Journal of Modern Physics, 2013, 4, 26-41
http://dx.doi.org/10.4236/jmp.2013.47A1004 Published Online July 2013 (http://www.scirp.org/journal/jmp)

Application of the Non-Local Physics in the Theory of Gravitational Waves and Big Bang

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Received April 7, 2013; revised May 10, 2013; accepted June 15, 2013

ABSTRACT

The theory of gravitational waves in the frame of non-local quantum hydrodynamics (NLQH) is considered. From calculations follow that NLQH equations for "empty" space have the traveling wave solutions belonging in particular to the soliton class. The possible influence and reaction of the background microwave radiation is taken into account. These results lead to the principal correction of the inflation theory and serve as the explanation for the recent discovery of the universe's cosmic microwave background anomalies. The simple analytical particular cases and numerical calculations are delivered. Proposal for astronomers—to find in the center domain of the hefty cold spot the smallest hot spot as the origin of the initial burst—Big Bang.

Keywords: Foundations of the Theory of Transport Processes; The Theory of Solitons; Generalized Hydrodynamic Equations; Foundations of Quantum Mechanics; Traveling Wave Solutions; Big Bang

1. Introduction

More than ten years ago, the accelerated cosmological expansion was discovered in direct astronomical observations at distances of a few billion light years, almost at the edge of the observable Universe. This acceleration should be explained because mutual attraction of cosmic bodies is only capable of decelerating their scattering. It means that we reach the revolutionary situation not only in physics but also in the natural philosophy on the whole. As result, new idea was introduced in physics about existing of a force with the opposite sign, which is called universal antigravitation. Its physical source is called as dark energy that manifests itself only because of postulated property of providing antigravitation.

It was postulated that the source of antigravitation is "dark matter" which inferred to exist from gravitational effects on visible matter. However, from the other side dark matter is undetectable by emitted or scattered electromagnetic radiation. It means that new essences—dark matter, dark energy—were introduced in physics only with the aim to account for discrepancies between measurements of the mass of galaxies, clusters of galaxies and the entire universe made through dynamical and general relativistic means, measurements based on the mass of the visible "luminous" matter. It could be reasonable if we are speaking about small corrections to the system of knowledge achieved by mankind to the time we are living. But mentioned above discrepancies lead to affirmation, that dark matter constitutes 80% of the matter in the universe, while ordinary matter makes up only 20%. Practically we are in front of the new challenge since Newton's *Mathematical Principles of Natural Philosophy* was published.

Dark matter was postulated by Swiss astrophysicist Fritz Zwicky [1,2] of the California Institute of Technology in 1933. He applied the virial theorem to the Coma cluster of galaxies and obtained evidence of unseen mass. Zwicky estimated the cluster's total mass based on the motions of galaxies near its edge and compared that estimate to one based on the number of galaxies and total brightness of the cluster. He found that there was about 400 times more estimated mass than was visually observable. The gravity of the visible galaxies in the cluster would be far too small for such fast orbits, so something extra was required. This is known as the "missing mass problem". Based on these conclusions, Zwicky inferred that there must be some non-visible form of matter, which would provide enough of the mass, and gravity to hold the cluster together.

Observations have indicated the presence of dark matter in the universe, including the rotational speeds of galaxies, gravitational lensing of background objects by galaxy clusters such as the Bullet Cluster, and the temperature distribution of hot gas in galaxies and clusters of galaxies.

The work by Vera Rubin (see for example [3,4]) revealed distant galaxies rotating so fast that they should fly apart. Outer stars rotated at essentially the same rate as inner ones (~254 km/s). This is in marked contrast to the solar system where planets orbit the sun with velocities that decrease as their distance from the centre increases. By the early 1970s, flat rotation curves were routinely detected. It was not until the late 1970s, however, that the community was convinced of the need for dark matter halos around spiral galaxies. The mathematical modeling (based on Newtonian mechanics and local physics) of the rotation curves of spiral galaxies was realized for the various visible components of a galaxy (the bulge, thin disk, and thick disk). These models were unable to predict the flatness of the observed rotation curve beyond the stellar disk. The inescapable conclusion, assuming that Newton's law of gravity (and the local physics description) holds on cosmological scales, that the visible galaxy was embedded in a much larger dark matter (DM) halo, which contributes roughly 50% - 90% of the total mass of a galaxy. As result other models of gravitation were involved in consideration—from "improved" Newtonian laws (such as modified Newtonian dynamics and tensor-vector-scalar gravity [5]) to the Einstein's theory based on the cosmological constant [6]. Einstein introduced this term as a mechanism to obtain a stable solution of the gravitational field equation that would lead to a static universe.

Computer simulations with taking into account the hypothetical DM in the local hydrodynamic description include usual moment equations plus Poisson equation with different approximations for the density of DM (ρ_{DM}) containing several free parameters. Computer simulations of cold dark matter (CDM) predict that CDM particles ought to coalesce to peak densities in galactic cores. However, the observational evidence of star dynamics at inner galactic radii of many galaxies, including our own Milky Way, indicates that these galactic cores are entirely devoid of CDM. No valid mechanism has been demonstrated to account for how galactic cores are swept clean of CDM. This is known as the "cuspy halo problem". As result, the restricted area of CDM influence introduced in the theory. As we can see that the concept of DM leads to many additional problems.

I do not intend to review the different speculations based on the principles of local physics. I see another problem. It is the problem of Oversimplification—but not "trivial" simplification of the important problem. The situation is much more serious—total Oversimplification based on principles of local physics, and obvious crisis, we see in astrophysics, simply reflects the general shortenings of the local kinetic transport theory. In other words returning to the previous problems—is it possible using only Newtonian gravitation law and non-local statistical description to forecast the Hubble expansion and flat gravitational curve of a typical spiral galaxy? Both questions have the positive answers [7].

Another extremely important class of cosmological problems is connected with gravitational waves, universe inflation and anomalies in the distribution of the universe's cosmic microwave background (CMB). This class of problems leads also to antigravitation but in absolutely another sense. In other words what is the origin of the Big Bang and evolution in the burst regime when "usual" matter and "dark" matter do not exist on principle?

Let us investigate the possibilities which deliver the unified generalized quantum hydrodynamics [8-12] for investigation of these problems. I deliver here some main ideas and deductions of the generalized Boltzmann physical kinetics and non-local physics. For simplicity, the fundamental methodic aspects are considered from the qualitative standpoint of view avoiding excessively cumbersome formulas. A rigorous description can be found, for example, in the monographs [8-11], see also [12-17].

Let us consider the transport processes in open dissipative systems and ideas of following transformation of generalized hydrodynamic description in quantum hydrodynamics which can be applied to the individual particle.

The kinetic description is inevitably related to the system diagnostics. Such an element of diagnostics in the case of theoretical description in physical kinetics is the concept of the physically infinitely small volume (**PhSV**). The correlation between theoretical description and system diagnostics is well-known in physics. Suffice it to recall the part played by test charge in electrostatics or by test circuit in the physics of magnetic phenomena. The traditional definition of **PhSV** contains the statement to the effect that the **PhSV** contains a sufficient number of particles for introducing a statistical description; however, at the same time, the **PhSV** is much smaller than the volume *V* of the physical system under consideration; in a first approximation, this leads to the local approach in investigating of the transport processes. It is assumed in classical hydrodynamics that local thermodynamic equilibrium is first established within the **PhSV**, and only after that the transition occurs to global thermodynamic equilibrium if it is at all possible for the system under study.

Let us consider the hydrodynamic description in more detail from this point of view. Assume that we have two neighboring physically infinitely small volumes $\mathbf{PhSV_1}$ and $\mathbf{PhSV_2}$ in a non-equilibrium system. Even the point-like particles (starting after the last collision near the boundary between two mentioned volumes) can change the distribution functions in the neighboring volume. The adjusting of the particles dynamic characteristics for translational degrees of freedom takes several collisions in the simplest case. As result, we have in the definite sense "the Knudsen layer" between these volumes. This fact unavoidably leads to fluctuations in mass and hence in other hydrodynamic quantities. Existence of such "Knudsen layers" is not connected with the choice of space nets and fully defined by the reduced description for ensemble of particles of finite diameters in the conceptual frame of open physically small volumes, therefore—with the chosen method of measurement. This entire complex of effects defines non-local effects in space and time.

The physically infinitely small volume (**PhSV**) is an *open* thermodynamic system *for any division of macroscopic system by a set of PhSVs*. But the Boltzmann equation (BE) [8,18,19]

$$Df/Dt = J^B, \quad (1.1)$$

where J^B is the Boltzmann collision integral and D/Dt is a substantive derivative, fully ignores non-local effects and contains only the local collision integral J^B. The foregoing nonlocal effects are insignificant only in equilibrium systems, where the kinetic approach changes to methods of statistical mechanics.

This is what the difficulties of classical Boltzmann physical kinetics arise from. Also a weak point of the classical Boltzmann kinetic theory is the treatment of the dynamic properties of interacting particles. On the one hand, as follows from the so-called "physical" derivation of BE, Boltzmann particles are regarded as material points; on the other hand, the collision integral in the BE leads to the emergence of collision cross sections.

Notice that the application of the above principles also leads to the modification of the system of Maxwell equations. While the traditional formulation of this system does not involve the continuity equation, its derivation explicitly employs the equation

$$\frac{\partial \rho^a}{\partial t} + \frac{\partial}{\partial \boldsymbol{r}} \cdot \boldsymbol{j}^a = 0, \quad (1.2)$$

where ρ^a is the charge per unit volume, and $\boldsymbol{j}^a$ is the current density, both calculated without accounting for the fluctuations. As a result, the system of Maxwell equations is written in the standard notation, namely

$$\frac{\partial}{\partial \boldsymbol{r}} \cdot \boldsymbol{B} = 0, \frac{\partial}{\partial \boldsymbol{r}} \cdot \boldsymbol{D} = \rho^a, \frac{\partial}{\partial \boldsymbol{r}} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \frac{\partial}{\partial \boldsymbol{r}} \times \boldsymbol{H} = \boldsymbol{j}^a + \frac{\partial \boldsymbol{D}}{\partial t} \quad (1.3)$$

contains

$$\rho^a = \rho - \rho^{fl}, \ \boldsymbol{j}^a = \boldsymbol{j} - \boldsymbol{j}^{fl}. \quad (1.4)$$

The ρ^{fl}, $\boldsymbol{j}^{fl}$ fluctuations calculated using the generalized Boltzmann equation are given, for example, in Refs. [9,12,13]. The violation of Bell's inequalities [20] is found for local statistical theories, and the transition to non-local description is inevitable.

The rigorous approach to derivation of kinetic equation relative to one-particle DF f (KE_f) is based on employing the hierarchy of Bogoliubov equations. Generally speaking, the structure of KE_f is as follows:

$$\frac{Df}{Dt} = J^B + J^{nl}, \quad (1.5)$$

where J^{nl} is the non-local integral term. An approximation for the second collision integral is suggested by me in *generalized* Boltzmann physical kinetics,

$$J^{nl} = \frac{D}{Dt}\left(\tau \frac{Df}{Dt}\right). \quad (1.6)$$

Here, τ is non-local relaxation parameter, in the simplest case—the mean time *between* collisions of particles, which is related in a hydrodynamic approximation with dynamical viscosity μ and pressure p,

$$\tau p = \Pi \mu, \quad (1.7)$$

where the factor Π is defined by the model of collision of particles: for neutral hard-sphere gas, $\Pi = 0.8$ [21, 22]. All of the known methods of the kinetic equation derivation relative to one-particle DF lead to approximation (1.6), including the method of many scales, the method of correlation functions, and the iteration method.

In the general case, the parameter τ is the non-locality parameter; in quantum hydrodynamics, its magnitude is correlated with the "time-energy" uncertainty relation [9,10,14,15]. Now we can turn our attention to the quantum hydrodynamic description of individual particles. The abstract of the classical Madelung's paper [23] contains only one phrase: "It is shown that the Schrödinger equation for one-electron problems can be transformed into the form of hydrodynamic equations". The following conclusion of principal significance can be done from the previous consideration [14,15]:

1) Madelung's quantum hydrodynamics is equivalent to the Schrödinger equation (SE) and leads to the description of the quantum particle evolution in the form of Euler equation and continuity equation. Quantum Euler equation contains additional potential of non-local origin which can be written for example in the Bohm form. SE is consequence of the Liouville equation as result of the local approximation of non-local equations.

2) Generalized Boltzmann physical kinetics leads to the strict approximation of non-local effects in space and time and *in the local limit* leads to parameter τ, which on the quantum level corresponds to the uncertainty principle "time-energy".

3) Generalized hydrodynamic equations (GHE) lead to

SE as a deep particular case of the generalized Boltzmann physical kinetics and therefore of non-local hydrodynamics.

In principle GHE needn't in using of the "time-energy" uncertainty relation for estimation of the value of the non-locality parameter τ. Moreover the "time-energy" uncertainty relation does not lead to the exact relations and from position of non-local physics is only the simplest estimation of the non-local effects. Really, let us consider two neighboring physically infinitely small volumes **PhSV$_1$** and **PhSV$_2$** in a non-equilibrium system. Obviously the time τ should tend to diminish with increasing of the velocities u of particles invading in the nearest neighboring physically infinitely small volume (**PhSV$_1$** or **PhSV$_2$**):

$$\tau = H/u^n . \tag{1.8}$$

But the value τ cannot depend on the velocity direction and naturally to tie τ with the particle kinetic energy, then

$$\tau = H/\left(mu^2\right), \tag{1.9}$$

where H is a coefficient of proportionality, which reflects the state of physical system. In the simplest case H is equal to Plank constant $\hbar$ and relation (1.9) becomes compatible with the Heisenberg relation. Possible approximations of τ—parameter in details in the monographs [8-10] are considered. But some remarks of the principal significance should be done.

It is known that Ehrenfest adiabatic theorem is one of the most important and widely studied theorems in Schrödinger quantum mechanics. It states that if we have a slowly changing Hamiltonian that depends on time, and the system is prepared in one of the instantaneous eigenstates of the Hamiltonian then the state of the system at any time is given by an the instantaneous eigenfunction of the Hamiltonian up to multiplicative phase factors.

The adiabatic theory can be naturally incorporated in generalized quantum hydrodynamics based on local approximations of non-local terms. In the simplest case if ΔQ is the elementary heat quantity delivered for a system executing the transfer from one state (the corresponding time moment is t_{in}) to the next one (the time moment t_e) then

$$\Delta Q = \frac{1}{\tau} 2\delta\left(\bar{T}\tau\right), \tag{1.10}$$

where $\tau = t_e - t_{in}$ and $\bar{T}$ is the average kinetic energy. For adiabatic case Ehrenfest supposes that

$$2\bar{T}\tau = \Omega_1, \Omega_2, \cdots \tag{1.11}$$

where $\Omega_1, \Omega_2, \cdots$ are adiabatic invariants. Obviously for Plank's oscillator (compare with (1.9))

$$2\bar{T}\tau = nh . \tag{1.12}$$

Then the adiabatic theorem and consequences of this theory deliver the general quantization conditions for non-local quantum hydrodynamics.

2. Generalized Quantum Hydrodynamic Equations

Strict consideration leads to the following system of the generalized hydrodynamic equations (GHE) [9,10,24] written in the generalized Euler form: continuity equation for species α

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\rho_\alpha - \tau_\alpha\left[\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\rho_\alpha \boldsymbol{v}_0\right)\right]\right\} \\ +\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho_\alpha \boldsymbol{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0\right) + \vec{\vec{I}}\cdot\frac{\partial p_\alpha}{\partial \boldsymbol{r}}\right.\right. \\ \left.\left. -\rho_\alpha \boldsymbol{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0 \times \boldsymbol{B}\right]\right\} = R_\alpha,\end{gathered} \tag{2.1}$$

and continuity equation for mixture

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\rho - \sum_\alpha \tau_\alpha\left[\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\rho_\alpha \boldsymbol{v}_0\right)\right]\right\} \\ +\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho \boldsymbol{v}_0 - \sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0\right) + \vec{\vec{I}}\cdot\frac{\partial p_\alpha}{\partial \boldsymbol{r}}\right.\right. \\ \left.\left. -\rho_\alpha \boldsymbol{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0 \times \boldsymbol{B}\right]\right\} = 0.\end{gathered} \tag{2.2}$$

Momentum equation for species

$$\begin{gathered}\frac{\partial}{\partial t}\left\{\rho_\alpha \boldsymbol{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0 + \frac{\partial p_\alpha}{\partial \boldsymbol{r}} - \rho_\alpha \boldsymbol{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0 \times \boldsymbol{B}\right]\right\} - \boldsymbol{F}_\alpha^{(1)}\left[\rho_\alpha - \tau_\alpha\left(\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial \boldsymbol{r}}\left(\rho_\alpha \boldsymbol{v}_0\right)\right)\right] \\ -\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha \boldsymbol{v}_0 - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0 + \frac{\partial p_\alpha}{\partial \boldsymbol{r}} - \rho_\alpha \boldsymbol{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0 \times \boldsymbol{B}\right]\right\} \times \boldsymbol{B} \\ +\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0 + p_\alpha \vec{\vec{I}} - \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0 + p_\alpha \vec{\vec{I}}\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha\left(\boldsymbol{v}_0\boldsymbol{v}_0\right)\boldsymbol{v}_0 + 2\vec{\vec{I}}\left(\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(p_\alpha \boldsymbol{v}_0\right)\right) + \frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\vec{\vec{I}} p_\alpha \boldsymbol{v}_0\right) - \boldsymbol{F}_\alpha^{(1)}\rho_\alpha \boldsymbol{v}_0\right.\right. \\ \left.\left. -\rho_\alpha \boldsymbol{v}_0 \boldsymbol{F}_\alpha^{(1)} - \frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\boldsymbol{v}_0 \times \boldsymbol{B}\right]\boldsymbol{v}_0 - \frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0\left[\boldsymbol{v}_0 \times \boldsymbol{B}\right]\right]\right\} = \int m_\alpha \boldsymbol{v}_\alpha J_\alpha^{st,el} \mathrm{d}\boldsymbol{v}_\alpha + \int m_\alpha \boldsymbol{v}_\alpha J_\alpha^{st,inel} \mathrm{d}\boldsymbol{v}_\alpha .\end{gathered} \tag{2.3}$$

Generalized moment equation for mixture

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho \boldsymbol{v}_0-\sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p_\alpha}{\partial \boldsymbol{r}}-\rho_\alpha \boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}-\sum_\alpha \boldsymbol{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial \rho_\alpha}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\left(\rho_\alpha \boldsymbol{v}_0\right)\right)\right]\\
&-\sum_\alpha \frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha \boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p_\alpha}{\partial \boldsymbol{r}}-\rho_\alpha \boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0\times\boldsymbol{B}\right]\right\}\times\boldsymbol{B}\\
&+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho \boldsymbol{v}_0\boldsymbol{v}_0+p\vec{\mathrm{I}}-\sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+p_\alpha\vec{\mathrm{I}}\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha\left(\boldsymbol{v}_0\boldsymbol{v}_0\right)\boldsymbol{v}_0+2\vec{\mathrm{I}}\left(\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(p_\alpha \boldsymbol{v}_0\right)\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\vec{\mathrm{I}}p_\alpha \boldsymbol{v}_0\right)\right.\right.\\
&\left.\left.-\boldsymbol{F}_\alpha^{(1)}\rho_\alpha \boldsymbol{v}_0-\rho_\alpha \boldsymbol{v}_0\boldsymbol{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\boldsymbol{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha \boldsymbol{v}_0\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right]\right\}=0
\end{aligned}\tag{2.4}$$

Energy equation for component

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha \boldsymbol{v}_0+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0\right)-\boldsymbol{F}_\alpha^{(1)}\cdot\rho_\alpha \boldsymbol{v}_0\right]\right\}\\
&+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha \boldsymbol{v}_0+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha \boldsymbol{v}_0+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0\right)\right.\right.\\
&+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0\boldsymbol{v}_0+\frac{7}{2}p_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\frac{1}{2}p_\alpha v_0^2\vec{\mathrm{I}}+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\vec{\mathrm{I}}+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\vec{\mathrm{I}}\right)-\rho_\alpha \boldsymbol{F}_\alpha^{(1)}\cdot\boldsymbol{v}_0\boldsymbol{v}_0-p_\alpha \boldsymbol{F}_\alpha^{(1)}\cdot\vec{\mathrm{I}}\\
&\left.\left.-\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{F}_\alpha^{(1)}-\frac{3}{2}\boldsymbol{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha \boldsymbol{F}_\alpha^{(1)}\right]\right\}\\
&-\left\{\rho_\alpha \boldsymbol{F}_\alpha^{(1)}\cdot\boldsymbol{v}_0-\tau_\alpha\left[\boldsymbol{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial}{\partial \boldsymbol{r}}\cdot p_\alpha\vec{\mathrm{I}}-\rho_\alpha \boldsymbol{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right)\right]\right\}\\
&=\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,el}\mathrm{d}\boldsymbol{v}_\alpha+\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,inel}\mathrm{d}\boldsymbol{v}_\alpha.
\end{aligned}\tag{2.5}$$

and after summation the generalized energy equation for mixture

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha \varepsilon_\alpha n_\alpha-\sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha \boldsymbol{v}_0+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0\right)-\boldsymbol{F}_\alpha^{(1)}\cdot\rho_\alpha \boldsymbol{v}_0\right]\right\}\\
&+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\boldsymbol{v}_0+\frac{5}{2}p \boldsymbol{v}_0+\boldsymbol{v}_0\sum_\alpha \varepsilon_\alpha n_\alpha-\sum_\alpha \tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0+\frac{5}{2}p_\alpha \boldsymbol{v}_0+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0\right)\right.\right.\\
&+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{v}_0\boldsymbol{v}_0+\frac{7}{2}p_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\frac{1}{2}p_\alpha v_0^2\vec{\mathrm{I}}+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\vec{\mathrm{I}}+\varepsilon_\alpha n_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\vec{\mathrm{I}}\right)-\rho_\alpha \boldsymbol{F}_\alpha^{(1)}\cdot\boldsymbol{v}_0\boldsymbol{v}_0-p_\alpha \boldsymbol{F}_\alpha^{(1)}\cdot\vec{\mathrm{I}}\\
&\left.\left.-\frac{1}{2}\rho_\alpha v_0^2\boldsymbol{F}_\alpha^{(1)}-\frac{3}{2}\boldsymbol{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]-\varepsilon_\alpha n_\alpha \boldsymbol{F}_\alpha^{(1)}\right]\right\}-\boldsymbol{v}_0\cdot\sum_\alpha \rho_\alpha \boldsymbol{F}_\alpha^{(1)}\\
&+\sum_\alpha \tau_\alpha \boldsymbol{F}_\alpha^{(1)}\cdot\left[\frac{\partial}{\partial t}\left(\rho_\alpha \boldsymbol{v}_0\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho_\alpha \boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial}{\partial \boldsymbol{r}}\cdot p_\alpha\vec{\mathrm{I}}-\rho_\alpha \boldsymbol{F}_\alpha^{(1)}-q_\alpha n_\alpha\left[\boldsymbol{v}_0\times\boldsymbol{B}\right]\right]=0.
\end{aligned}\tag{2.6}$$

Here $\boldsymbol{F}_\alpha^{(1)}$ are the forces of the non-magnetic origin, $\boldsymbol{B}$ –magnetic induction, $\vec{\mathrm{I}}$ –unit tensor, q_α –charge of the α –component particle, p_α –static pressure for α –component, ε_α –internal energy for the particles of α –component, $\boldsymbol{v}_0$ –hydrodynamic velocity for mixture. For calculations in the self-consistent electro-magnetic field the system of non-local Maxwell equations should be added (see (1.3)).

It is well known that basic Schrödinger Equation (SE) of quantum mechanics firstly was introduced as a quantum mechanical postulate. The obvious next step should be done and was realized by E. Madelung in 1927—the derivation of special hydrodynamic form of SE after introduction wave function Ψ as

$$\Psi(x,y,z,t)=\alpha(x,y,z,t)\mathrm{e}^{\mathrm{i}\beta(x,y,z,t)}. \tag{2.7}$$

Using (2.7) and separating the real and imagine parts of SE one obtains

$$\frac{\partial\alpha^2}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{\alpha^2\hbar}{m}\frac{\partial\beta}{\partial\mathbf{r}}\right)=0, \tag{2.8}$$

and Equation (2.8) immediately transforms in continuity equation if the identifications in the Madelung's notations for density ρ and velocity $\boldsymbol{v}$

$$\rho=\alpha^2=\Psi\Psi*, \tag{2.9}$$

$$\boldsymbol{v}=\frac{\partial}{\partial\mathbf{r}}(\beta\hbar/m) \tag{2.10}$$

introduce in Equation (2.8). Identification for velocity (2.10) is obvious because for 1D flow with constant values p, E_k

$$v=\frac{\partial}{\partial x}(\beta\hbar/m)=\frac{\hbar}{m}\frac{\partial}{\partial x}\left[-\frac{1}{\hbar}(E_k t-px)\right]=\frac{1}{m}\frac{\partial}{\partial x}(px)=v_\phi, \tag{2.11}$$

where v_ϕ is phase velocity. The existence of the condition (2.10) means that the corresponding flow has potential

$$\Phi=\beta\hbar/m. \tag{2.12}$$

As result two effective quantum hydrodynamic equations take place:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\cdot(\rho\boldsymbol{v})=0, \tag{2.13}$$

$$\frac{\partial\mathbf{v}}{\partial t}+\frac{1}{2}\frac{\partial}{\partial\mathbf{r}}v^2=-\frac{1}{m}\frac{\partial}{\partial\mathbf{r}}\left(U-\frac{\hbar^2}{2m}\frac{\Delta\alpha}{\alpha}\right). \tag{2.14}$$

But

$$\frac{\Delta\alpha}{\alpha}=\frac{\Delta\alpha^2}{2\alpha^2}-\frac{1}{\alpha^2}\left(\frac{\partial\alpha}{\partial\mathbf{r}}\right)^2, \tag{2.15}$$

and the relation (2.15) transforms (2.14) in particular case of the Euler motion equation

$$\frac{\partial\mathbf{v}}{\partial t}+\left(\boldsymbol{v}\cdot\frac{\partial}{\partial\mathbf{r}}\right)\boldsymbol{v}=-\frac{1}{m}\frac{\partial}{\partial\mathbf{r}}U^*, \tag{2.16}$$

where introduced the efficient potential

$$U^*=U-\frac{\hbar^2}{4m\rho}\left[\Delta\rho-\frac{1}{2\rho}\left(\frac{\partial\rho}{\partial\mathbf{r}}\right)^2\right]. \tag{2.17}$$

Additive quantum part of potential can be written in the so called Bohm form

$$\frac{\hbar^2}{2m\sqrt{\rho}}\Delta\sqrt{\rho}=\frac{\hbar^2}{4m\rho}\left[\Delta\rho-\frac{1}{2\rho}\left(\frac{\partial\rho}{\partial\mathbf{r}}\right)^2\right]. \tag{2.18}$$

Then

$$U^*=U+U_{qu}$$

$$=U-\frac{\hbar^2}{2m\sqrt{\rho}}\Delta\sqrt{\rho}=U-\frac{\hbar^2}{4m\rho}\left[\Delta\rho-\frac{1}{2\rho}\left(\frac{\partial\rho}{\partial\mathbf{r}}\right)^2\right]. \tag{2.19}$$

Some remarks:

1) SE transforms in hydrodynamic form without additional assumptions. But numerical methods of hydrodynamics are very good developed. As result at the end of seventieth of the last century we realized the systematic calculations of quantum problems using quantum hydrodynamics (see for example [8]);

2) SE reduces to the system of continuity equation and the particular case of the Euler equation with the additional potential proportional to $\hbar^2$. The physical sense and the origin of the Bohm potential are established later in [14,15];

3) SE (obtained in the frame of the theory of classical complex variables) cannot contain the energy equation on principle. As result in many cases the palliative approach is used when for solution of dissipative quantum problems the classical hydrodynamics is used with the insertion of the additional Bohm potential in the system of hydrodynamic equations;

4) The system of the generalized quantum hydrodynamic equations contains energy equation written for unknown dependent value which can be specified as quantum pressure p_α of non-local origin;

5) Generalized hydrodynamic equations (GHE) (2.1)-(2.6) can be written in the spherical coordinate system [11,25].

3. Propagation of Plane Gravitational Waves in Vacuum with Cosmic Microwave Background (CMB)

Newtonian gravity propagates with the infinite speed. This conclusion is connected only with the description in the frame of local physics. Usual affirmation—general relativity (GR) reduces to Newtonian gravity in the weak-field, low-velocity limit. In literature you can find criticism of this affirmation because the conservation of angular momentum is implicit in the assumptions on which GR rests. Finite propagation speeds and conservation of angular momentum are incompatible in GR. Therefore, GR was forced to claim that gravity is not a force that propagates in any classical sense, and that aberration does not apply. But here I do not intend to join to this widely discussed topic using only unified non-local model.

Let us apply generalized quantum hydrodynamic Equations (2.1)-(2.6) for investigation of the gravitational wave propagation in vacuum using non-stationary 1D

Cartesian description.

Call attention to the fact that Equations (2.1)-(2.6) contain two forces of gravitational origin, $\boldsymbol{F}$—the force acting on the unit volume of the space and $\boldsymbol{g}$—the force acting on the unit mass. As result we have from Equations (2.1)-(2.6):

(continuity equation)

$$\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\cdot(\rho \boldsymbol{v}_0)\right]\right\}+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho \boldsymbol{v}_0-\tau\left[\frac{\partial}{\partial \boldsymbol{r}}(\rho \boldsymbol{v}_0)+\frac{\partial}{\partial \boldsymbol{r}}\cdot(\rho \boldsymbol{v}_0\boldsymbol{v}_0)+\ddot{\mathrm{I}}\cdot\frac{\partial p}{\partial \boldsymbol{r}}-\boldsymbol{F}\right]\right\}=0, \tag{3.1}$$

(continuity equation, 1D case)

$$\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_0)\right]\right\}+\frac{\partial}{\partial x}\left\{\rho v_0-\tau\left[\frac{\partial}{\partial t}(\rho v_0)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial x}-F\right]\right\}=0, \tag{3.2}$$

(momentum equation)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho \boldsymbol{v}_0-\tau\left[\frac{\partial}{\partial t}(\rho \boldsymbol{v}_0)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho \boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial p}{\partial \boldsymbol{r}}-\boldsymbol{F}\right]\right\}\\&-\boldsymbol{g}\left[\rho-\tau\left(\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}(\rho \boldsymbol{v}_0)\right)\right]\\&+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\rho \boldsymbol{v}_0\boldsymbol{v}_0+p\ddot{\mathrm{I}}-\tau\left[\frac{\partial}{\partial t}\left(\rho \boldsymbol{v}_0\boldsymbol{v}_0+p\ddot{\mathrm{I}}\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho(\boldsymbol{v}_0\boldsymbol{v}_0)\boldsymbol{v}_0\right.\right.,\\&\left.\left.+2\ddot{\mathrm{I}}\left(\frac{\partial}{\partial \boldsymbol{r}}\cdot(p\boldsymbol{v}_0)\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\ddot{\mathrm{I}}p\boldsymbol{v}_0\right)-\boldsymbol{F}\boldsymbol{v}_0-\boldsymbol{v}_0\boldsymbol{F}\right]\right\}=0\end{aligned} \tag{3.3}$$

(momentum equation, 1D case)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho v_0-\tau\left[\frac{\partial}{\partial t}(\rho v_0)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial t}-F\right]\right\}\\&-g\left[\rho-\tau\left(\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_0)\right)\right]\\&+\frac{\partial}{\partial x}\left\{\rho v_0^2+p-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^2+p\right)+\frac{\partial}{\partial x}\left(\rho v_0^3+3pv_0\right)\right.\right.\\&\left.\left.-2Fv_0\right]\right\}=0,\end{aligned} \tag{3.4}$$

(energy equation) (please see Equation (3.5) below)

(energy equation, 1D case) (please see Equation (3.6) below)

Nonlinear evolution Equations (3.1)-(3.6) contain forces $\boldsymbol{F}$, $\boldsymbol{g}$ acting on space and masses including cross-term (see for example the last line in Equation (3.6)). The relation $\boldsymbol{F}=\rho\boldsymbol{g}$ comes into being only after the mass appearance as a result of the Big Bang.

The cosmic microwave background (CMB) radiation is an emission of black body thermal energy coming from all parts of the sky. CMB saves the character traces of the initial burst evolution. If the quantum density ρ tends to zero the first term in the third line of the energy Equation (3.6) can be used for estimation of the initial fluctuation and for the investigation of the influence of this initial fluctuation on the following evolution.

$$5\frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho}F-\frac{\partial}{\partial x}\frac{p^2}{\rho}\right)\right\}\approx \pm L v_0\frac{\partial F}{\partial x}, \tag{3.7}$$

where L is the character length parameter reflecting the fluctuation influence on the initial physical system. Then

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p-\tau\left[\frac{\partial}{\partial t}\left(\frac{\rho v_0^2}{2}+\frac{3}{2}p\right)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho v_0^2\boldsymbol{v}_0+\frac{5}{2}p\boldsymbol{v}_0\right)-\boldsymbol{F}\cdot\boldsymbol{v}_0\right]\right\}+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\boldsymbol{v}_0+\frac{5}{2}p\boldsymbol{v}_0-\tau\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_0^2\boldsymbol{v}_0+\frac{5}{2}p\boldsymbol{v}_0\right)\right.\right.\\&\left.\left.+\frac{\partial}{\partial \boldsymbol{r}}\cdot\left(\frac{1}{2}\rho v_0^2\boldsymbol{v}_0\boldsymbol{v}_0+\frac{7}{2}p\boldsymbol{v}_0\boldsymbol{v}_0+\frac{1}{2}pv_0^2\ddot{\mathrm{I}}+\frac{5}{2}\frac{p^2}{\rho}\ddot{\mathrm{I}}\right)-\boldsymbol{F}\cdot\boldsymbol{v}_0\boldsymbol{v}_0-p\boldsymbol{g}\cdot\ddot{\mathrm{I}}-\frac{1}{2}v_0^2\boldsymbol{F}-\frac{3}{2}\boldsymbol{g}p\right]\right\}\\&-\left\{\boldsymbol{F}\cdot\boldsymbol{v}_0-\tau\left[\boldsymbol{g}\cdot\left(\frac{\partial}{\partial t}(\rho \boldsymbol{v}_0)+\frac{\partial}{\partial \boldsymbol{r}}\cdot\rho \boldsymbol{v}_0\boldsymbol{v}_0+\frac{\partial}{\partial \boldsymbol{r}}\cdot p\ddot{\mathrm{I}}-\boldsymbol{F}\right)\right]\right\}=0,\end{aligned} \tag{3.5}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho v_0^2+3p-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^2+3p\right)+\frac{\partial}{\partial x}\left(\rho v_0^3+5pv_0\right)-2Fv_0\right]\right\}\\&+\frac{\partial}{\partial x}\left\{\rho v_0^3+5pv_0-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^3+5pv_0\right)+\frac{\partial}{\partial x}\left(\rho v_0^4+8pv_0^2\right)-2Fv_0^2-v_0^2F\right]\right\}\\&+5\frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho}F-\frac{\partial}{\partial x}\frac{p^2}{\rho}\right)\right\}-2\left\{Fv_0-\tau g\left(\frac{\partial}{\partial t}(\rho v_0)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial x}-F\right)\right\}=0,\end{aligned} \tag{3.6}$$

$$\frac{\partial}{\partial t}\left\{\rho v_0^2+3p-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^2+3p\right)+\frac{\partial}{\partial x}\left(\rho v_0^3+5pv_0\right)\right.\right.$$
$$\left.\left.-2Fv_0\right]\right\}$$
$$+\frac{\partial}{\partial x}\left\{\rho v_0^3+5pv_0-\tau\left[\frac{\partial}{\partial t}\left(\rho v_0^3+5pv_0\right)+\frac{\partial}{\partial x}\left(\rho v_0^4+8pv_0^2\right)\right.\right.$$
$$\left.\left.-3Fv_0^2\right]\right\}$$
$$\pm Lv_0\frac{\partial F}{\partial x}-2\left\{Fv_0-\tau g\left(\frac{\partial}{\partial t}\left(\rho v_0\right)+\frac{\partial}{\partial x}\left(\rho v_0^2\right)+\frac{\partial p}{\partial x}-F\right)\right\}$$
$$=0. \tag{3.8}$$

The system of Equations (3.2), (3.4) and (3.8) can be transformed as follows (u—velocity in the x—direction):
(continuity equation, 1D case)

$$\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}=0, \tag{3.9}$$

(momentum equation, 1D case)

$$\frac{\partial p}{\partial x}-\frac{\partial}{\partial t}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}-2\tau\left(\frac{\partial p}{\partial x}-F\right)\frac{\partial u}{\partial x}$$
$$-\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+3p\frac{\partial u}{\partial x}\right]\right\}=0, \tag{3.10}$$

(energy equation, 1D case)

$$3\frac{\partial p}{\partial t}+3u\frac{\partial p}{\partial x}+5p\frac{\partial u}{\partial x}+2u\left(\frac{\partial p}{\partial x}-F\right)$$
$$-\frac{\partial}{\partial t}\left\{\tau\left[3\frac{\partial p}{\partial t}+3u\frac{\partial p}{\partial x}+5p\frac{\partial u}{\partial x}+2u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\}$$
$$-\frac{\partial}{\partial x}\left\{\tau\left[5\frac{\partial}{\partial t}(pu)+5u\frac{\partial}{\partial x}(pu)+11up\frac{\partial u}{\partial x}\right]\right\}$$
$$-6\tau\left(\frac{\partial p}{\partial x}-F\right)u\frac{\partial u}{\partial x}\pm Lu\frac{\partial F}{\partial x}=0, \tag{3.11}$$

Non-local equations are closed system of three differential equations with three dependent variables. In this case *no needs* to use the additional Poisson equation leading to Newton gravitational description. If non-locality parameter τ is equal to zero the mentioned system becomes unclosed.

Let us introduce the length scale ξ_0, quantum pressure scale p_0, the force scale F_0, the velocity scale u_0 and approximation for non-local parameter

$$\tau=\frac{Hp_0}{F_0|u|}. \tag{3.12}$$

The length scale is taken as $\xi_0=p_0/F_0$, then H is dimensionless parameter. The principles of the τ—approximation are discussed in [7-10], here I remark only that the approximation (3.12) is compatible with the Heisenberg relation (see also (1.8), (1.9)).

Let us introduce the coordinate system moving along the positive direction of x-axis in 1D space with velocity $C=u_0$ equal to phase velocity of considering object

$$\xi=x-Ct. \tag{3.13}$$

Taking into account the De Broglie relation we should wait that the group velocity u_g is equal $2u_0$. In moving coordinate system all dependent hydrodynamic values are function of (ξ,t). We investigate the possibility of the object formation of the soliton type. For this solution there is no explicit dependence on time for coordinate system moving with the phase velocity u_0. Write down the system of Equations (3.9)-(3.11) in the moving coordinate system:
(continuity equation, 1D case)

$$\frac{\partial}{\partial\xi}\left\{\tau\left[\frac{\partial p}{\partial\xi}-F\right]\right\}=0, \tag{3.14}$$

(momentum equation, 1D case)

$$\frac{\partial p}{\partial\xi}-2\frac{\partial u}{\partial\xi}\tau\left[\frac{\partial p}{\partial\xi}-F\right]-3\frac{\partial}{\partial\xi}\left\{\tau p\frac{\partial u}{\partial\xi}\right\}=0, \tag{3.15}$$

(energy equation, 1D case)

$$2u\left(\frac{\partial p}{\partial\xi}-F\right)+5p\frac{\partial u}{\partial\xi}-4\tau\left(\frac{\partial p}{\partial\xi}-F\right)u\frac{\partial u}{\partial\xi}$$
$$-6u\frac{\partial}{\partial\xi}\left\{\tau p\frac{\partial u}{\partial\xi}\right\}-11\tau p\left(\frac{\partial u}{\partial\xi}\right)^2\pm Lu\frac{\partial F}{\partial\xi}=0, \tag{3.16}$$

Let us write down these equations in the dimensionless form, where dimensionless symbols are marked by tildes, using the introduced scales and approximation (3.12) for the non-local parameter. The mentioned equations take the form

$$\frac{\partial}{\partial\tilde\xi}\left\{\frac{1}{\sqrt{\tilde u^2}}\left[\frac{\partial\tilde p}{\partial\tilde\xi}-\tilde F\right]\right\}=0, \tag{3.17}$$

$$\frac{\partial\tilde p}{\partial\tilde\xi}-2\frac{\partial\tilde u}{\partial\tilde\xi}\frac{H}{\sqrt{\tilde u^2}}\left[\frac{\partial\tilde p}{\partial\tilde\xi}-\tilde F\right]-3\frac{\partial}{\partial\tilde\xi}\left\{\frac{H}{\sqrt{\tilde u^2}}\tilde p\frac{\partial\tilde u}{\partial\tilde\xi}\right\}=0, \tag{3.18}$$

$$2\tilde u\left(\frac{\partial\tilde p}{\partial\tilde\xi}-\tilde F\right)+5\tilde p\frac{\partial\tilde u}{\partial\tilde\xi}-4\frac{H}{\sqrt{\tilde u^2}}\tilde u\left(\frac{\partial\tilde p}{\partial\tilde\xi}-\tilde F\right)\frac{\partial\tilde u}{\partial\tilde\xi}$$
$$-6\tilde u\frac{\partial}{\partial\tilde\xi}\left\{\frac{H}{\sqrt{\tilde u^2}}\tilde p\frac{\partial\tilde u}{\partial\tilde\xi}\right\}-11\frac{H}{\sqrt{\tilde u^2}}\tilde p\left(\frac{\partial\tilde u}{\partial\tilde\xi}\right)^2\pm\frac{L}{\xi_0}\tilde u\frac{\partial\tilde F}{\partial\tilde\xi}=0, \tag{3.19}$$

4. Results of Mathematical Modeling

Now we are ready to display the results of the mathematical modeling realized with the help of Maple (the versions Maple 9 or more can be used).

First of all from Equations (3.17)-(3.19) it is possible to make analytical estimates using the condition

$$\frac{\partial \tilde{p}}{\partial \tilde{\xi}} - \tilde{F} = 0, \quad (4.1)$$

In this case Equation (3.17) is satisfied identically and Equations (3.18), (3.19) can be written as follows

$$\frac{\partial \tilde{p}}{\partial \tilde{\xi}} - 3\frac{\partial}{\partial \tilde{\xi}}\left\{\frac{H}{\sqrt{\tilde{u}^2}}\tilde{p}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right\} = 0, \quad (4.2)$$

$$5\tilde{p}\frac{\partial \tilde{u}}{\partial \tilde{\xi}} - 6\tilde{u}\frac{\partial}{\partial \tilde{\xi}}\left\{\frac{H}{\sqrt{\tilde{u}^2}}\tilde{p}\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right\} - 11\frac{H}{\sqrt{\tilde{u}^2}}\tilde{p}\left(\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right)^2 \pm \frac{L}{\xi_0}\tilde{u}\frac{\partial \tilde{F}}{\partial \tilde{\xi}} = 0, \quad (4.3)$$

Multiplying Equation (4.2) by $(-2u)$ and adding to (4.3) one obtains

$$5\tilde{p}\frac{\partial \tilde{u}}{\partial \tilde{\xi}} - 2\tilde{u}\frac{\partial \tilde{p}}{\partial \tilde{\xi}} - 11\frac{H}{\sqrt{\tilde{u}^2}}\tilde{p}\left(\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right)^2 \pm \frac{L}{\xi_0}\tilde{u}\frac{\partial \tilde{F}}{\partial \tilde{\xi}} = 0, \quad (4.4)$$

or

$$5\frac{\partial \tilde{u}}{\partial \tilde{\xi}} - 2\tilde{u}\frac{\partial \ln \tilde{p}}{\partial \tilde{\xi}} - 11\frac{H}{\sqrt{\tilde{u}^2}}\left(\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right)^2 \pm \frac{L}{\xi_0}\frac{\tilde{u}}{\tilde{p}}\frac{\partial \tilde{F}}{\partial \tilde{\xi}} = 0, \quad (4.5)$$

Omitting the derivative of the logarithmic function and the last term in Equation (4.5) one obtains the equation

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{\xi}} = 2.2H\left(\frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right)^2, \quad (4.6)$$

which non-trivial solution is

$$\tilde{u} = \tilde{u}\left(\tilde{x} = 0, \tilde{t} = 0\right)\exp\left(\frac{\tilde{x} - \tilde{u}\tilde{t}}{2.2H}\right) \quad (4.7)$$

for waves propagating in the positive direction of x axis. It can be shown that for $\tilde{\xi} = \tilde{x} + \tilde{u}\tilde{t}$ exists the solution

$$\tilde{u} = \tilde{u}\left(\tilde{x} = 0, \tilde{t} = 0\right)\exp\left(\frac{\tilde{x} + \tilde{u}\tilde{t}}{2.2H}\right). \quad (4.8)$$

It means that qualitative consideration leads to traveling waves with exponential evolution and no surprise that the solution of full system (3.17)-(3.19) defines solitons.

I emphasize that:

1) Relations (4.7), (4.8) reflecting the exponential law of the perturbation evolution, are the particular case of the generalized *H*—theorem proved by me (see [9,26];

2) If $\tilde{u} = \tilde{u}\left(\tilde{x} = 0, \tilde{t} = 0\right) = 0$, then (this follows from Equations (4.1) and (4.2)) $\tilde{p} = \text{const}$, $\tilde{F} = 0$;

3) The physical system is at rest until the appearance of external perturbations.

The system of Equations (3.17)-(3.19) has the great possibilities of mathematical modeling as result of changing of two parameters H and $\tilde{L} = L/\xi_0$ and six Cauchy conditions describing the character features of initial perturbations which lead to the soliton formation. Maple program contains Maple's notations—for example the expression $D(u)(0) = 0$ means in the usual notations $\left(\partial\tilde{u}/\partial\tilde{\xi}\right)(0) = 0$, independent variable t responds to $\tilde{\xi}$.

We begin with investigation of the problem of principle significance—is it possible after a perturbation (defined by Cauchy conditions) to obtain the object of the soliton's kind as result of the self-organization? With this aim let us consider the initial perturbations:

$$u(0) = 1,\ p(0) = 1,\ f(0) = 1,\ D(u)(0) = 0,$$
$$D(p)(0) = 0, D(f)(0) = 0.$$

The following Maple notations on figures are used: *u*—velocity $\tilde{u}$, *p*—pressure $\tilde{p}$, and *f*—the self consistent force $\tilde{F}$. Explanations placed under all following figures. In the soliton regime the solution exists only in the restricted domain of the 1D space and the obtained object in the moving coordinate system $\left(\tilde{\xi} = \tilde{x} - \tilde{t}\right)$ has the constant velocity $\tilde{u} = 1$ for all parts of the object. In this case the domain of the solution existence defines the character soliton size. The following numerical results reflect two principally different regime of the physical system evolution. The distinctive features of evolution are defined by the sign plus or minus in front of the term D in the energy equation,

$$D = \pm\frac{L}{\xi_0}\tilde{u}\frac{\partial \tilde{F}}{\partial \tilde{\xi}} = \pm\tilde{L}\tilde{u}\frac{\partial \tilde{F}}{\partial \tilde{\xi}}. \quad (4.9)$$

The term *D* reflects the interaction of physical system with the surrounding media and defines the value of perturbation. The Maple inscription L on figures corresponds to the dimensionless value $\tilde{L}$ including the sign in front of $\tilde{L}$.

Therefore, Regime I is characterized by the negative sign in front of $\tilde{L}$. The mentioned calculations are displayed in **Figures 1-12**.

One can see that the first regime is characterized by the force directed basically (in front of the wave, **Figures 2**, **5**, **8** and **11**) against the direction of the wave propagation. This fact leads to the effect of attraction. Domains of the solution existence in regime I:

1) For $H = 1$, $\tilde{L} = -1$. $\rightarrow$ (−0.7298; 1.1618);

2) For $H = 1$, $\tilde{L} = -0.1$. $\rightarrow$ (−0.1571; 0.6347);

3) For $H = 1$, $\tilde{L} = -0.01$. $\rightarrow$ (−0.0265; 0.5327);

4) For $H = 1$, $\tilde{L} = -0.001$. $\rightarrow$ (−0.003801; 0.5186).

Regime II is characterized by the positive sign in front of ***L***. The mentioned calculations are displayed in **Figures 13-22**.

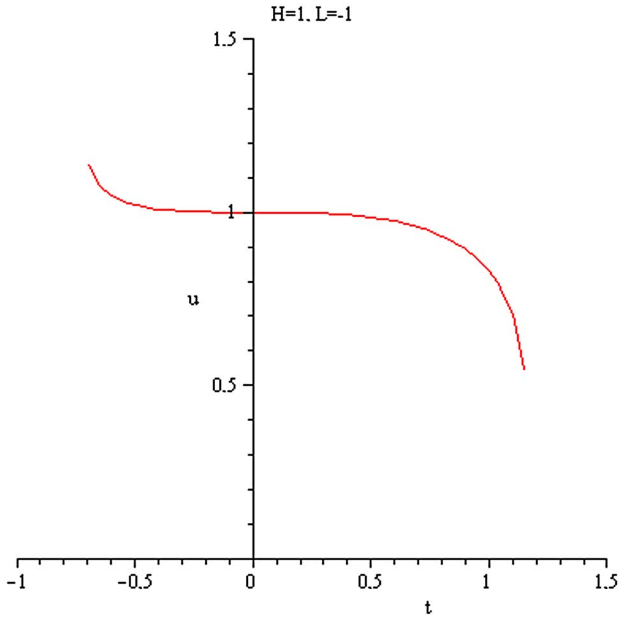

Figure 1. ***u*—velocity** $\tilde{u}$, $H = 1$, $\tilde{L} = -1$.

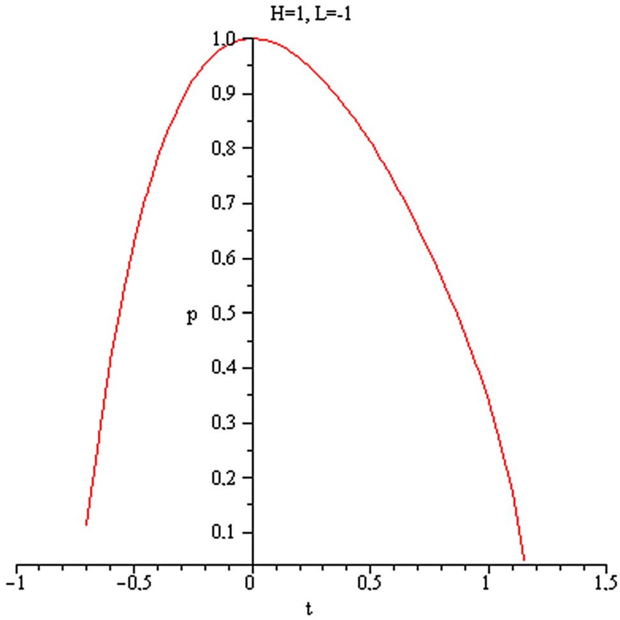

Figure 3. ***p*—pressure** $\tilde{p}$, $H = 1$, $\tilde{L} = -1$.

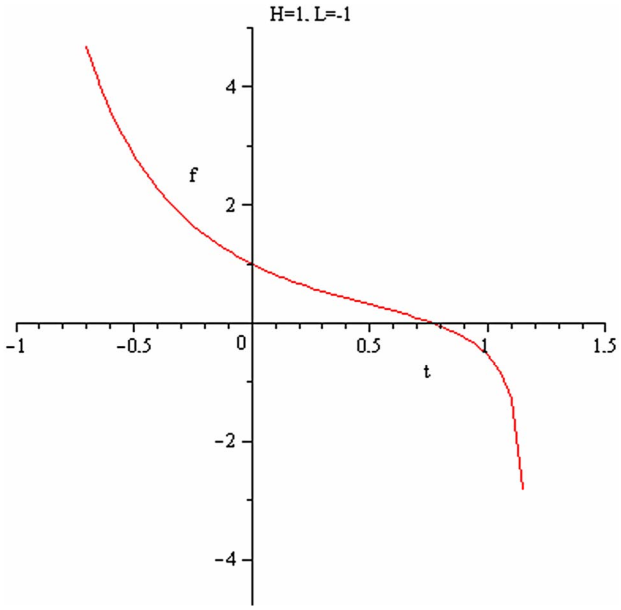

Figure 2. ***f*—the self-consistent force** $\tilde{F}$, $H = 1$, $\tilde{L} = -1$.

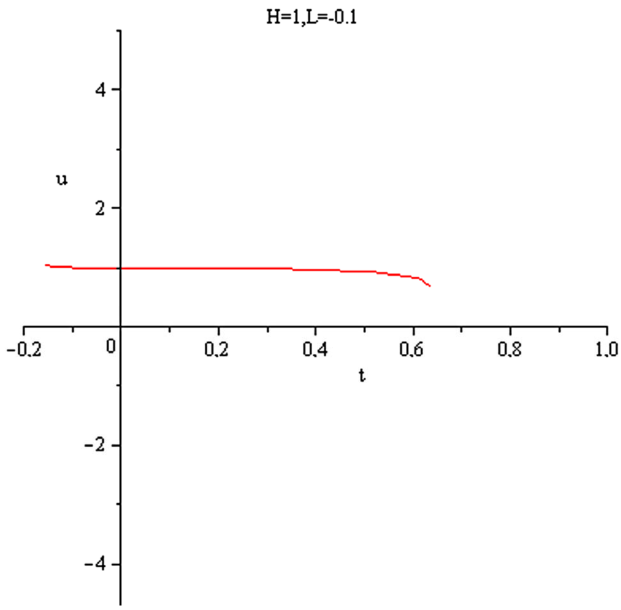

Figure 4. ***u*—velocity** $\tilde{u}$, $H = 1$, $\tilde{L} = -0.1$.

The second regime is characterized by the force directed along the direction of the wave propagation. This fact leads to the effect of anti-attraction during the Big Bang, (**Figures 13**, **17**, **19** and **21**). The term "antigravitation" is deeply embedded in the physical literature but this term unlikely applicable for the vacuum explosion. As follow from **Figures 15** and **16** the fist regime of traveling waves corresponds only to the early life of evolution and later gives way to regime of the very intensive explosion which details should be investigated using non-stationary 3D models on the basement of Equations (2.1)-(2.6). For example domain of the solution existence for the case is shown in **Figures 15** and **16**:
$t = \tilde{\xi}_{\lim} \leq 34.582$.

Important to notice that Hubble expansion can be explained as result of the matter self-catching in the frame of the Newtonian law of gravitation [7].

As you see during all investigations we needn't to use the theory Newtonian gravitation for solution of nonlinear non-local evolution equations (EE). In contrast with the local physics this approach in the frame of quantum non-local hydrodynamics leads to the closed mathemati-

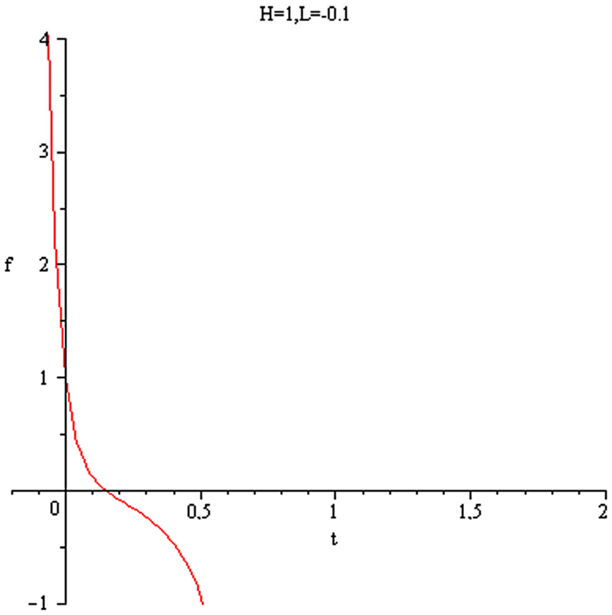

Figure 5. ***f*****—the self-consistent force** $\tilde{F}$ **;** $H = 1$**,** $\tilde{L} = -0.1$ **.**

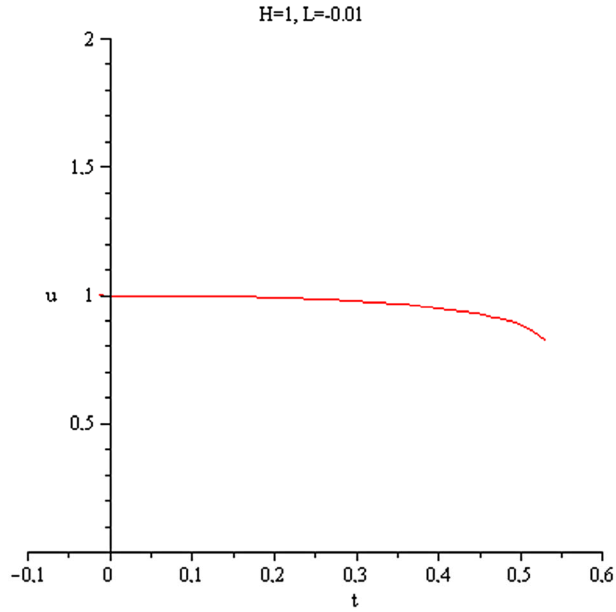

Figure 7. ***u*****—velocity** $\tilde{u}$ **,** $H = 1$**,** $\tilde{L} = -0.01$ **.**

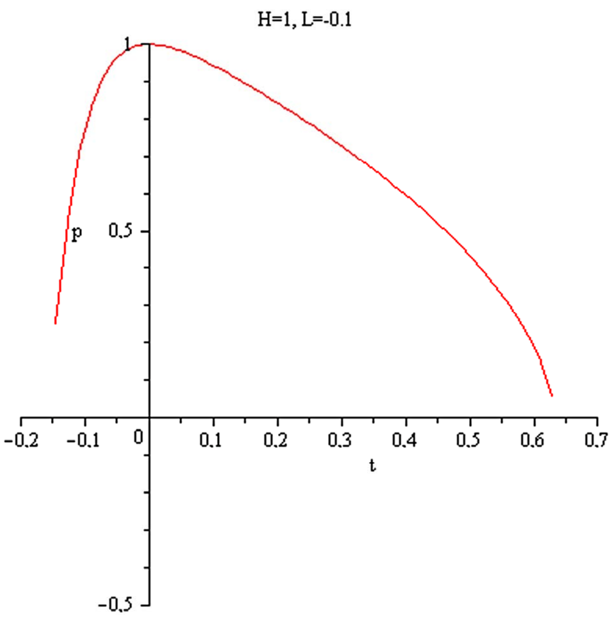

Figure 6. ***p*****—pressure** $\tilde{p}$ **;** $H = 1$**,** $\tilde{L} = -0.1$ **.**

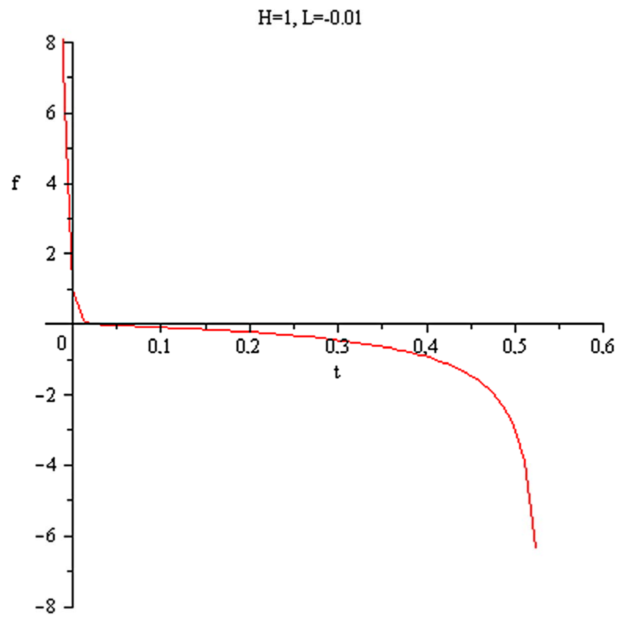

Figure 8. ***f*****—the self-consistent force** $\tilde{F}$ **;** $H = 1$**,** $\tilde{L} = -0.01$ **.**

cal description for the physical system under consideration.

If the matter is absent, the gravitational evolution of the system in space and time is containing in EE only so to speak on the "genetic level"; it means the origin of the EE derivation in the macroscopic case for massive system.

Better to speak about evolution of "originating vacuum" (OV) which description in time and 3D space on the level of quantum hydrodynamics demands only quantum pressure p, the self-consistent force $\boldsymbol{F}$ (acting on unit of the space volume) and velocity $\boldsymbol{v}_0$. The perturbations of OV lead to two different processes—travelling waves including the soliton formation (regime I) and the explosion of the system (regime II, the Big Bang regime). Both regimes can be incorporated in one scenario.

As follow from calculations (see **Figures 13-22**) the most intensive explosion effect achieves for the smallest perturbations with the positive sign in front of $\tilde{L}$. From the mathematical point of view we have the typical Hadamard instability leading to the Big Bang. Moreover two regimes differ from one another by the directions of

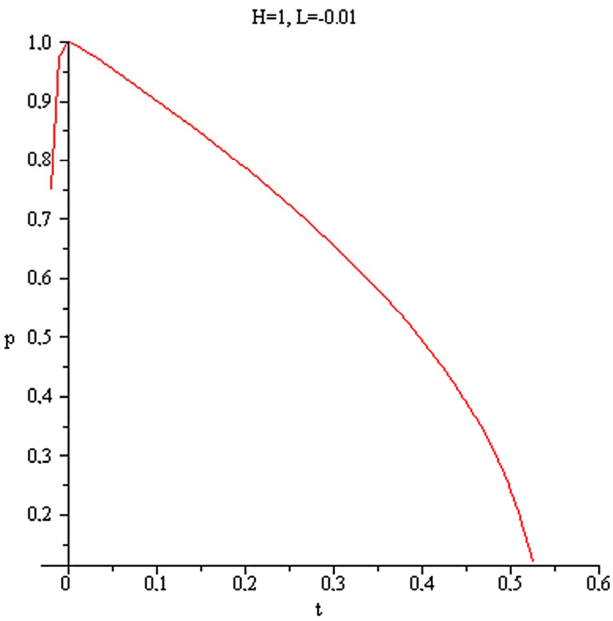

Figure 9. ***p*****—pressure** $\tilde{p}$ **;** $H = 1$, $\tilde{L} = -0.01$ **.**

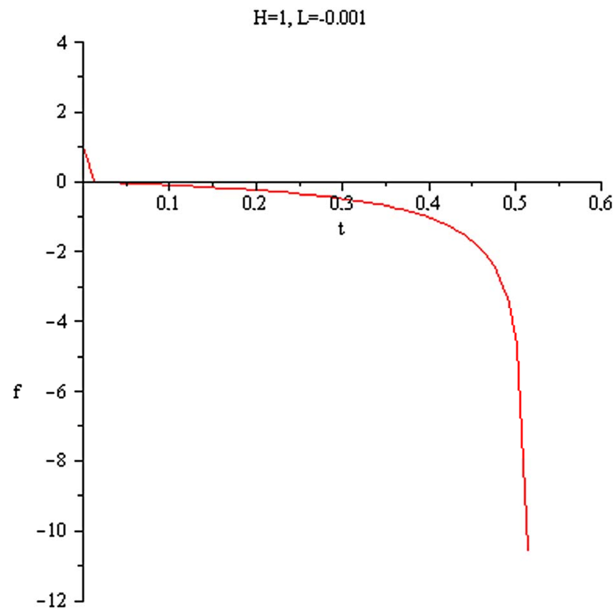

Figure 11. ***f*****—the self-consistent force** $\tilde{F}$ **;** $H = 1$, $\tilde{L} = -0.001$ **.**

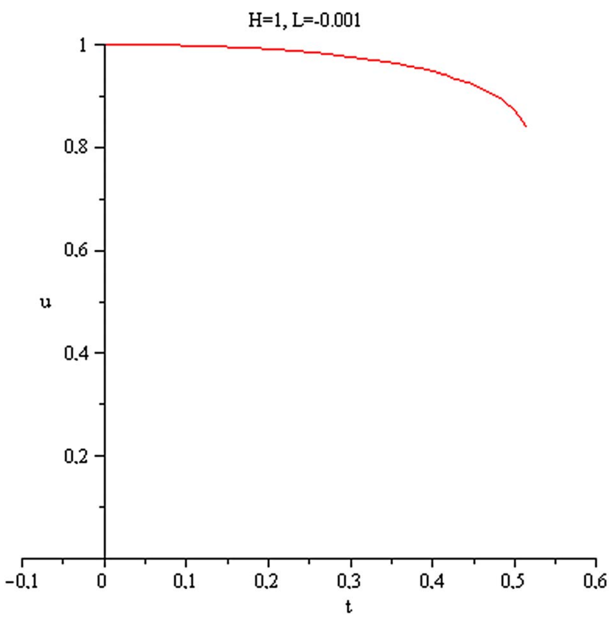

Figure 10. ***u*****—velocity** $\tilde{u}$ **,** $H = 1$, $\tilde{L} = -0.001$ **.**

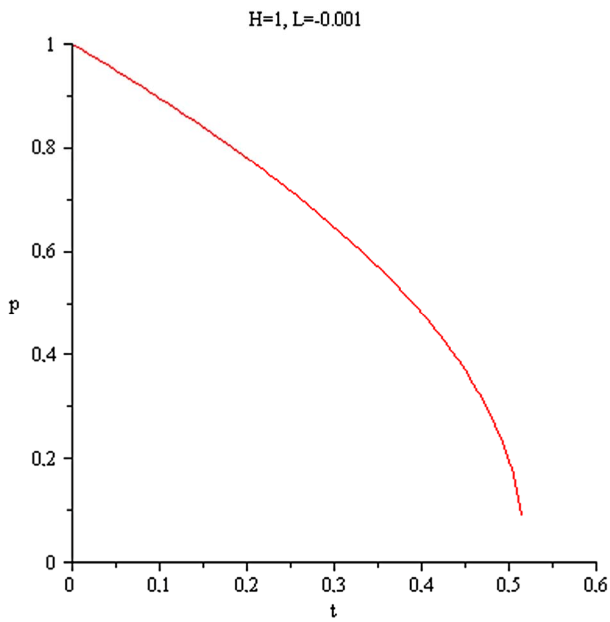

Figure 12. ***p*****—pressure** $\tilde{p}$ **;** $H = 1$, $\tilde{L} = -0.001$ **.**

forces F.

After the Big Bang and interaction of OV with the created matter the microwave background radiation should contain the traces of the travelling waves evolution realized as regime I. Let us look at the last measurements realized in the frame of the Planck programme. The temperature variations don't appear to behave the same on large scales as they do on small scales, and there are some particularly large features, such as a hefty cold spot, that were not predicted by basic inflation models.

From the position of the developed theory it is no surprise. Really, look at the Planck space observatory's map (**Figure 23**) of the universe's cosmic microwave background. This map is in open Internet access (see for example SPACE.com Staff. Date: 21 March 2013 Time: 11:15 AM ET).

It was reported that CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380,000 years old. It shows tiny temperature fluctuations that correspond to regions of slightly different densities, representing the seeds of all future structure: the stars and galaxies of today.

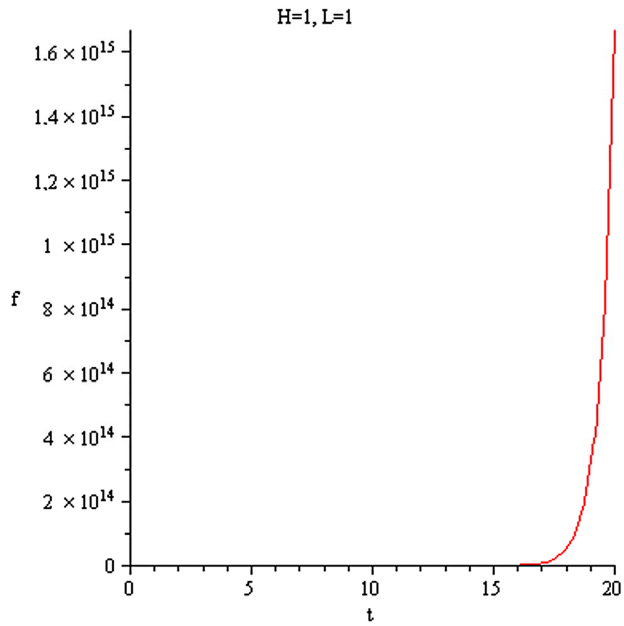

Figure 13. ***f*—the self-consistent force $\tilde{F}$, $H=1$, $\tilde{L}=1$.**

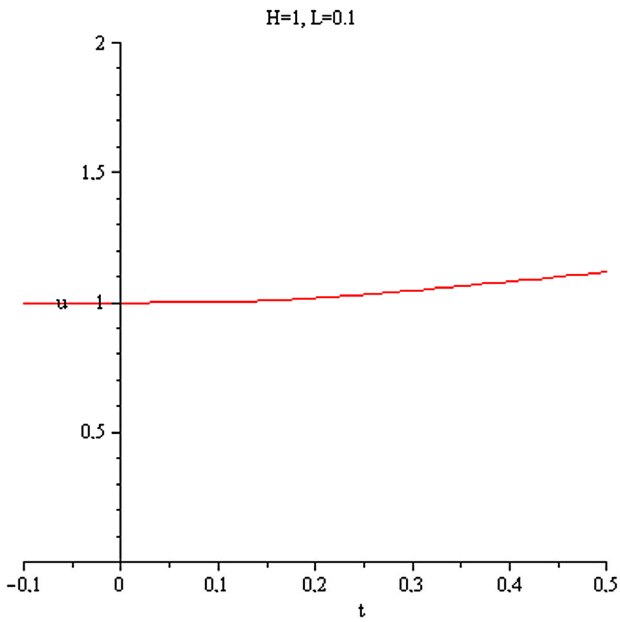

Figure 15. ***u*—velocity $\tilde{u}$, $H=1$, $\tilde{L}=0.1$.**

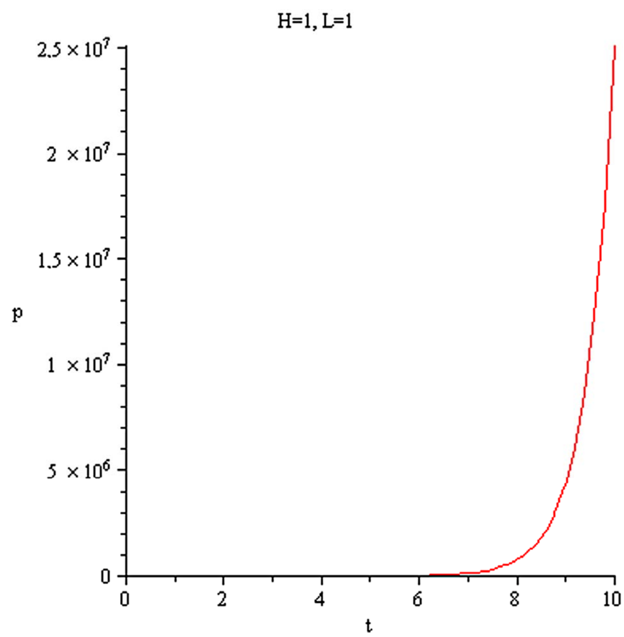

Figure 14. ***p*—pressure $\tilde{p}$; $H=1$, $\tilde{L}=1$.**

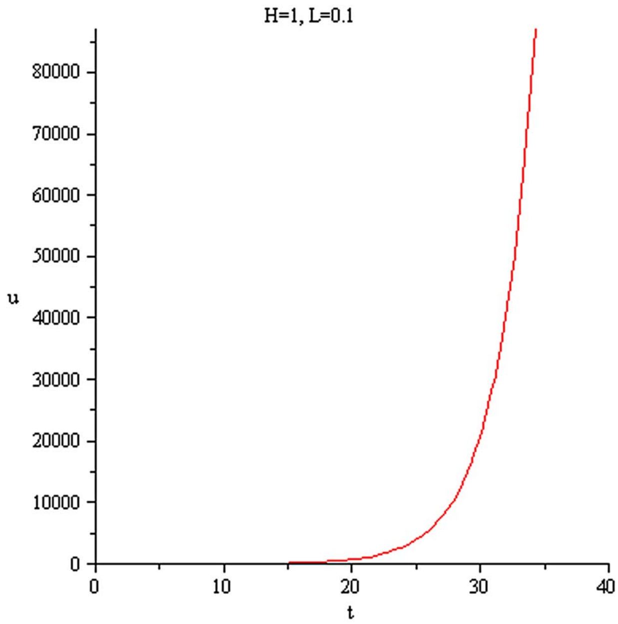

Figure 16. ***u*—velocity $\tilde{u}$, $H=1$, $\tilde{L}=0.1$.**

From the position of the developed theory Planck's all-sky map contains the regular traces of traveling waves as the alternation of the "hot" (red) and "cold" (blue) strips. In **Figure 23** the Planck space observatory staff shows the "mysterious" hefty cold spot as the blue small area bounded by the white circle.

From the position of the developed theory *it is the area reflecting the initial explosion of OV. In this case the center domain of the mentioned hefty cold spot should contain the smallest hot spot as the origin of the initial burst*.

I hope this fact will be established by astronomers after following more precise observations.

5. Conclusions

During all investigations we needn't to use the theory Newtonian gravitation for solution of nonlinear non-local evolution equations. In contrast with the local physics this approach in the frame of quantum non-local hydrodynamics leads to the closed mathematical description

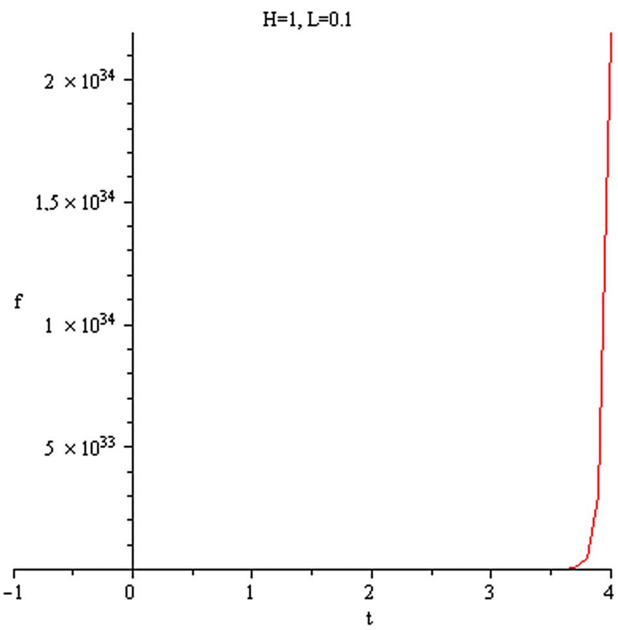

Figure 17. ***f*****—the self-consistent force** $\tilde{F}$ **;** $H=1$**,** $\tilde{L}=0.1$**.**

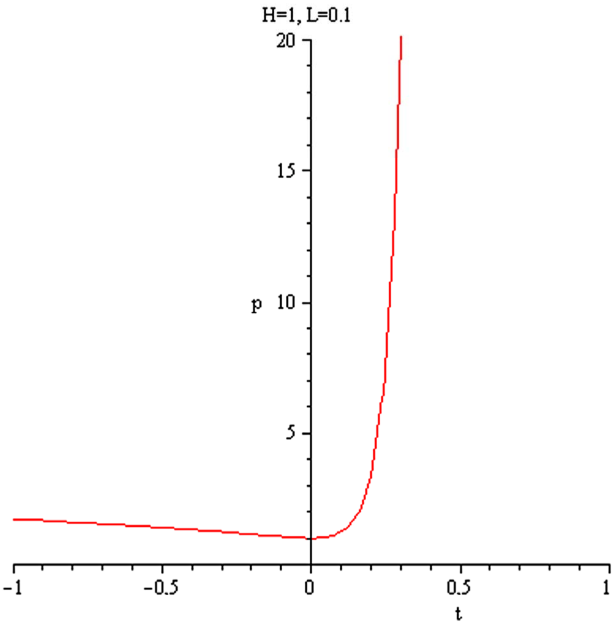

Figure 18. ***p*****—pressure** $\tilde{p}$ **;** $H=1$**,** $\tilde{L}=0.1$**.**

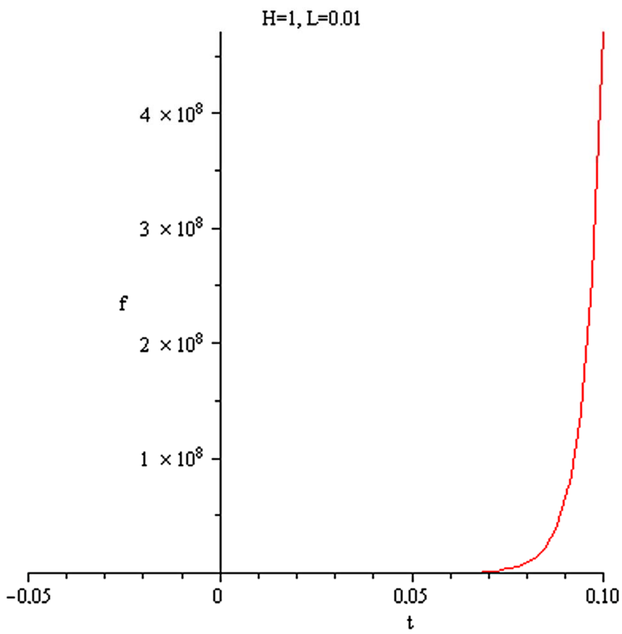

Figure 19. ***f*****—the self-consistent force** $\tilde{F}$ **,** $H=1$**,** $\tilde{L}=0.01$**.**

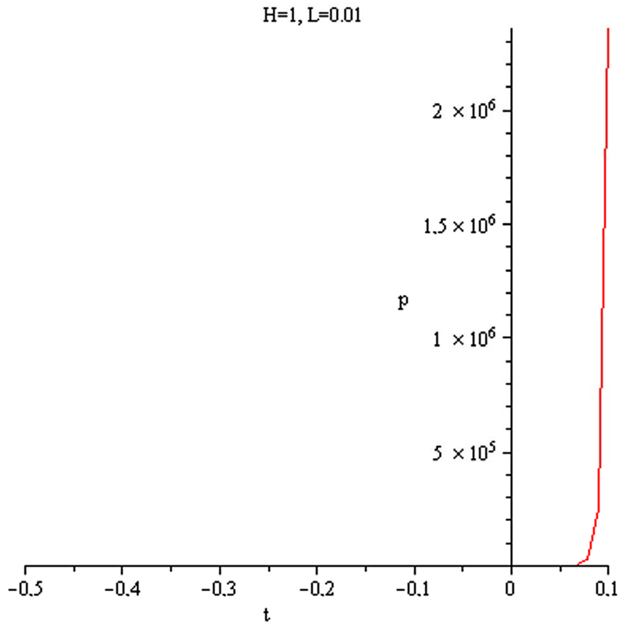

Figure 20. ***p*****—pressure** $\tilde{p}$ **;** $H=1$**,** $\tilde{L}=0.01$**.**

for the physical system under consideration. If the matter is absent, non-local evolution equations have nevertheless non-trivial solutions corresponding evolution of "originating vacuum" (OV) which description in time and 3D space on the level of quantum hydrodynamics demands only quantum pressure *p*, the self-consistent force $\boldsymbol{F}$ (acting on unit of the space volume) and velocity $\boldsymbol{v}_0$.

The perturbations of OV lead to two different processes—travelling waves including the soliton formation (regime I) and the explosion of the system (regime II, the Big Bang regime). Both regimes can be incorporated in one scenario. From the mathematical point of view we have the typical Hadamard instability (the smaller is an initial perturbation the greater is the burst intensity) leading to the Big Bang. Two regimes differ from one another by the directions of forces $\boldsymbol{F}$.

Finally some words concerning the following investigations. Numerical calculations, realized in the spherical coordinate system for the dependent variables (r—radius,

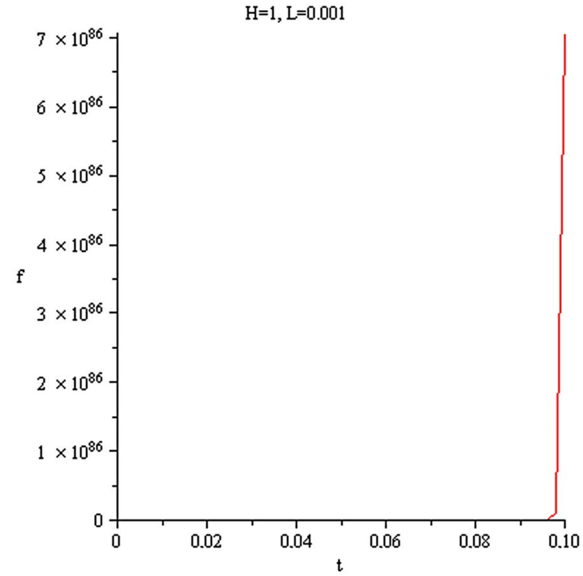

Figure 21. ***f*—the self-consistent force $\tilde{F}$, $H = 1$, $\tilde{L} = 0.001$.**

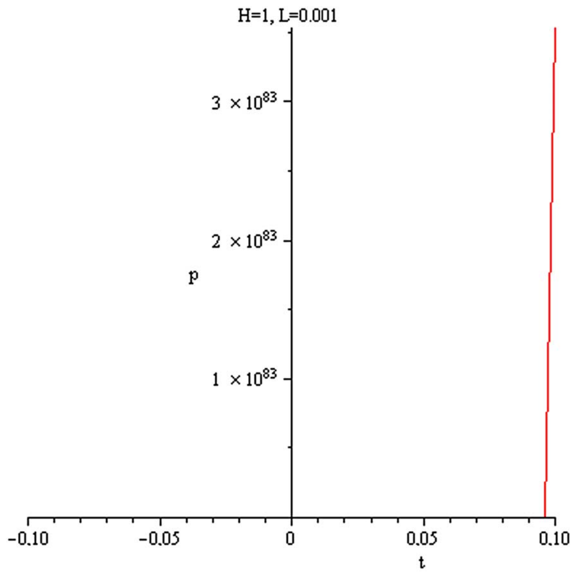

Figure 22. ***p*—pressure $\tilde{p}$; $H = 1$, $\tilde{L} = 0.001$.**

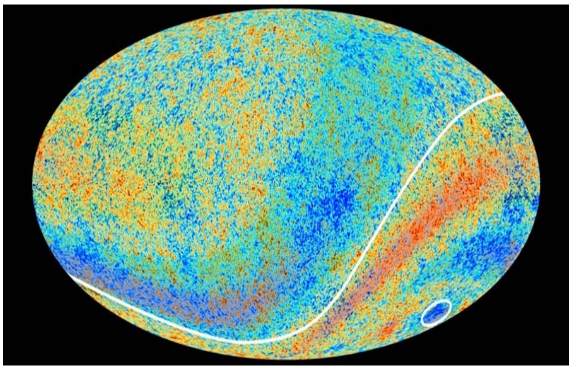

Figure 23. Planck space observatory's map of the universe's cosmic microwave background.

t —time) cannot change principal results of the shown calculations in the Cartesian coordinate system. Increasing of the character distances between "cold" and "hot" zones (see **Figure 23**) is obliged to the burst configuration closed to the spherical form. But some other effects obviously need in 3D non-stationary calculations. This remark relates first of all to so called "dark flow" describing a possible non-random component of the peculiar velocity of galaxy clusters.

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