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Dynamical Phase Transitions in Quantum Systems

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Abstract

Many years ago Bohr characterized the fundamental differences between the two extreme cases of quantum mechanical many-body problems known at that time: between the compound states in nuclei at extremely high level density and the shell-model states in atoms at low level density. It is shown in the present paper that the compound nucleus states at high level density are the result of a dynamical phase transition due to which they have lost any spectroscopic relation to the individual states of the nucleus. The last ones are shell-model states which are of the same type as the shell-model states in atoms. Mathematically, dynamical phase transitions are caused by singular (exceptional) points at which the trajectories of the eigenvalues of the non-Hermitian Hamilton operator cross. In the neighborhood of these singular points, the phases of the eigenfunctions are not rigid. It is possible therefore that some eigenfunctions of the system align to the scattering wavefunctions of the environment by decoupling (trapping) the remaining ones from the environment. In the Schrödinger equation, nonlinear terms appear in the neighborhood of the singular points.

Keywords: Non-Hermitian Quantum Physics, Dynamical Phase Transitions, Phase Rigidity of Eigenfunctions, Nonlinear Schrodinger Equation, Exceptional Points

1. Introduction

In 1936, Niels Bohr wrote in the address delivered on January 27 before the Copenhagen Academy [1,2]: *In the atom and in the nucleus we have indeed to do with two extreme cases of mechanical many-body problems for which a procedure of approximation resting on a combination of one-body problems, so effective in the former case, loses any validity in the latter where we, from the very beginning, have to do with essential collective aspects of the interplay between the constituent particles.* About 20 years later, it could be shown that the spectra of nuclei at low excitation energy are described well on the basis of the *shell model* [3], *i.e.* on the basis of an approximation *resting on a combination of one-body problems*. The shell closures in nuclei differ from those in atoms since the symmetries of nuclear forces differ from those of the forces in atoms. In the following years, the *collective aspects of the interplay between the constituent particles* were considered in nuclear physics studies only partly, *i.e.* they were reduced, above all, to the two-body residual interaction in the framework of the shell model. These studies provided very good results for light and medium-mass nuclei as well as for the low-lying states of heavy nuclei. In the case of the narrow compound-nucleus resonances, the contradictions to some basic assumptions of the statistical description were ignored. Here, most studies are performed by using the Gaussian orthogonal ensemble although the differences of this ensemble to a two-body (random) ensemble are not discussed thoroughly. Also the influence of particle decay thresholds is almost not considered in these papers.

As a result of nuclear physics studies during many years, we have to accept today that the resonance states at low and high level density differ fundamentally from one another. In light nuclei, most resonance states are at low excitation energy of the nucleus, where the level density is small. The lifetimes of the resonance states are often near to the limit for single-particle (or alpha) decay. All resonance states show individual spectroscopic features.

The situation in heavy nuclei is completely different. The first (elastic) threshold for particle decay is at about 8 MeV excitation energy of the nucleus where the level density is extremely high. In a small energy region above this threshold, the so-called neutron (compound nucleus) resonances are identified. They are extremely long-lived

corresponding to decay widths of the order of eV. The central part of the spectrum is described well by statistical methods, e.g. by the Gaussian orthogonal ensemble. The single resonance states decay according to a power law [4,5] and show chaotic features [6]. Much less discussed in literature are the so-called single-particle resonances in heavy nuclei the widths of which are of the order of magnitude of MeV. Their energy is mostly just below the elastic decay threshold, and their width at energies above the threshold (see [7] for the energy dependence of the widths) is much larger than the widths of the long-lived states. In the cross section, they appear as a smooth background for the very narrow neutron resonances. The time scales of these two different types of resonance states are well separated from one another: up to 10^6 neutron resonances are overlapped by one single-particle resonance. The Feshbach *unified theory of nuclear reactions* gives a good description of this situation [8,9].

In medium-mass nuclei, the first (elastic) decay threshold is at a comparably low excitation energy of the nucleus where the level density is still relatively low. These nuclei are characterized by overlapping resonance states with different lifetimes. An example are the isobaric analogue resonances which are described well by the doorway picture. According to this picture, the doorway states coexist with long-lived compound nucleus resonance states (see also [10]). The doorway states being comparably short-lived, are coupled directly to the decay channels and to the narrow compound nucleus resonance states. The narrow resonance states, however, are assumed to be coupled to the continuum *only* via the doorway states. This model characterizes the transition from the regime at low level density to that at high level density. Unfortunately, this transition can not be controlled by a parameter since the nuclear forces are too strong. According to [7], it may be considered to be a *dynamical phase transition*.

This interpretation is supported by experimental results obtained some years ago for the mean compound-nucleus lifetime in proton induced reactions on *Ni* isotopes by using the crystal blocking technique [11]. The mean lifetime is determined at the energies E = 5.65 and 6.50 MeV. It is significantly longer at the higher bombarding energy than at the lower one, contrary to expectations of a purely statistical theory, and is much smaller than expected. Furthermore, the data suggest an interpretation in terms of some form of intermediate structure resulting from the local spreading of a comparatively simple configuration [11]. This picture obtained experimentally, fits in with the spectroscopic redistribution processes appearing (according to the theory of open quantum systems) in the regime of overlapping resonances [7,12].

In the following, dynamical phase transitions in quantum systems will be discussed. They are directly related to the existence of exceptional points the mathematical properties of which are known for more than 40 years [13]. Their role in physical systems is discussed only recently. They are identified also in theoretical studies for nuclei under realistic conditions [14]. It will be shown in the present paper that the above cited statement by Niels Bohr is true in spite of its seeming contradiction to the shell structure of nuclei. The narrow compound nucleus resonances in heavy nuclei (well known at that time) are the result of a dynamical phase transition. They are characterized by *essential collective aspects of the interplay between the constituent particles* and not by *a combination of one-body problems*. Exceptional points play an important role in this transition.

In Section 2, the mathematical properties of exceptional points are sketched. They are singular points appearing at the crossing points of eigenvalue trajectories. The scattering on many-level systems is considered in Section 3. The exceptional points influence not only resonance states but also discrete states the energy of which is beyond the window coupled directly to the continuum of scattering wavefunctions. Resonance trapping and dynamical phase transitions are discussed in Section 4. Here, also the experimental verification of the resonance trapping phenomenon is mentioned. In contrast to nuclei, governed by the strong nuclear forces, the appearance of a dynamical phase transition can be controlled by means of a parameter in many other quantum systems. In Section 5, three examples will be considered, in which a dynamical phase transition is experimentally shown to exist. Some outlook is given in the last section.

2. Definition and Mathematical Properties of Exceptional Points

Many years ago, Kato [13] introduced the notation *exceptional points* for singularities appearing in the perturbation theory for linear operators. Consider a family of operators of the form

$$T(\varsigma) = T(0) + \varsigma T' \tag{1}$$

where ς is a scalar parameter, $T(0)$ is the unperturbed operator and $\varsigma T'$ is the perturbation. Then the number of eigenvalues of $T(\varsigma)$ is independent of ς with the exception of some special values of ς where (at least) two eigenvalues coalesce. These special values of ς are the exceptional points. An example is the operator

$$T(\varsigma) = \begin{pmatrix} 1 & \varsigma \\ \varsigma & -1 \end{pmatrix}. \quad (2)$$

In this case, the two values $\varsigma = \pm$ i give the same eigenvalue 0.

Operators of the type (2) appear in the description of physical systems, for example in the theory of open quantum systems [7]. In this case, they represent a symmetric 2×2 Hamiltonian describing a two-level system with the unperturbed energies ε_1 and ε_2 and the interaction ω between the two levels,

$$H(\omega) = \begin{pmatrix} \varepsilon_1 & \omega \\ \omega & \varepsilon_2 \end{pmatrix}. \quad (3)$$

In an open quantum system, two states can interact directly (corresponding to a first-order term) as well as via an environment (second-order term) [7]. In the present paper, we consider the case that the direct interaction is contained in the energies ε_k ($k = 1, 2$) . Then ω contains exclusively the coupling of the states via the environment which, in the case of an open quantum system, consists of the continuum of scattering wavefunctions into which the system is embedded. This allows us to study environmentally induced effects in open quantum systems in a very clear manner.

The eigenvalues of the operator $H(\omega)$ are

$$\varepsilon_{1,2} = \frac{\varepsilon_1 + \varepsilon_2}{2} \pm Z \; ; \; Z = \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4\omega^2}. \quad (4)$$

The two eigenvalue trajectories cross when $Z = 0$, *i.e.* when

$$\frac{\varepsilon_1 - \varepsilon_2}{2\omega} = \pm i \, . \quad (5)$$

At these *crossing points*, the two eigenvalues coalesce,

$$\varepsilon_1 = \varepsilon_2 \equiv \varepsilon_0 . \quad (6)$$

The crossing points may be called therefore exceptional points.

However, there are some essential differences between the exceptional points considered in the mathematical literature and the crossing points which appear in physical systems. The differences arise from the fact that the crossing points are points in the continuum of scattering wavefunctions (which represents the environment). They are therefore of measure 0 and can not be observed directly. However, they influence the behavior of the eigenvalue trajectories $\varepsilon_k(\alpha)$ (where α is a certain parameter) in their neighborhood in a non-negligible manner. Thus, the most interesting features of the exceptional (crossing) points in physical systems are not the properties at the crossing points themselves. Much more interesting are their effects onto the eigenvalue trajectories $\varepsilon_k(\alpha)$ in a finite parameter range around the critical value $\alpha = \alpha_{cr}$ (at which two trajectories cross) and, above all, the behavior of the eigenvalue trajectories in approaching the crossing point, $\varepsilon_k(\alpha) \to \varepsilon_k(\alpha_{cr})$. The phenomenon of *avoided level crossing* is known in physical systems since many years [15,16]. It occurs not only for discrete states but also for narrow resonance states [17]. In the scattering theory, the crossing points appear as double poles of the *S* matrix.

The Hamilton operator *H* describing an open quantum system is non-Hermitian. The eigenfunctions ϕ_k of such an operator are biorthogonal,

$$\langle \phi_k^* \mid \phi_l \rangle = \delta_{k,l} \, . \quad (7)$$

From these equations follows [7]

$$\langle \phi_k \mid \phi_k \rangle \equiv A_k \geq 1 \quad (8)$$

and

$$\langle \phi_k \mid \phi_{l\neq k} \rangle = -\langle \phi_{l\neq k} \mid \phi_k \rangle \equiv B_k^l \; ; \; \mid B_k^l \mid \geq 0 \, . \quad (9)$$

At the crossing point

$$A_k^{(cr)} \to \infty \quad \mid B_k^{l\,(cr)} \mid \to \infty \quad (10)$$

and the relation between the eigenfunctions ϕ_1 and ϕ_2 of the operator (3) is

$$\phi_1^{cr} \to \pm i \, \phi_2^{cr} \quad \phi_2^{cr} \to \mp i \, \phi_1^{cr} . \quad (11)$$

According to the last relation (11), the two eigenfunctions are linearly dependent of one another at the crossing point such that the number of eigenfunctions of H is reduced at this point. This result shows once more that the crossing point is an exceptional point in the sense defined by Kato [13].

Let us now consider the consequences of the biorthogonality relations (7) and (8) for the two borderline cases characteristic of neighboring resonance states.

1) The two levels are distant from one another. Then the eigenfunctions are (almost) orthogonal

$$\langle \phi_k^* \mid \phi_k \rangle \approx \langle \phi_k \mid \phi_k \rangle = A_k \approx 1 . \quad (12)$$

2) The two levels cross. Then the two eigenfunctions are linearly dependent according to (11) and

$$\langle \phi_k \mid \phi_k \rangle = A_k \to \infty \, . \quad (13)$$

according to (10).

The two relations (12) and (13) show that the phases of the two eigenfunctions relative to one another change when the crossing point is approached. This can be expressed quantitatively by defining the *phase rigidity* r_k of the eigenfunctions ϕ_k ,

$$r_k \equiv \frac{\langle \phi_k^* \mid \phi_k \rangle}{\langle \phi_k \mid \phi_k \rangle} = A_k^{-1} . \quad (14)$$

According to (12) and (13) holds

$$1 \geq r_k \geq 0 . \tag{15}$$

The non-rigidity r_k of the phases of the eigen-functions of H follows also from the fact that $\langle \phi_k^* \mid \phi_k \rangle$ is a complex number (in difference to the norm $\langle \phi_k \mid \phi_k \rangle$ which is a real number) such that the normalization condition (7) can be fulfilled only by the additional postulation Im $\langle \phi_k^* \mid \phi_k \rangle = 0$ (what corresponds to a rotation).

It should be mentioned finally, that the Schrödinger equation describing an open quantum system, becomes *nonlinear* near an exceptional point [7,17]. The most important part of the nonlinear contributions is contained in

$$\begin{aligned} &(H_0 - \varepsilon_k) \mid \phi_k \rangle - \langle \phi_k \mid W \mid \phi_k \rangle \mid \phi_k \mid^2 \mid \phi_k \rangle = 0 ; \\ &W \equiv -\begin{pmatrix} 0 & \omega \\ \omega & 0 \end{pmatrix} \end{aligned} \tag{16}$$

with $H = H_0 - W$ in (3). This equation is a nonlinear Schrödinger equation. The degree of nonlinearity is determined by the value $A_k \equiv \mid \phi_k \mid^2$, Equation (8), *i.e.* by the biorthogonality of the eigenfunctions of the non-Hermitian Hamilton operator H_{eff}. It is the larger the closer the crossing (exceptional) point is approached.

3. Scattering on Many-Level Systems

In an open many-level quantum system, the states of the system B can interact via the common environment C. Hence, the Hamilton operator consists of a first-order and a second-order interaction term,

$$H_{eff} = H_B + V_{BC} \frac{1}{E^+ - H_C} V_{CB} \tag{17}$$

with

$$\begin{aligned} &\mathrm{Re} \left\{ \left\langle \Phi_i^B \mid H_{eff} \mid \Phi_j^B \right\rangle \right\} = \left\langle \Phi_i^B \mid H_B \mid \Phi_j^B \right\rangle + \\ &\frac{1}{2\pi} \sum_c P \int_{E_{thr}^l}^{E_{thr}^h} \mathrm{d}E' \, \frac{\hat{\gamma}_i^c \hat{\gamma}_j^c}{E - E'} \end{aligned} \tag{18}$$

$$\mathrm{Im} \left\{ \left\langle \Phi_i^B \mid H_{eff} \mid \Phi_j^B \right\rangle \right\} = -\frac{1}{2} \sum_c \hat{\gamma}_i^c \hat{\gamma}_j^c . \tag{19}$$

Here, the Φ_k^B are eigenfunctions of H_B, $(H_B - E_k^B)\Phi_k^B = 0$, and the scattering wave functions ξ_E^c follow from $(H_C - E)\, \xi_E^c = 0$. Further, P denotes the principal value integral. The V_{BC}, V_{CB} stand for the interaction between the two subspaces B and C and

$$\hat{\gamma}_k^c = \sqrt{2\pi} \left\langle \xi_E^c \mid V \mid \Phi_k^B \right\rangle \tag{20}$$

is the coupling matrix element between the wavefunction Φ_k^B of the discrete state k of the system and the scattering wavefunction ξ_E^c of the environment. The direct (first-order) interaction V between two states is included in H_B and its eigenfunctions Φ_k^B.

According to [7], we are looking for the *exact solution* of the problem. The Hamiltonian H_{eff} is non-Hermitian, generally. We calculate not only Im $\left\{ \left\langle \Phi_i^B \mid H_{eff} \mid \Phi_j^B \right\rangle \right\}$, but also Re $\left\{ \left\langle \Phi_i^B \mid H_{eff} \mid \Phi_j^B \right\rangle \right\}$, and that by including the principal value integral and without any statistical assumptions. The Schrödinger equation reads

$$(H_{eff} - z_k)\Phi_k = 0 \tag{21}$$

with the eigenvalues z_k and eigenfunctions Φ_k of the Hamilton operator H_{eff}. In detail:

1) The states inside the energy window are coupled directly to the environment such that the effective Hamilton operator H_{eff} is non-Hermitian, *i.e.* the principal value integral in (18) as well as the residuum (19) have to be calculated. The eigenvalues are complex,

$$z_k = E_k - \frac{i}{2}\Gamma_k \tag{22}$$

in general, and the eigenfunctions Φ_k are complex and biorthogonal,

$$\left\langle \Phi_i^* \mid \Phi_j \right\rangle = \delta_{i,j} , \tag{23}$$

compare (7). The coupling matrix elements between the Φ_k and the ξ_c^E are

$$\gamma_k^c = \sqrt{2\pi} \left\langle \xi_c^E \mid V \mid \Phi_k \right\rangle \tag{24}$$

in analogy to (20).

2) The states outside the energy window are not coupled directly to the environment such that the effective Hamiltonian H_{eff} is Hermitian at the energy of the states, *i.e.* only the principal value integral in (18) has to be calculated. At the energy of the states, the eigenvalues $z_k = E_k$ are real, *i.e.* $\Gamma_k = 0$, and the Φ_k are orthogonal in the standard manner,

$$\left\langle \Phi_i \mid \Phi_j \right\rangle = \delta_{i,j} . \tag{25}$$

The coupling matrix elements between the Φ_k and the ξ_c^E vanish at the energy of the state. They are, however, different from zero at energies inside the window coupled directly to the environment and contribute to the principle value integral.

Thus, the non-Hermitian Hamilton operator H_{eff} of the open system provides, according to the boundary conditions, resonance or discrete states. It is interesting to remark that the spectroscopic properties of mirror nuclei differ from one another. An example are the nuclei ^{17}O and ^{17}F with 8 (9) protons and 9 (8) neutrons in the first (second) case. The differences arise

from the different positions of the energetically lowest decay channel (being a neutron (proton) decay channel) [12].

The individual states Φ_k of the many-level system have different spectroscopic properties and hence depend on parameters in a different manner. They may therefore cross or avoid crossing. The most interesting effects appear in the very neighborhood of the crossing points where the contributions of all the other states to the crossing phenomenon need not to be considered. Hence, the exceptional points defined in (2) and (3) play an important role also in the many-level system. The phases of the eigenfunctions Φ_k of H_{eff} are not rigid in approaching a crossing point, and the *phase rigidity* ρ of the eigenfunctions Φ_k may be defined. From (15) follows

$$1 \geq \rho \geq 0\,, \tag{26}$$

for details see [7].

The value ρ of the phase rigidity is surely the most interesting difference between Hermitian (with the Hamiltonian H_B) and non-Hermitian (with the Hamiltonian H_{eff}) quantum physics. While the phases are rigid ($\rho = 1$) in the first case, they may vary according to (26) in the second case. It is possible therefore that some wavefunctions Φ_k of the system align with the scattering wavefunctions of the environment while the other states decouple (more or less) from the environment. This phenomenon, called *resonance trapping*, is nothing but width bifurcation caused by exceptional points (see Section 2). In this manner, the non-Hermitian quantum physics is able to describe environmentally induced effects.

Also the *nonlinearities* in the neighborhood of exceptional points can be seen when the scattering problem on a many-level system is considered [7]. For example, the S matrix at a double pole (corresponding to an exceptional point) in the one-continuum case reads

$$S = 1 - 2i\frac{\Gamma_0}{E - E_0 + \frac{i}{2}\Gamma_0} - \frac{\Gamma_0^2}{(E - E_0 + \frac{i}{2}\Gamma_0)^2} \tag{27}$$

where the notation (6) is used and $\varepsilon_0 \equiv E_0 - \frac{i}{2}\Gamma_0$. At the exceptional point, the cross section vanishes due to interferences. The minimum is however washed out in the neighborhood of the double pole. In any case, the resonance is broader than a Breit-Wigner resonance according to (27).

4. Resonance Trapping and Dynamical Phase Transitions

Some years ago, the question has been studied [18] whether or not the resonance trapping phenomenon is related to some type of phase transition. The study is performed by using the toy model

$$H_{eff}^{toy} = H_0 + i\alpha VV^+ \tag{28}$$

in the one-channel case and with the assumption that (almost) all crossing (exceptional) points accumulate in one point [19]. The control parameter α is a real number. It has been found that resonance trapping may be understood, in this case, as a second-order phase transition. The calculations are performed for a linear chain consisting of a finite number N of states. The state in the center of the spectrum traps the other ones and becomes a collective state in a global sense: it contains components of almost all basic states of the system, also of those which are not overlapped by it. The normalized width Γ_0 / N of this state can be considered as the order parameter: it increases linearly as a function of α, and the first derivative of Γ_0 / N jumps at the critical value $\alpha = \alpha^{cr}$. The two phases of the system differ by the number of localized states. In the case considered, this number is N at $\alpha < \alpha^{cr}$, and $N-1$ at $\alpha > \alpha^{cr}$.

Much more interesting is the realistic case with the Hamiltonian (17). In this case, trapping of resonance states occurs in the regime of overlapping resonances hierarchically, *i.e.* one by one [7]. The crossing points do *not* accumulate in one point, but are distributed over a certain range of the parameter: a *dynamical* phase transition takes place in a finite parameter range inside the regime of overlapping resonances [7]. Also in this case, almost all resonance states are involved in the phase transition of the system and, furthermore, the number N of localized states is reduced. That means, the subspace of localized states splits into two parts under the influence of the environment. One part contains the few short-lived states which are (more or less) aligned to the scattering states of the environment, while the other part contains the trapped, long-lived and well localized states. Both time scales are well separated from one another. A theoretical example are the short-lived whispering gallery modes in a small microwave cavity with convex boundary which coexist with many long-lived states, for details see [20-22]. An experimental example are the isobaric analogue resonances in medium-mass nuclei.

The dynamical phase transitions are surely the most interesting feature of non-Hermitian quantum physics. Physically, they are environmentally induced, as can be seen from the Hamiltonian (3) or (17), see also [7,23]. Mathematically, this phenomenon is directly related to the existence of exceptional (crossing) points. In detail:

- The phases of the eigenfunctions of the non-

Hermitian Hamilton operator are not rigid in approaching the exceptional (crossing) point: $r_k < 1$ in the regime of overlapping resonances.

- Due to $r_k < 1$, some resonance states may align with the scattering states of the environment while other ones decouple from the environment (width bifurcation).
- The short-lived aligned resonance states lose, to a great deal, their localization and make the system (almost) transparent.
- The long-lived trapped resonance states are well localized and show chaotic features.
- The spectroscopic relation between the localized states at low level density (without resonance overlapping) and those at high level density (with overlapping short-lived and long-lived resonances) is lost.

The two phases of the system below and beyond the dynamical phase transition are characterized by the following properties. In one of the phases, the states have individual spectroscopic features. Here, the real parts (energies) of the eigenvalue trajectories avoid crossing as function of a certain control parameter, while the imaginary ones (widths) can cross. In the other phase, the narrow resonance states are superimposed with a smooth background and the individual spectroscopic features of the states are lost. The narrow resonance states and, respectively, the corresponding discrete states show chaotic features. They do not cross in energy, but show level repulsion. The real parts (energies) of the eigenvalue trajectories of narrow resonance states can cross with those of the broad states since the narrow and broad states exist at well separated time scales. In the transition region, the different time scales corresponding to the short-lived and long-lived resonance states are formed. In this regime, the cross section is enhanced due to the (at least partial) alignment of some states with the scattering states of the environment [7]. An example is the enhanced transmission through quantum dots in the regime of overlapping resonances [24,25].

About 10 years ago, the counterintuitive resonance trapping phenomenon is tested experimentally [26]. The experiment is based on the equivalence of the electromagnetic spectrum for flat cavities to the quantum mechanical spectrum of the corresponding system. This equivalence holds also when the system is opened by coupling the discrete states of the cavity to an attached waveguide. In the experiment [26], a microwave Sinai cavity with an attached waveguide with variable slit width was used.

The result of this experimental study agrees with theory: the widths of all resonance states first increase with increasing coupling strength to the channels (continuum of scattering wavefunctions) but finally decrease again for most of the states. Thus, the dynamical phase transition has been directly traced in this experiment.

The appearance of dynamical phase transitions can explain some puzzles that are observed experimentally and cannot be explained theoretically in the framework of conventional Hermitian quantum theory. Some examples will be sketched in Section 5. The dynamical phase transitions are responsible also for the observation of the *two extreme cases of mechanical many-body problems* mentioned by Niels Bohr [1,2]. Shell model states with individual spectroscopic features may appear only at low level density in nuclei as well as in atoms. At high level density, however, the long-lived resonance states cannot be described by *a combination of one-body problems*. They are the result of resonance trapping (characteristic of the dynamical phase transition) and have almost nothing in common with shell model states. They rather are states of an ensemble of long-lived resonance states that is overlapped by a short-lived resonance state.

5. Dynamical Phase Transitions in Experimental Results

5.1. Phase Lapses

In experiments [27-29] on Aharonov-Bohm rings containing a quantum dot in one arm, both the phase and the magnitude of the transmission amplitude $T = |T| e^{i\beta}$ of the dot can be extracted. The obtained results caused much discussion since they do not fit into the standard understanding of the transmission process. As a function of the plunger gate voltage V_g, a series of well-separated transmission peaks of rather similar width and height has been observed in many-electron dots and, according to expectations, the transmission phases $\beta(V_g)$ increase continuously by π across every resonance. In contrast to expectations, however, β always jumps sharply downwards by π in each valley between any two successive peaks. These jumps called phase lapses, were observed in a large succession of valleys for every many-electron dot studied. Only in few-electron dots, the expected so-called mesoscopic behavior is observed, *i.e.* the phases are sensitive to details of the dot configuration. The problem is considered theoretically, in the framework of conventional Hermitian quantum physics, in many papers over many years, however without solving it.

In [30], the phase lapses observed experimentally at high level density are related to the trapped resonance states resulting from the dynamical phase transition. In accordance to this picture, only the resonance states at low level density show individual spectroscopic features.

At high level density, the observed resonances arise from trapped states. They show level repulsion, have vanishing spectroscopic relation to the open decay channels (*i.e.* small decay widths), and phase lapses appear. It follows further, that any theoretical study on the basis of conventional Hermitian quantum physics is unable to explain the experimental results convincingly. In other words: the experimentally observed phase lapses can be considered to be a proof for the dynamical phase transitions occurring in mesoscopic systems.

5.2. Quantum Dynamical Phase Transition in the Spin Swapping Operation

A swapping gate in a two-spin system exchanges the degenerate states $|\uparrow,\downarrow\rangle$ and $|\downarrow,\uparrow\rangle$. Experimentally, this is achieved by turning on and off the spin-spin interaction b that splits the energy levels and induces an oscillation with a natural frequency ω. An interaction $\hbar/\tau_{SE}$ with an environment of neighboring spins degrades this oscillation within a decoherence time scale τ_ϕ. The experimental frequency ω is expected to be roughly proportional to $b/\hbar$ and the decoherence time τ_ϕ proportional to τ_{SE}. In [31], experimental data are presented that show drastic deviations in both ω and τ_ϕ from this expectation. Beyond a critical interaction with the environment, the swapping freezes and the decoherence rate drops as $1/\tau_\phi \propto (b/\hbar)^2 \tau_{SE}$. That means, the relaxation decreases when the coupling to the environment increases. The transition between these two quantum dynamical phases occurs when $\omega \propto \sqrt{(b/\hbar)^2 - (k/\tau_{SE})^2}$ becomes imaginary (where k depends only on the anisotropy of the system-environment interaction, $0 \le k \le 1$). The experimental results are interpreted by the authors as an environmentally induced quantum dynamical phase transition occurring in the spin swapping operation [31-34].

Further theoretical studies within the Keldysh formalism showed that τ_ϕ is a non-trivial function of the system-environment interaction rate τ_{SE}, indeed: it is $1/\tau_\phi \propto 1/\tau_{SE}$ at low τ_{SE} (according to the Fermi golden rule) but $1/\tau_\phi \propto \tau_{SE}$ at large τ_{SE}. This theoretical result is in (qualitative) agreement with the experimental results. In [35], the dynamical phase transition in the spin swapping operation is related to the existence of an exceptional point.

The dynamical phase transition observed experimentally in the spin swapping operation and described theoretically within the Keldysh formalism shows qualitatively the same features as the dynamical phase transitions discussed in the present paper on the basis of the resonance trapping phenomenon (width bifurcation).

5.3. Loss Induced Optical Transparency in Complex Optical Potentials

Recently, the prospect of realizing complex *PT* symmetric potentials within the framework of optics has been suggested [36-38]. It is based on the fact that the optical wave equation is formally equivalent to the quantum mechanical Schrödinger equation. One expects therefore that *PT* symmetric optical lattices show a behavior which is qualitatively similar to that discussed for open quantum systems in the present paper.

Experimental studies showed, indeed, a phase transition that leads to a loss induced optical transparency in specially designed non-Hermitian guiding potentials [39 -41]: the output transmission first decreases, attains a minimum and then increases with increasing loss. The phase transition is related, in these papers, to *PT* symmetry breaking. In a following theoretical paper [42], the Floquet-Bloch modes are investigated in *PT* symmetric complex periodic potentials. As a result, the modes are skewed (nonorthogonal) and nonreciprocal. That means, they show the same features as modes of an open quantum system under the influence of exceptional points. A detailed discussion of this analogy is given in [23]. The optical realization of relativistic non-Hermitian quantum mechanics is considered in [43]. Here, the *PT* symmetry breaking of the Dirac Hamiltonian is shown to be related to resonance narrowing what is nothing but resonance trapping.

The title of one of the papers published in Nature Physics [41] to this topic reads: *Broken symmetry makes light work*. It is exactly this property which characterizes the phase transition in complex optical potentials. However, the situation in open quantum systems is qualitatively the same: in the dynamical phase transition, the spectroscopic relation to the individual resonance states (including all symmetries) is broken and the system becomes transparent, see e.g. Section 4.

6. Summary

Dynamical phase transitions are a phenomenon characteristic of quantum systems at high level density. Mathematically, this phenomenon can be described in the non-Hermitian quantum physics since the phases of the eigenfunctions of a non-Hermitian operator are, in general, not rigid. The non-rigidity is large in the neighborhood of exceptional points (crossing points of the eigenvalue trajectories). Here, the Schrödinger equation is

nonlinear. Physically, the dynamical phase transitions are environmentally induced.

It is interesting to see that quantum systems behave according to expectations only at low level density. Here, the states show individual spectroscopic features. After passing the transition region with overlapping resonances by further variation of the control parameter, the behavior of the system becomes counterintuitive: the narrow resonance states decouple more or less from the continuum of scattering states, and the number of localized states decreases. The decoupling (*resonance trapping*) occurs due to the alignment of a few states of the system to the scattering states of the environment. This is an effect to which all states of the system contribute, see Section 4 and [18].

In his address [1,2], Niels Bohr compared the trapped resonance states in nuclei (compound nucleus resonances) at high level density with the low-lying resonance states of single-particle (shell model) nature in atoms. These states differ from one another exactly in the manner described by him. In the first case, the states are beyond the dynamical phase transition of the system while they are below the transition in the second case. The resonance states at low level density (below the dynamical phase transition) in nuclei as well as in atoms have individual spectroscopic properties described well by the shell model. The narrow states in nuclei at high level density, however, are described with adequate accuracy by a statistical ensemble containing the interaction between all particles, e.g. by the Gaussian orthogonal ensemble. There is no need to consider, in the center of the spectrum, the relation to a two-body random ensemble. The short-lived and long-lived resonance states are formed under the influence of the environment in the transition region with many overlapping resonances.

According to the results represented in the present paper, dynamical phase transitions in quantum systems occur due to the existence of exceptional (crossing) points. They are therefore a generic property emerging in the regime of overlapping resonances where the resonance states lose any spectroscopic relation to the individual resonance states of the system. Correspondingly, dynamical phase transitions are found experimentally in different systems.

Knowing the mathematical properties of the exceptional points it is possible, on the one hand, to explain (at least qualitatively) some experimental results which could not be understood in the framework of the conventional Hermitian quantum physics in spite of much effort. On the other hand, quantum systems can be manipulated systematically for applications. This includes also the interesting topic of non-Hermitian quantum physics which results from the formal equivalence of the optical wave equation in PT symmetric optical lattices to the quantum mechanical Schrödinger equation. This equivalence allows to receive much new information on quantum systems. In any case, further theoretical and experimental studies in the field of non-Hermitian quantum physics will broaden our understanding of quantum mechanics. Moreover, the results are expected to be of great value for applications.

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