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H- and H_2-Cordial Labeling of Some Graphs

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ABSTRACT

In this paper we prove that the join of two path graphs, two cycle graphs, Ladder graph and the tensor product $P_n \otimes P_2$ are H_2-cordial labeling. Further we prove that the join of two wheel graphs W_n and W_m, $n+m \equiv 0 \pmod 4$ admits a H-cordial labeling.

Keywords: H-Cordial; H_2-Cordial; Join of Two Graphs

1. Introduction

All graphs considered here are finite, simple and undirected. The origin of graph labelings can be attributed to Rosa. Several types of graph labeling have been investigated both from a purely combinatorial perspective as well as from an application point of view. Any graph labeling will have the following three common characteristics. A set of numbers from which vertex labels are chosen, $v_f(i)$ number of vertices of G having label i under f. $e_f(i)$ = number of edges of G having label i under f^*.

The concept of cordial labeling was introduced by I. Cahit, who called a graph G is cordial if there is a vertex labeling $f: v(G) \to \{0,1\}$ such that the induced labeling $f^*: E(G) \to \{0,1\}$ defined by

$f^*(xy) = |f(x) - f(y)|$, for all edges $xy \in E(G)$ and with the following inequalities holding

$|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

In [1] introduced the concept of H-cordial labeling. [1] calls a graph H-cordial if it is possible to label the edges with the numbers from the set $\{1,-1\}$ in such a way that, for some k, at each vertex v the sum of the labels on the edges incident with v is either k or $-k$ and the inequalities $|v_f(k) - v_f(-k)| \le 1$ and $|e_f(1) - e_f(-1)| \le 1$ are also satisfied where $v(i)$ and $e(j)$ are respectively, the number of vertices labeled with i and the number of edges labeled with j. He calls a graph H_n-cordial if it is possible to label the edges with the numbers from the set $\{\pm1, \pm2, \cdots, \pm n\}$ in such a way that at each vertex v, the sum of the labels on the edges incident with v is in the set $\{\pm1, \pm2, \cdots, \pm n\}$ and the inequalities $|v_f(i) - v_f(-i)| \le 1$ and $|e_f(i) - e_f(-i)| \le 1$ are also satisfied for each i with $1 \le i \le n$.

In [1] proved that $k_{n,n}$ is H-Cordial if and only if $n > 2$ and "n" is even; and $k_{m,n}, m \ne n$ is H-cordial if and only if $n \equiv 1 \pmod 4$, m is even and $m > 2$, $n > 2$.

In [2] proved that k_n is H-Cordial if and only if $n \equiv 0$ or $3 \pmod 4$ and $n \ne 3$.

W_n is H-cordial if and only if n is odd. k_n is not H_2-cordial if $n \equiv 1 \pmod 4$. Also [2] proved that every wheel has an H_2-cordial labeling.

In [3] several variations of graph labeling such as graceful, bigraceful, harmonious, cordial, equitable, humming etc. have been introduced by several authors. For definitions and terminologies in graph theory we refer to [4].

1.1. Definition: The join $G = G_1 + G_2$ of graph G_1 and G_2 with disjoint point sets V_1 and V_2 and edge sets E_1 and E_2 denoted by $G = G_1 + G_2$ is the graph union $G_1 \cup G_2$ together with all the edges joining v_1, v_2. If G_1 is (p_1,q_1) graph and G_2 is (p_2,q_2) graph then $G_1 + G_2$ is $(p_1 + p_2, q_1 + q_2 + p_1 + p_2)$.

1.2. Definition: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The Cartesian product of G_1 and G_2 which is denoted by $G_1 \times G_2$ is the graph with vertex set $v = v_1 \times v_2$ consisting of vertices $V = \{u = (u_1, u_2), v = (v_1, v_2)/u$ and v are adjacent in $G_1 \times G_2$ whenever $u_1 = v_1$ and u_2 adjacent to v_2 or u_1 adjacent to v_1 and $u_2 = v_2\}$.

1.3. Definition: The tensor product $G = G_1 \otimes G_2$ of graphs G_1 and G_2 with disjoint point sets v_1 and v_2 and edge sets E_1 and E_2 is the graph with vertex set $V_1 \times V_2$ such that (u_1, u_2) is adjacent to (v_1, v_2) whenever $\{u_1, v_1\} \in E_1$ and $\{u_2, v_2\} \in E_2$. If G_1 is (p_1, q_1) graph and G_2 is (p_2, q_2) graph, then $G_1 \otimes G_2$ is a $(P_1P_2, 2q_1q_2)$.

In this paper we have investigated some results on H- and H_2-cordial labeling for join of two graphs, Cartesian product and tensor product of some graphs.

2. Main Results

2.1. *Theorem*: The join of two path graphs P_n and P_m admits a H_2-cordial labeling when $n+m \equiv 1,2\,(\bmod 4)$.

***Proof*:** Let $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_m$ are the two vertex sets of the path graphs P_n and P_m. The edge set E_1 and E_2 is the graph union of P_n and P_m together with all the edges joining the vertex sets v_i and u_i, $i = 1, 2, \cdots, n$.

Define the edge labeling

$$f : E(G) \to \{1, -1\}$$

The edge matrix of $P_n + P_m$ is given in **Table 1**.

In view of the above labeling pattern we give the poof as follows:

1) When $n+m \equiv 1(\bmod 4)$

Consider the join of two path graphs P_3 and P_2.

Using **Table 1** the edge label matrix of $P_3 + P_2$ is given by

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \\ \end{array} \begin{array}{c} \begin{array}{cc} u_1 & u_2 \end{array} \\ \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \\ \begin{array}{cc} -1 & -1 \end{array} \end{array} \begin{array}{c} \\ 1 \\ 1-1 \\ -1 \\ \\ \end{array}$$

with respect to the above labeling total number of vertices labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by $v_f(1) = n-4$, $v_f(-1) = n-4$, $v_f(2) = n-3$ and $v_f(-2) = n-4$.

$\therefore |v_f(1) - v_f(-1)| + |v_f(2) - v_f(-2)| = |1|$, differ by one.

The total number of edges labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by

$$e_f(1) = \frac{n-1}{2},\ e_f(-1) = \frac{n+1}{2},\ e_f(2) = e_f(-2) = 0$$

$\therefore |e_f(1) - ef(-1)| + |e_f(2) - ef(-2)| = |1|$, differ by one.

Hence the join of two path graphs P_3 and P_2 admits a H_2-cordial labeling.

2) When $n+m \equiv 2(\bmod 4)$

Consider the join of two path graphs P_4 and P_2.

Using **Table 1**, the edge label matrix of $P_4 + P_2$ is given by

Table 1. Edge matrix of $P_n + P_m$.

$$\begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_m \end{array} \\ \begin{bmatrix} v_1u_1 & v_1u_2 & \cdots & v_1u_m \\ v_2u_1 & v_2u_2 & \cdots & v_2u_m \\ \vdots & \vdots & & \vdots \\ v_nu_1 & v_nu_2 & \cdots & v_nu_m \end{bmatrix} \\ \begin{array}{cccc} u_1u_2 & u_1u_2, u_2u_3 & \cdots & u_{m-1}u_m \end{array} \end{array} \begin{array}{c} \\ v_1v_2 \\ v_1v_2, v_2v_3 \\ \vdots \\ v_{n-1}v_n \\ \\ \end{array}$$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cc} u_1 & u_2 \end{array} \\ \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \\ \begin{array}{cc} 1 & 1 \end{array} \end{array} \begin{array}{c} \\ 1 \\ 1-1 \\ -1-1 \\ -1 \\ \\ \end{array}$$

In view of the above labeling pattern the total number of edges labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by $e_f(1) = n/2, e_f(-1) = n/2, e_f(2) = 0, e_f(-2) = 0$

$\therefore |e_f(1) - e_f(-1)| + |e_f(2) - e_f(-2)| = 0$, differ by zero.

The total number of vertices labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by

$v_f(1) = n-4, v_f(-1) = n-4, v_f(2) = n-5$ and $v_f(-2) = n-5$.

$\therefore |v_f(1) - v_f(-1)| + |v_f(2) - v_f(-2)| = 0$, differ by zero.

Thus in each cases we have

$|v_f(1) - v_f(-1)| + |v_f(2) - v_f(-2)| \le 1$ and

$|e_f(1) - e_f(-1)| + |e_f(2) - e_f(-2)| \le 1$.

Hence the join of two path graphs P_4 and P_2 admits a H_2-cordial labeling of graphs.

In **Figure 1** illustrates the H_2-cordial labeling on $P_4 + P_2$. Among the twelve edges, six edges receive the label +1 and the other six edges receive the label -1. The induced vertex labels are given in the figure.

2.2. *Theorem*: The join of two cycle graphs C_n and C_m admits a H_2-cordial labeling when

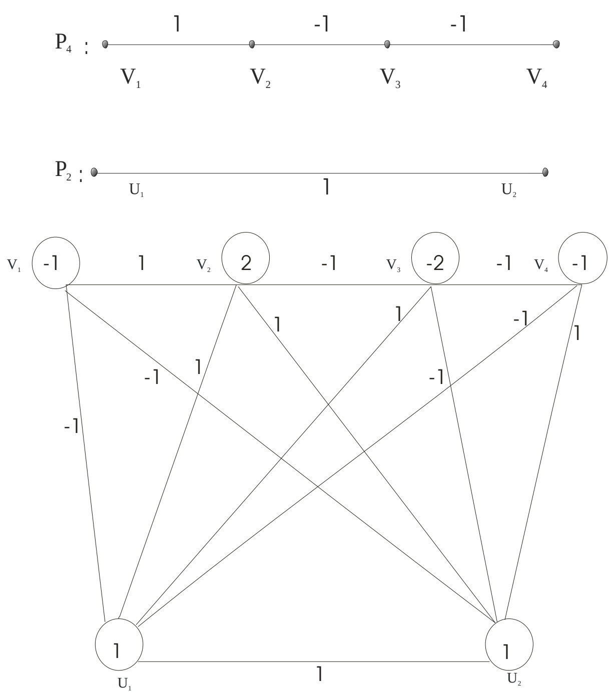


Figure 1. H_2-cordial labeling on $P_4 + P_2$.

$n+m\equiv 1,3(\bmod 4)$, $n,m\geq 3$.

***Proof*:** Let $v_1, v_2,\cdots,v_n$ and $u_1, u_2,\cdots,u_m$ are the vertex set of cycles c_n and c_m respectively. The edge sets E_1 and E_2 is the graph union of c_n and c_m together with all the edges joining the vertex sets $v_1, v_2,\cdots,v_n$ and $u_1, u_2,\cdots,u_m$.

We note that $|V(G)| = p_1 + p_2$ and

$|E(G)| = q_1 + q_2 + p_1 p_2$.

Define $f : E(G)\to\{1,-1\}$.

The edge matrix table of $c_n + c_m$ is given in **Table 2**.

In view of the above labeling pattern we give the proof as follows.

Case (1) when $n+m\equiv 3(\bmod 4)$, $n,m\geq 3$.

Consider the join of two cycle graphs c_3 and c_4.

Using **Table 2** the edge lable matrix of c_3 and c_4 is given by

$$\begin{array}{cc c}
 & \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} & \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix} & \begin{array}{c} -11 \\ -11 \\ 1 \end{array} \\
 & \begin{array}{cccc} -11 & -11 & 1-1 & -11 \end{array} &
\end{array}$$

In view of the above labeling pattern the total number of edges labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by

$$e_f(1)=\frac{n+1}{2},\ e_f(-1)=\frac{n+2}{2},\ e_f(2)=0,\ e_f(-2)=0.$$

$\therefore |e_f(1)-e_f(-1)|+|e_f(2)-e_f(-2)|=|1|$, differ by one.

The total number vertices labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by

$v_f(1)=n-5, v_f(-1)=n-5, v_f(2)=n-5$ and

$v_f(-2)=n-6$.

$\therefore |v_f(1)-v_f(-1)|+|v_f(2)-v_f(-2)|=|1|$, differ by one.

Thus in each cases we have

$|v_f(1)-v_f(-1)|+|v_f(2)-v_f(-2)|\leq 1$ and

$\therefore |e_f(1)-e_f(-1)|+|e_f(2)-e_f(-2)|\leq 1$.

Hence the join of two cycle graphs c_3 and c_4 admits a H_2-cordial labeling.

Table 2. Edge matrix of $c_n + c_m$.

$$\begin{array}{cc c}
 & \begin{array}{cccccc} u_1 & u_2 & u_3 & \cdots & & u_m \end{array} & \\
\begin{array}{c} v_1 \\ v_2 \\ \vdots \\ v_n \end{array} & \begin{bmatrix} v_1u_1 & v_1u_2 & v_1u_3 & \cdots & & v_1u_m \\ v_2u_1 & v_2u_2 & v_2u_3 & \cdots & & v_2u_m \\ \vdots & \vdots & \vdots & & & \vdots \\ v_nu_1 & v_nu_2 & \cdots & \cdots & & v_nu_m \end{bmatrix} & \begin{array}{c} v_1v_2, v_1v_n \\ v_1v_2, v_2v_3 \\ \vdots \\ v_1v_n, v_{n-1}v_n \end{array} \\
 & \begin{array}{cccc} u_1u_2, u_1u_m & u_1u_2, u_2u_3 & \cdots & \cdots\ u_1u_m, u_{m-1}u_m \end{array} &
\end{array}$$

Case (2) when $n+m\equiv 1(\bmod 4)$, $n,m>3$.

Consider the join of two cycle graphs c_5 and c_4.

Using **Table 2** the edge label matrix of $c_5 + c_4$ is given by

$$\begin{array}{cc c}
 & \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix} & \begin{array}{c} 11 \\ 1-1 \\ -11 \\ 1-1 \\ 1-1 \end{array} \\
 & \begin{array}{cccc} 1-1 & 1-1 & -11 & -11 \end{array} &
\end{array}$$

In view of the above labeling pattern the total number of edges labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by

$$e_f(1)=\frac{n-1}{2},\ e_f(-1)=\frac{n+1}{2},\ e_f(2)=e_f(-2)=0.$$

$\therefore |e_f(1)-e_f(-1)|+|e_f(2)-e_f(-2)|=0$, differ by zero.

The total number of vertices labeled with $1'^s, -1'^s, 2'^s$ and $-2'^s$ are given by

$v_f(1)=n-7, v_f(-1)=n-7$

$v_f(2)=n-6, v_f(-2)=n-7$

$\therefore |v_f(1)-v_f(-1)|+|v_f(2)-v_f(-2)|=|1|$, differ by one.

Thus in each cases we have

$\therefore |v_f(1)-v_f(-1)|+|v_f(2)-v_f(-2)|\leq 1$ and

$\therefore |e_f(1)-e_f(-1)|+|e_f(2)-e_f(-2)|\leq 1$.

Hence the join of two cycle graphs c_5 and c_4 admits a H_2-cordial labeling.

In **Figure 2** illustrates the H_2-cordial labeling on $C_5 + C_4$. Among the 29 edges, fifteen edges receive the label +1 and the remaining fourteen edges receive the label -1. The induced vertex labels are given in the figure.

2.3. *Theorem*: The join of two wheel graphs W_n and W_m admits a H-cordial labeling when $n+m\equiv 0(\bmod 4)$.

***Proof*:** Let $v_1, v_2,\cdots,v_n$ and $u_1, u_2,\cdots,u_m$ are the vertex set of the wheel graph W_n and W_m. The edge set E_1 and E_2 is the graph union of W_n and W_m together with all the edges joining the vertex set $v_1, v_2,\cdots,v_n$ and $u_1, u_2,\cdots,u_m$. We note that $|V(G)| = p_1 + p_2$ and $|E(G)| = q_1 + q_2 + p_1 p_2$.

Define $f : E(G)\to\{1,-1\}$

The edge matrix is given in **Table 3**.

In the view of the above labeling pattern we give the proof as follows:

when $n+m\equiv 0(\bmod 4)$

Consider the join of two wheel graphs W_4 and W_4. Using **Table 3** the edge label matrix of $W_4 + W_4$ is given by

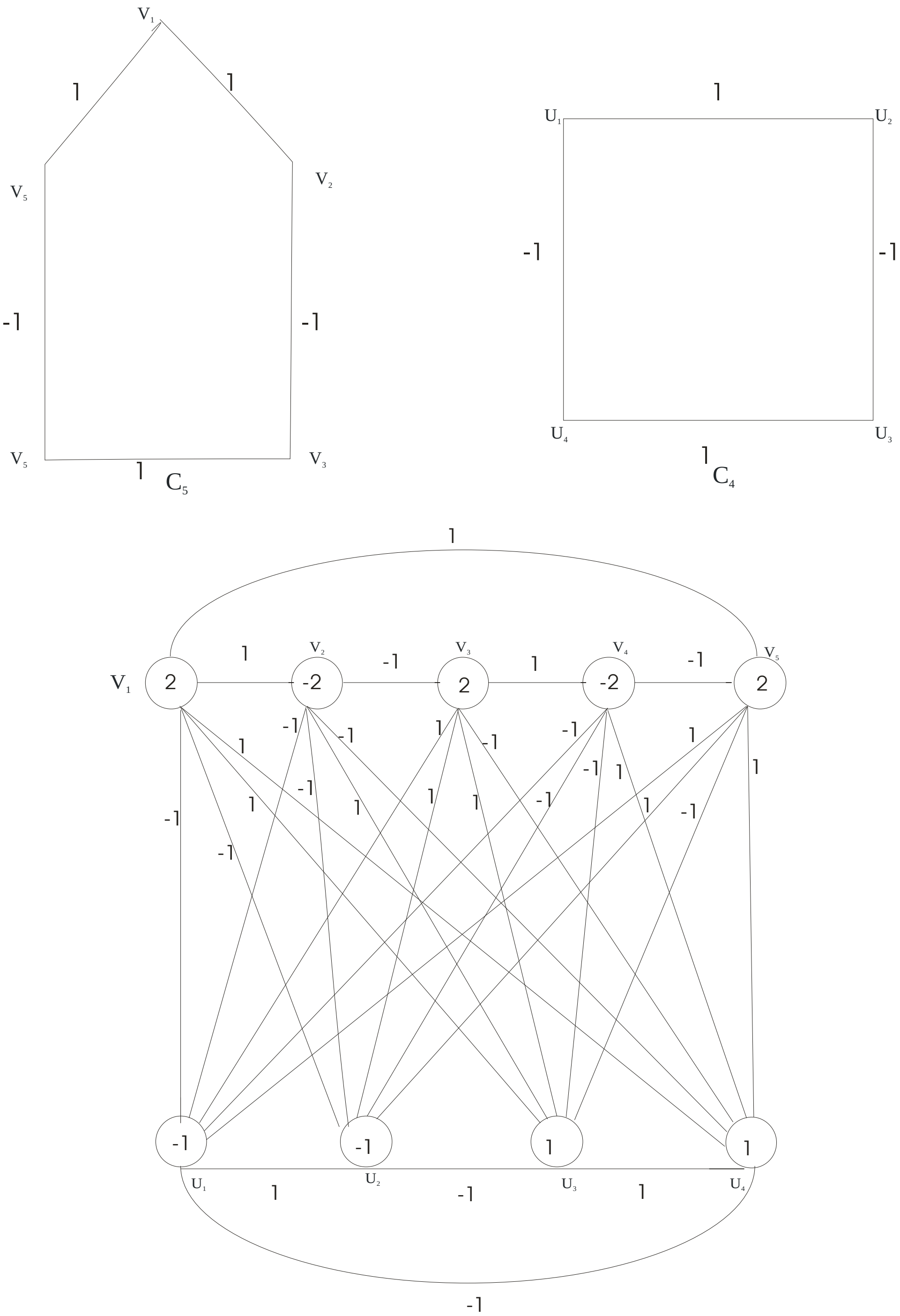


Figure 2. H_2-cordial labeling on $C_5 + C_4$.

Table 3. Edge matrix of $W_n + W_m$.

$$
\begin{array}{ccccccc}
 & u_1 & u_2 & \cdots & u_m & \\
\begin{matrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{matrix} & \multicolumn{4}{c}{\begin{bmatrix} v_1u_1 & v_1u_2 & \cdots & v_1u_m \\ v_2u_1 & v_2u_2 & \cdots & v_2u_m \\ \vdots & \vdots & & \vdots \\ v_nu_1 & v_nu_2 & \cdots & v_nu_m \end{bmatrix}} & \begin{matrix} v_1v_2, v_1, v_3 \cdots v_1v_n \\ v_1v_2, v_2, v_3 \cdots v_2v_n \\ \vdots \\ v_1v_n, v_2v_n \cdots v_{n-1}v_n \end{matrix} \\
 & u_1u_2, u_1u_3 \cdots u_1u_m & u_1u_2, u_2u_3 \cdots u_1u_m & \cdots & u_1u_mu_2u_m \cdots u_{m-1}u_m &
\end{array}
$$

$$\begin{array}{cccccc} & u_1 & u_2 & u_3 & u_4 & \\ v_1 & -1 & -1 & -1 & 1 & 11-1 \\ v_2 & -1 & 1 & 1 & -1 & 1-1-1 \\ v_3 & 1 & -1 & 1 & -1 & 1-11 \\ v_4 & 1 & 1 & 1 & -1 & -1-11 \\ & -1-11 & -1-11 & -1-11 & 1\ 11 & \end{array}$$

In view of the above labeling pattern we give the proof as follows.

The total number of edges labeled with $1's$ and $-1's$ are given by $e_f(1) = n/2$, $e_f(-1) = n/2$

$\therefore \left|e_f(1) - e_f(-1)\right| = 0$, differ by zero. The total number of vertices labeled with $1's$ and $-1's$ are given by $v_f(1) = n/2$ and $v_f(-1) = n/2$

$\therefore \left|v_f(1) - v_f(-1)\right| = 0$, differ by zero.

Thus in each cases we have $\left|v_f(1) - v_f(-1)\right| \le 1$ and $\left|e_f(1) - e_f(-1)\right| \le 1$.

Hence the join of two wheel graphs w_4 and w_4 admits a H-cordial labeling.

In **Figure 3** illustrates the H-cordial labeling on W_4 + W_4. Among the twenty eight edges, fourteen edges receive the label +1 and the other fourteen edges receive the label -1. The induced vertex labels are shown in the figure.

2.4 ***Theorem***: $L_n = P_n \times P_2$ (also known as ladder graph) is H_2-Cordial labeling for even *n*.

Proof: Let *G* be the graph $P_n \times P_2$ where *n* is even and $V(G) = \{V_{ij} \quad i = 1, 2, \cdots, n \text{ and } j = 1, 2\}$ be the vertices of *G*.

We note that $|V(G)| = 2n$ and $|E(G)| = 3n - 2$.

Define $f : E(G) \to \{1, -1\}$ as follows

Case (1) When $n \equiv 0 \pmod 4$

For $1 \le i, k \le n-1$

$f(v_{i_1}, v_{k_1}) = 1$

For $n-1 < i, k \le n$

$f(v_{i_1}, v_{k_1}) = -1$

For $1 \le i, k \le n-1$

$f(v_{i2,}, v_{k2}) = 1$

For $n-1 < i, k \le n$

$f(v_{i2,}, v_{k2}) = -1$

For $1 \le i \le n-1$

$f(v_{i1}, v_{i2}) = -1$

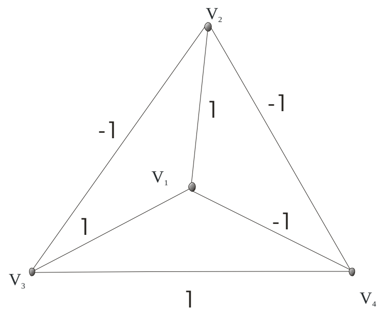
For $n-1 < i \le n$

$f(v_{i1}, v_{i2}) = 1$

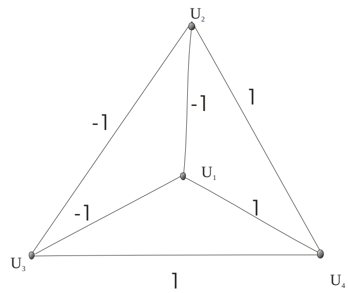
Case (2) when $n \equiv 2 \pmod 4$

For $1 \le i, k \le n-2$

$f(v_{i1}, v_{k1}) = 1$



W_4



W_4

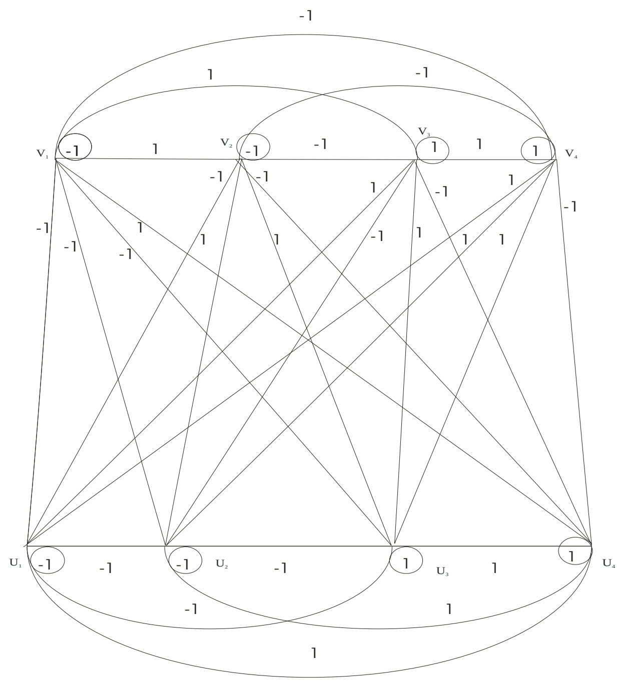


Figure 3. H-cordial labeling on W_4 + W_4.

$f(v_{i2}, v_{k2}) = 1$

For $n-2 < i, k \le n$

$f(v_{i1}, v_{k1}) = -1$

$f(v_{i2}, v_{k2}) = -1$

For $1 \le i \le n-2$

$f(v_{i1}, v_{i2}) = -1$

For $n-2<i\le n$,

$f\left(v_{i1},v_{i2}\right)=1$

In view of the above defined labeling pattern we give the proof as follows.

The total number of edges labeled with $1's,-1's,2's$ and $-2's$ are given by $e_f\left(1\right)=n/2$ $e_f\left(-1\right)=n/2$, $e_f\left(2\right)=e_f\left(-2\right)=0$.

$\therefore\left|e_f\left(1\right)-e_f\left(-1\right)\right|+\left|e_f\left(2\right)-e_f\left(-2\right)\right|=\left|0\right|$, differ by zero.

The total number of vertices labeled with $1's$ and $-1's$, $2's$ and $-2's$ are given by

$v_f\left(1\right)=v_f\left(-1\right)=\left(n-4\right)/2;$

$v_f\left(-2\right)=\left(n-4\right)/2,v_f\left(2\right)=\left(n-4\right)/2.$

$\therefore\left|v_f\left(1\right)-v_f\left(-1\right)\right|+\left|v_f\left(2\right)-v_f\left(-2\right)\right|=0$ differ by zero.

Thus in each cases we have

$$\left|v_f\left(1\right)-v_f\left(-1\right)\right|+\left|v_f\left(2\right)-v_f\left(-2\right)\right|\le 1$$

$$\left|e_f\left(1\right)-e_f\left(-1\right)\right|+\left|e_f\left(2\right)-e_f\left(-2\right)\right|\le 1$$

Hence the ladder graph $P_n\times P_2$ admits a H_2-cordial labeling.

In **Figure 4** illustrates the H_2-cordial labeling on $P_4\times P_2$. Among the ten edges, five edges receive the label +1 and other five edges receive the label -1. The induced vertex labels are shown in the figure.

2.5 *Theorem*: The tensor product $P_n\otimes P_2$ is H_2-cordial labeling.

***Proof*:** Let *G* be the graph $P_n\otimes P_2$ and $V\left(G\right)=\left\{u_iv_j \quad i=1,2,\cdots,n \text{ and } j=1,2\right\}$ be the vertices of *G*.

We note that $\left|V\left(G\right)\right|=2n$ and $\left|E\left(G\right)\right|=2n-2$

Define $f:E\left(G\right)\to\left\{1,-1\right\}$ two cases are to be considered.

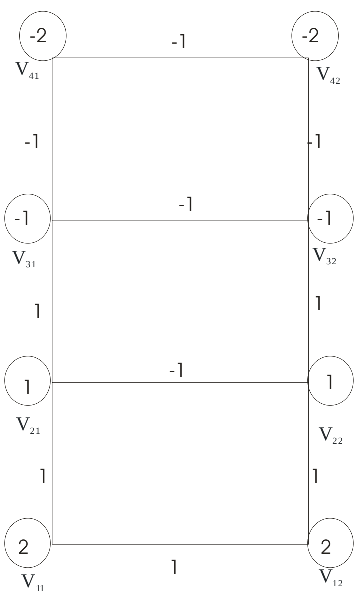


Figure 4. H_2-cordial labeling on $P_4\times P_2$.

Case (1). When *n* is even

For $1\le i\le n$

$$f\left(u_iv_1,u_{i+1}v_2\right)\begin{cases}=1, & \text{if } i\equiv 1\left(\text{mod }2\right)\\ =-1, & \text{if } i\equiv 1\left(\text{mod }2\right)\end{cases}$$

For $1\le i\le n$

$$f\left(u_iv_2,u_{i+1}v_1\right)\begin{cases}=-1, & \text{if } i\equiv 1\left(\text{mod }2\right)\\ =1, & \text{if } i\equiv 1\left(\text{mod }2\right)\end{cases}$$

Case (2) When *n* is odd

For $1\le i\le n$

$$f\left(u_iv_1,u_{i+1}v_2\right)\begin{cases}=1, & \text{if } i\equiv 1,3\left(\text{mod }4\right)\\ =-1, & \text{if } i\equiv 0,2\left(\text{mod }4\right)\end{cases}$$

For $1\le i\le n$

$$f\left(u_iv_2,u_{i+1}v_1\right)\begin{cases}=-1, & \text{if } i\equiv 1,3\left(\text{mod }4\right)\\ =1, & \text{if } i\equiv 0,2\left(\text{mod }4\right)\end{cases}$$

In view of the above defined labeling pattern we give the proof as follows.

The total number of edges labeled with $1's,-1's,2's$ and $-2's$ are given by $e_f\left(1\right)=n/2$ $e_f\left(-1\right)=n/2$, $e_f\left(2\right)=e_f\left(-2\right)=0$.

$\therefore\left|e_f\left(1\right)-e_f\left(-1\right)\right|+\left|e_f\left(2\right)-e_f\left(-2\right)\right|=\left|0\right|$, differ by zero.

The total number of vertices labeled with $1's,-1's,2's$ and $-2's$ are given by

$v_f\left(1\right)=v_f\left(-1\right)=\left(n-8\right)/2;$

$v_f\left(-2\right)=\left(n-4\right)/2,v_f\left(2\right)=\left(n-4\right)/2.$

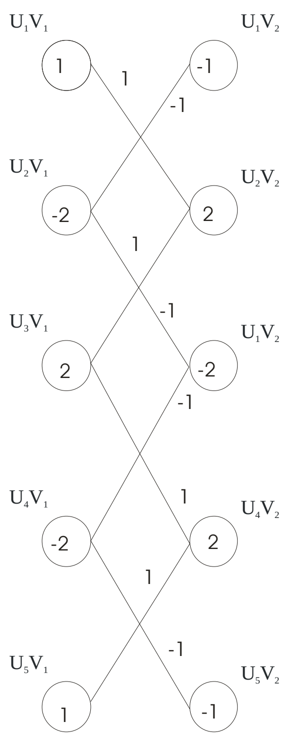


Figure 5. H_2-cordial labeling on $P_4\otimes P_2$.

$\therefore \left|v_f(1)-v_f(-1)\right|+\left|v_f(2)-v_f(-2)\right|=0$ differ by zero.

Thus in each cases we have

$\left|v_f(1)-v_f(-1)\right|+\left|v_f(2)-v_f(-2)\right|\le 1$

$\left|e_f(1)-e_f(-1)\right|+\left|e_f(2)-e_f(-2)\right|\le 1$

Hence the tensor product $P_n \otimes P_2$ admits a H_2-cordial labeling.

In **Figure 5** illustrates the H_2-cordial labeling on $P_5 \otimes P_2$. Among the eight edges four edges receive the label +1 and the other four edges receive the label -1. The induced vertex labels are shown in the figure.

3. Concluding Remarks

Here we investigate H- and H_2-cordial labeling for join of path graphs, cycle graphs, wheel graphs, Cartesian product and tensor product. Similar results can be derived for other graph families and in the context of different graph labeling problem is an open area of research.

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