

# Estimation of GPS Carrier Phase Multipath Signals Based on Site Environment

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**Abstract.** This paper presents a method that is based on Electromagnetic Modeling (EM) technique for modelling GPS carrier phase multipath signals. A commercial software plus modules developed in-house are used for modeling and processing carrier phase multipath error parameters. Static multipath modeling experiments show that up to about 35% carrier phase errors and about 25% 3D positioning errors can be reduced.

**Key words:** GPS, multipath, electromagnetic modeling (EM)

## 1 Introduction

Multipath is a dominant error source that limits the widespread use of Global Positioning System (GPS) in precise surveying and deformation monitoring. Multipath is the phenomenon in which a signal arrives at an antenna via several paths due to signal reflection and diffraction. GPS carrier phase measurements are affected by the multipath signals that can significantly affect the quality of data used for static and kinematic positioning applications. In the receiver, multipath is characterized by four parameters, all of which are relative to the direct signal. They are amplitude of the reflected signal relative to the direct signal, path delay, phase of the reflected signal relative to the direct signal and phase rate (Braasch, 1999).

Research has been conducted to reduce GPS carrier phase multipath effects based on hardware approaches. Multipath Estimating Delay Lock Loop (MEDLL) technique has been developed for estimating multipath signals to mitigate both code and carrier phase errors (Townsend et al., 1995). Moelker (1997) developed Multiple Signal Classification (MUSIC) technique with multiple antennas to mitigate code multipath and with a MEDLL receiver. Garin et al. (1997) simulated multipath

error by Advanced Strobe Correlator. Ray et al. (1998) derived spatial correlation of the multipath error between multiple closely-spaced antennas. Böder et al. (2001) developed a method for the absolute field calibration of multipath by decorrelating the multipath through controlled motion of a robot.

Data processing techniques have also been used to identify multipath errors. Multipath was identified by looking at the difference between L1 and L2 phase observations (Georgiadou et al., 1988). Axelrad et al. (1994) and Sleewaegen (1997) used signal-to-noise ratio (SNR) information to estimate multipath. Van Nee (1994) analyzed carrier lock loop behaviour in the presence of multipath. Walker et al. (1996) and Hannah et al. (1998) used parabolic equation method for propagation modeling of GPS signals and multipath. Xia (2001) and Satirapod et al. (2003) used wavelet algorithm to reduce multipath. Lau et al. (2003) modeled modernized GPS signals to mitigate the multipath from nearby reflective objects mainly. Zheng et al. (2005) used Vondrak Filter and Cross-Validation method for filtering of GPS multipath signal.

Modeling of carrier phase multipath signals using Electromagnetic Modeling technique is another useful approach especially in the urban environment. Both long and short delay multipath can be determined from the known satellite-reflector-antenna geometry. GPS multipath signal propagation model is possible to be visualized in an urban 3D model. Parameters used for characterizing multipath can also be estimated using EM technique.

## 2 Multipath Effects on Carrier Phase

The magnitudes of the direct and multipath phasors are given by

$$D=R(\tau) \quad (1)$$

$$M=\alpha R(\tau-\delta) \quad (2)$$

where  $R(\tau)$  is the prn code autocorrelation function as a function of lag  $\tau$ ;  $\delta$  is the multipath delay relative to the direct signal and  $\alpha$  is the ratio of multipath signal strength to direct signal strength.

Carrier phase multipath error  $\varepsilon_M$  due to  $n$  multipath signals is described as a function of the excess signal path (multipath delay), the ratio of the direct signal amplitude to the indirect signal amplitude (damping factor) and the carrier wavelength. By assuming negligible the multipath relative Doppler, the expression for the error in the carrier phase due to  $n$  specular multipath signals is given by the equation, see Mora-Castro et al. (1998):

$$\varepsilon_M = \frac{\lambda}{2\pi} \cdot \arctan \left[ \frac{\sum_i \alpha_i \sin(\frac{d_i}{\lambda} \cdot 2\pi)}{1 + \sum_i \alpha_i \cos(\frac{d_i}{\lambda} \cdot 2\pi)} \right] \quad (3)$$

where  $\alpha_i$  is the damping factor (ratio of the multipath amplitude to the constant part of the direct signal),  $d_i$  is the multipath delay (m) and  $\lambda$  is the carrier wavelength (m).

Note that the damping factor  $\alpha_i$  is not a constant. It depends on the coefficient of reflection of the surface and the antenna gain pattern. For a given antenna and reflecting surface, it will also vary with the elevation of direct and reflected signal, see Bétaille (2003). The reflected signals are always delayed relative to the direct signal as they travel longer paths in an environment. The maximum possible multipath errors on carrier phase observations can reach  $0.25 \lambda$ , i.e. 4.7 cm or 6.1 cm for L1 or L2 respectively, see Van Nee (1994).

### 3 The Method of Electromagnetic Modeling

Computational electromagnetic modeling (EM) offers powerful techniques to solve a variety of electromagnetic problems as they can calculate the solution to this kind of problem based on a full-wave analysis. EM can also be used to study the propagation of GPS signals. The size of domain is one of the important factors for some of the EM modeling techniques. Special attention about EM techniques domain size limitation should be paid before applying the appropriate EM technique for modeling of GPS signal propagation.

Although different EM technique can be used to study the GPS propagation theoretically, the size of the domain is one of the important factors for some of the EM techniques. Because of the large size of the buildings compared to the wavelength at wireless frequencies, direct numerical solvers of Maxwell's equations, such as the finite element and finite difference methods, involve

too many unknowns to be feasible at this time. From the practical point of view, the high-frequency techniques like Geometrical Theory of Diffraction (GTD)/ Uniform Theory of Diffraction (UTD) and the parabolic equation method are more suitable for modeling GPS signal propagation. Some research on propagation modeling of GPS signals using parabolic equation method can be found in Walker et al. (1996) and Hannah et al. (1998).

In this paper, the GTD/UTD technique (Kouyoumjian et al., 1974; McNamara et al., 1990) with multiple-image theory and a ray launching technique are used for modeling of GPS multipath signals. The GTD/UTD technique is a high-frequency method, which is suitable to apply when the structures are larger than the wavelength. Characterizing GPS multipath effects in a pre-defined simple multipath scenario based on the GTD technique in signal propagation model have been carried out by Mora-Castro et al. (1998). For the GPS L1 and L2 frequencies, the wavelengths are about 0.19 cm and 0.24 cm respectively which are much smaller than the building structure in urban environment. Since terrain information is required for determining appropriate diffraction coefficients, 3D building models are useful in applying the method.

In free space, the GTD formulation for the electric field in the direction  $(\theta, \phi)$  in the far field of the transmitting antenna at a distance  $r$  is

$$E_{GTD}(r, \theta, \phi) = \left( A_\theta(\theta, \phi) \hat{e}_\theta + A_\phi(\theta, \phi) \hat{e}_\phi \right) \frac{e^{-j\beta r}}{r} \quad (4)$$

where

$$A_\theta(\theta, \phi) = \sqrt{\frac{P_T \eta_0}{2\pi}} g_\theta(\theta, \phi)$$

$$A_\phi(\theta, \phi) = \sqrt{\frac{P_T \eta_0}{2\pi}} g_\phi(\theta, \phi) \quad (6)$$

$$g_\theta(\theta, \phi) = |G_\theta(\theta, \phi)|^2 e^{j\psi_\theta} \quad (7)$$

|                          |                                                                                                                                                       |
|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| $G_\theta(\theta, \phi)$ | is the $\theta$ component of the gain of the transmitting antenna                                                                                     |
| $\psi_\theta$            | is the relative phase of the $\theta$ component of the far zone electric field, and with an analogous definition for $g_\phi \times \beta = \omega/c$ |
| $P_T$                    | is the time-averaged power radiated by the transmitter                                                                                                |
| $r$                      | is the distance from the transmitter to the field point                                                                                               |

The multiple-image theory and the ray launching technique are used for identifying the propagation paths.

The locations of multiple image sources are determined directly in order to find out the point of reflection in a wall or diffraction at an edge. Those rays either will reach the receiver or will be blocked by buildings or lose because of multiple reflections, are identified. Local ray-fixed coordinate system and edge-fixed coordinate system with appropriate dyadic reflection or diffraction coefficients are required for each reflection or diffraction.

For example, there is an incident wave  $E^i(Q)$ , which has reflection or diffraction at point S given by  $E^{r,d}(s)$ , where

$$E^{r,d}(s) = E^i(Q) \cdot H(Q, S) \cdot e^{-jks} \quad (8)$$

$$H(Q, S) = \begin{cases} R \cdot A_s & \text{for reflection} \\ D \cdot A_d & \text{for diffraction} \end{cases} \quad (9)$$

|           |                                                     |
|-----------|-----------------------------------------------------|
| R         | is dyadic reflection coefficient                    |
| D         | is dyadic diffraction coefficient                   |
| $A_s$     | is spreading factor for a reflection from a surface |
| $A_d$     | is spreading factor for a diffraction at an edge    |
| k         | is propagation constant $= (2\pi/\lambda)$          |
| s         | is distance from S                                  |
| $\lambda$ | is wavelength                                       |

Because of the large number of combinations of multiple reflections, ray launching technique (Glassner, 1989) is used to simplify the formulation. This technique will consider those images which are involved in a particular ray only while other images will be ignored.

$$\hat{R}(\rho) = \hat{R}_o + \hat{R}_d \cdot \rho \quad (10)$$

where

|             |                                               |
|-------------|-----------------------------------------------|
| $\hat{R}_o$ | is a vector denoting the ray's origin         |
| $\hat{R}_d$ | is a unit vector in the ray direction         |
| $\rho$      | represents the distance from the ray's origin |

$\hat{R}_d$  scans from 0 to  $2\pi$  with angular increment  $\theta_s$ . If an interaction between a ray and an object occurs, the direction of reflected rays, the surface normals, the location of images and the reflecting planes are computed. A reception circle at the receiver point having a radius of  $\theta_s \cdot d/2$ , where d is the total distance of the propagation path from the satellite to the receiver, is constructed to determine if there is any reflected ray reaches the receiver. The reason for choosing a radius of  $\theta_s \cdot d/2$  is that  $\theta_s \cdot d$  is the distance swept out at the end of the ray when the launch angle is changed by  $\theta_s$ , provided the set of reflecting surfaces remains the same. More details on this method can be found in Tan et al. (1995a, 1995b).

Computation time is directly proportional to the size of  $\theta_s$ . Possibility rays will be missed if  $\theta_s$  is set too large.

Normally,  $0.2^\circ$  is chosen as  $\theta_s$ . Ray will reach the receiver if the ray intersects the circle. Three-dimensional path is determined by computing all the corresponding images, surface normals and reflecting planes to do the backward ray-tracing for the exact path from the satellite to the receiver. The exact point of diffraction at the corner will depend on the position of the receiver, and is determined by using the generalized Fermat's Principle, see McNamara et al. (1990).

#### 4 Experiment and Analysis

Tests were carried out on the roof of a building on the campus of the Hong Kong Polytechnic University in September 2004. Two receivers, both of which are of Leica System 530 model, 02 and 03 were used for the experiment. The antennas, both of which are of AT 502 model, were placed on the concrete pillars which were about 1.270 m in height as shown in Figure 1. The distance between the two receivers was about 3.020 m. Since the distance between the two receivers was short, almost all GPS errors but multipath and noise were eliminated. Double difference residuals calculated consisted of carrier phase noise and multipath error mainly.



Fig. 1 Test Area

Signal propagation model was used for accounting the characteristics of different multipath signals. An urban environment digital model, which was generated by the CAD and the Wireless Insite software, was required for modeling the multipath environment. The structure was decomposed into flat faces because of its simpler reflection and diffraction geometrical relationship. The footprints of the buildings covering an area of about 0.5 square kilometers were shown in Figure 2. A three-dimensional view of the study area together with a geo-referenced aerial photograph was shown in Figure 3. Figure 4 shows the typical propagation paths from satellites.

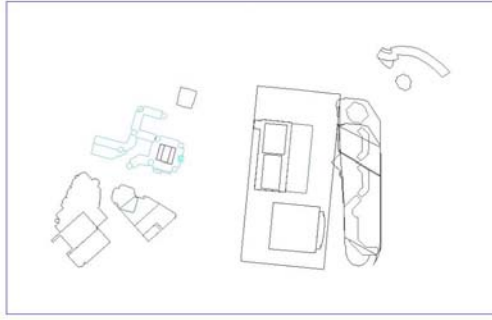


Fig. 2 Plan view of Test Area

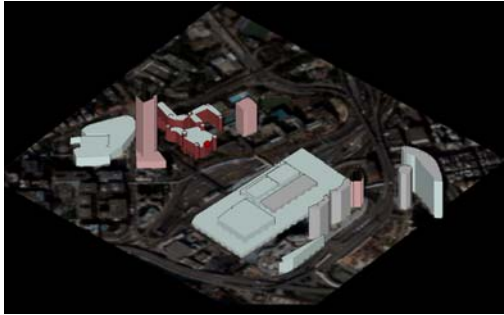


Fig. 3 3-D view of Test Area

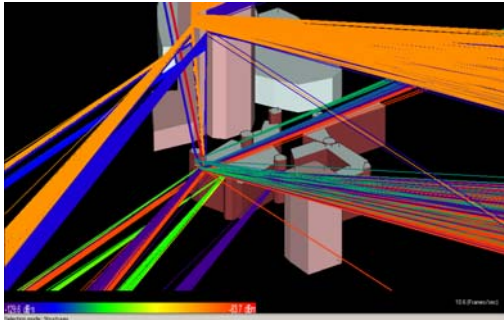


Fig. 4 Propagation paths from satellites

Carrier phase noise is random in nature, while the multipath error is oscillatory where the amplitude depends upon the material and the surface structure of the reflector as well as the distance between the reflector and the antenna, see Ray et al. (1998). Parameters such as time delay, path delay and damping factor were required for computing the phase multipath error. Those parameters were produced by GTD/UTD technique and outputted in ASCII text format for further processing by MatLAB. In this project, the modeling results were based on L1 GPS frequency, which is 1.575 GHz. Estimated carrier phase multipath errors for satellite 19, satellite 20 on two receivers were then calculated and shown in Figure 5 and Figure 6 respectively. Double difference carrier phase multipath errors for satellite 19 and satellite 20 were shown in Figure 7 and Figure 8 respectively. The reference satellite used in forming the double difference observables was satellite 1.

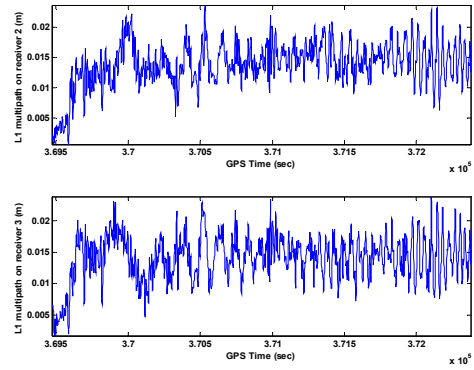


Fig. 5 L1 multipath for SV 19 on two receivers

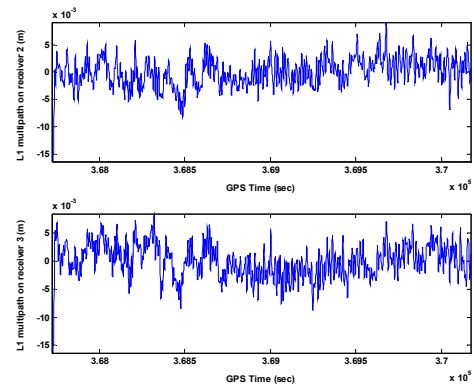


Fig. 6 L1 multipath for SV 20 on two receivers

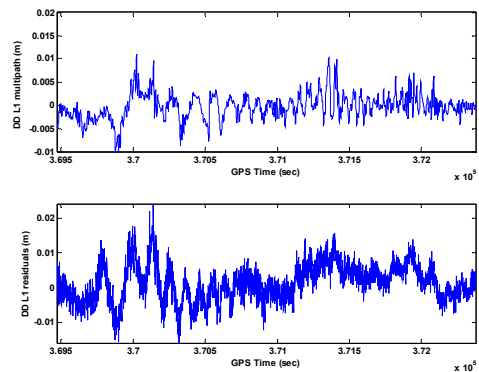


Fig. 7 DD L1 multipath for SV 19

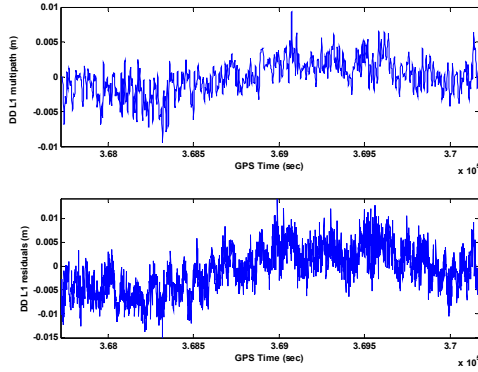


Fig. 8 DD L1 multipath for SV 20

Data collected in the previous day in almost the same period (around 236 seconds later than the following day) showed the similar trend (see Figure 9). This indicated that the DD residuals were due to multipath mainly. Table 1 shows the results of carrier phase errors calculated based on the residuals before and after applying multipath mitigation.

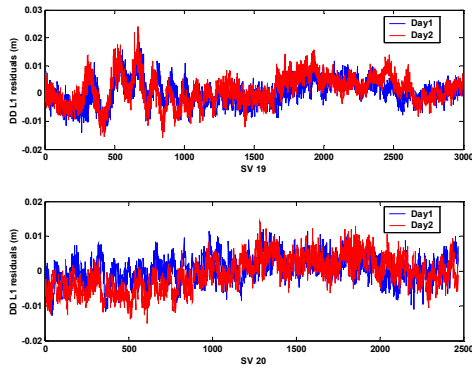


Fig. 9 L1 DD residuals in two consecutive days for SV 19 and SV 20

Tab. 1 Carrier phase DD statistics

| SV | Before Correction |          | After Correction |          | Improvement (%) |
|----|-------------------|----------|------------------|----------|-----------------|
|    | Mean (mm)         | RMS (mm) | Mean (mm)        | RMS (mm) |                 |
| 19 | 1.3               | 5.6      | 1.6              | 4.1      | 26.8            |
| 20 | -0.8              | 4.4      | -0.7             | 2.9      | 34.1            |

From the results, we could see that there is about 27% improvement of carrier phase errors for Satellite 19 after multipath mitigation. 34.1% improvement of carrier phase errors could be achieved for Satellite 20. Note that the damping factor, as we mentioned before, was not a constant. Figures 10 and 11 show the damping factors for satellite 19 and satellite 20 respectively.

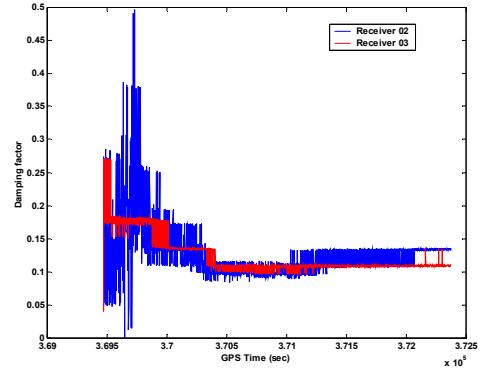


Fig. 10 Damping factor for SV 19 on two receivers

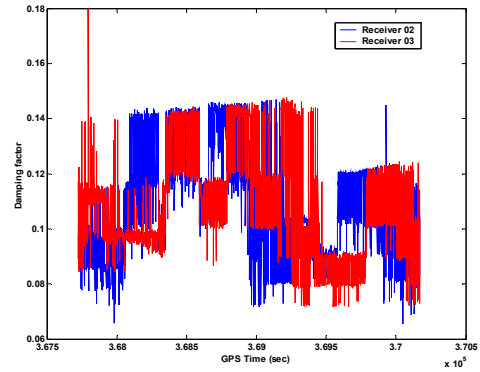


Fig. 11 Damping factor for SV 20 on two receivers

Position accuracy based on double difference solution after mitigating the carrier phase multipath was investigated. The impact of multipath mitigation on deviation of coordinates between GPS time 367723 and 372466 at 1 second epoch interval was studied. Deviations of coordinates before multipath mitigation and after multipath mitigation were shown in Figures 12, 13 and 14 respectively.

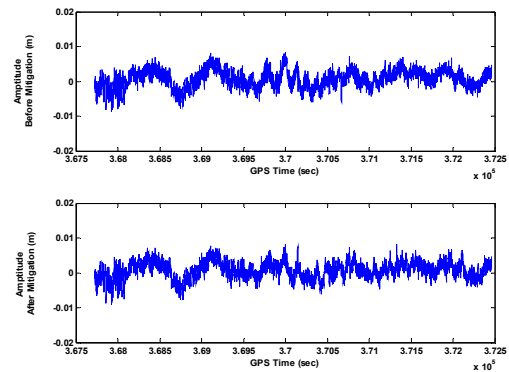


Fig. 12 Deviation of x-coordinates before and after multipath mitigation

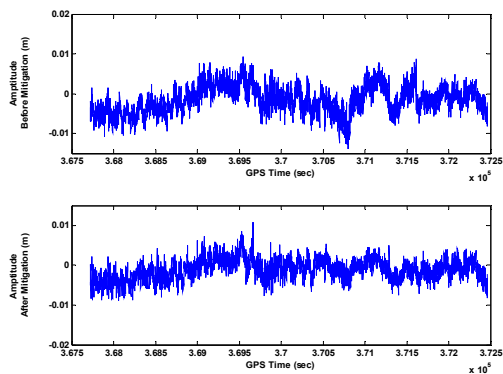


Fig. 13 Deviation of y-coordinates before and after multipath mitigation

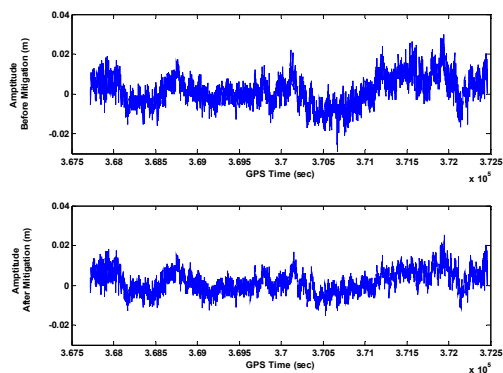


Fig. 14 Deviation of z-coordinates before and after multipath mitigation

From the above figures, we could see that there was a slight improvement in the x-coordinates. More apparent improvements were obtained in y- and z-coordinates. Table 2 gives some statistics of the results.

Tab. 2 Statistics results of positioning errors before and after multipath mitigation

| Correc-ti-on         | Standard Deviation (mm) |      |      |          |
|----------------------|-------------------------|------|------|----------|
|                      | x                       | y    | z    | 3D Error |
| Coordinate Deviation |                         |      |      |          |
| Before               | 2.5                     | 3.4  | 7.5  | 8.6      |
| After                | 2.4                     | 2.4  | 5.6  | 6.5      |
| Improvement (%)      | 4                       | 29.4 | 25.3 | 24.4     |

## 5 Conclusions

GPS carrier phase multipath has been modeled in a 3D urban environment. Parameters used for characterizing the multipath relative to the direct signal were estimated with the EM technique. Test results have shown that 26.8% and 34.1% errors in the carrier phase observations can be modelled and removed with the method. About

25% of the 3D positioning errors in static mode can be modelled and reduced.

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