + 16b - 4b^2 - b^3},$$

$$i = 0,1.$$

Note that superscript * is used to denote the equilibrium market outcomes in the standard quantity-setting competition without the price-raising effect.

Then we obtain the following equilibrium differences between the quantities and capacity levels of firms 0 and 1:

$$q_i^* - x_i^* = -\frac{b^2(a - m)}{32 + 16b - 4b^2 - b^3} < 0,$$

$$\forall b \in (-1,0) \cup (0,1),$$

$$i = 0,1.$$

Thus, we find that in the standard quantity-setting competition with differentiated goods and without the price-raising effect, firms 0 and 1 both choose over-capacity irrespective of the relation between the products.

Equilibrium Outcomes in a Standard Price-Setting Competition with Differentiated Goods

Similar to the standard quantity-setting competition without the price-raising effect, in the price-setting competition without the price raising effect, we give the following equilibrium outcomes including the quantities and capacity levels of firms 0 and 1:

$$q_i^{**} = \frac{(16 - 9b^2 + b^4)(a - m)}{32 + 16b - 28b^2 - 9b^3 + 9b^4 + b^5 - b^6},$$

$$x_i^{**} = \frac{2(8 - 6b^2 + b^4)(a - m)}{32 + 16b - 28b^2 - 9b^3 + 9b^4 + b^5 - b^6},$$

$$i = 0,1.$$

Note that superscript ** is used to denote the equilibrium market outcomes in the standard quantity-setting competition without the price-raising effect.

Thus, we obtain the following equilibrium differences between the quantities and capacity levels of both firms:

$$q_i^{**} - x_i^{**} = \frac{b^2(3 - b^2)(a - m)}{32 + 16b - 28b^2 - 9b^3 + 9b^4 + b^5 - b^6} > 0,$$

$$\forall b \in (-1,0) \cup (0,1), i = 0,1.$$

Thus, in the standard price-setting competition without the price-raising effect, we find that both firms 0 and 1 choose under-capacity irrespective of the relation between the products.

Equilibrium Outcomes of Firms 0 and 1 in Quantity-Setting Competition

In this subsection, we give the equilibrium following market outcomes, including the equilibrium price levels and profits of both firms 0 and 1 in the quantity-setting competition with the price-raising effect as the unilateral externality:

$$p_0^q =$$
$$\frac{2m(195 + 73\theta - 11\theta^2 - \theta^3) + a(169 + 273\theta + 67\theta^2 + 3\theta^3)}{559 + 419\theta + 45\theta^2 + \theta^3}$$

$$p_1^q = \frac{a(13 + 3\theta)^2 + m(390 + 341\theta + 36\theta^2 + \theta^3)}{559 + 419\theta + 45\theta^2 + \theta^3},$$

$$\Pi_0^q = \frac{2(a - m)^2(13 + 18\theta + \theta^2)^2(97 + 30\theta + \theta^2)}{(559 + 419\theta + 45\theta^2 + \theta^3)^2},$$

$$\Pi_1^q = \frac{2(a - m)^2(13 + 3\theta)^2(97 + 30\theta + \theta^2)}{(559 + 419\theta + 45\theta^2 + \theta^3)^2}.$$

Equilibrium Outcomes of Firms 0 and 1 in Price-Setting Competition

In this subsection, we give the equilibrium following market outcomes, including the equilibrium price levels and profits of both firms 0 and 1 in the price-setting competition with the price-raising effect as the unilateral externality:

$$p_0^p = \frac{m\left(64 + 184\theta + 208\theta^2 + 77\theta^3 - 11\theta^4 - 9\theta^5 - \theta^6\right) + 2a\left(8 + 40\theta + 76\theta^2 + 77\theta^3 + 43\theta^4 + 11\theta^5 + \theta^6\right)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6},$$

$$p_1^p = \frac{a\left(4 + 6\theta + 5\theta^2 + \theta^3\right)^2 + m\left(64 + 216\theta + 284\theta^2 + 163\theta^3 + 38\theta^4 + 3\theta^5\right)}{80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6},$$

$$\Pi_0^p = \frac{4\left(a - m\right)^2 \left(1 + \theta\right)^3 \left(2 + 5\theta + \theta^2\right)^2 \left(28 + 52\theta + 37\theta^2 + 10\theta^3 + \theta^4\right)}{\left(80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6\right)^2},$$

$$\Pi_1^p = \frac{\left(a - m\right)^2 \left(4 + 6\theta + 5\theta^2 + \theta^3\right)^2 \left(28 + 80\theta + 89\theta^2 + 47\theta^3 + 11\theta^4 + \theta^5\right)}{\left(80 + 264\theta + 360\theta^2 + 231\theta^3 + 75\theta^4 + 13\theta^5 + \theta^6\right)^2}.$$

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