*Journal of Modern Physics*, **2012, 3, 604-609**
http://dx.doi.org/10.4236/jmp.2012.37082 Published Online July 2012 (http://www.SciRP.org/journal/jmp)

# Nonlinear Propagation of Dust-Ion-Acoustic Waves in a Degenerate Dense Plasma

**M. S. Zobaer, N. Roy, A. A. Mamun**
Department of Physics, Jahangirnagar University, Dhaka, Bangladesh
Email: salim.zobaer@yahoo.com, niparoybd@gmail.com, mamun_phys@yahoo.co.uk

Received April 28, 2012; revised May 25, 2012; accepted June 16, 2012

## ABSTRACT

Nonlinear propagation of dust-ion-acoustic waves in a degenerate dense plasma (with the constituents being degenerate, for both the limits non-relativistic or ultra-relativistic) have been investigated by the reductive perturbation method. The Korteweg de-Vries (K-dV) equation and Burger's equation have been derived, and the numerical solutions of those equations have been analyzed to identify the basic features of electrostatic solitary and shock structures that may form in such a degenerate dense plasma. The implications of our results in compact astrophysical objects, particularly, in white dwarfs, have been briefly discussed.

**Keywords:** Degenerate Plasma; Dust-Ion-Acoustic Waves; K-dV Equation; Burzer's Equation

In present days, most theoretical concerns are to understand the environment of the compact objects having their interiors supporting themselves via degenerate pressure. The degenerate pressure, which arises due to the combine effect of Pauli's exclusion principle (Wolfgang Ernst Pauli, 1925) and Heisenberg's uncertainty principle (Werner Heisenberg, 1927), depends only on the fermion number density, but not on it's temperature. This degenerate pressure has a vital role to study the electrostatic perturbation in matters existing in extreme conditions [1-7]. The extreme conditions of matter are caused by significant compression of the interstellar medium. High density of degenerate matter in these compact objects (which are, in fact, "relics of stars") is one of these extreme conditions. These interstellar compact objects, having ceased burning thermonuclear fuel and thereby no longer generate thermal pressure, are contracted significantly, and as a result, the density of their interiors becomes extremely high to provide non-thermal pressure through degenerate pressure of their constituent particles and particle-particle interaction. The observational evidence and theoretical analysis imply that these compact objects support themselves against gravitational collapse by degenerate pressure.

The degenerate electron number density in such a compact object is so high (e.g. in white dwarfs it can be of the order of $10^{30}$ cm$^{-3}$, even more [8]) that the electron Fermi energy is comparable to the electron mass energy and the electron speed is comparable to the speed of light in vacuum. The equation of state for degenerate electrons in such interstellar compact objects are mathematically explained by Chandrasekhar [4] for two limits, namely non-relativistic and ultra-relativistic limits. The interstellar compact objects provide us cosmic laboratories for studying the properties of the medium (matter), as well as waves and instabilities [9-22] in such a medium at extremely high densities (degenerate state) for which quantum as well as relativistic effects become important [9,21]. The quantum effects on linear [16,18,22] and nonlinear [17,20] propagation of electrostatic and electromagnetic waves have been investigated by using the quantum hydrodynamic (QHD) model [9,21], which is an extension of classical fluid model in a plasma, and by using the quantum magneto-hydrodynamic (QMHD) model [16-20], which involve spin $-\frac{1}{2}$ and one-fluid MHD equations.

Recently, a number of theoretical investigations have also been made of the nonlinear propagation of electrostatic waves in degenerate quantum plasma by a number of authors, e.g. Hass [23], Misra and Samanta [24], Mistra *et al.* [25] etc. However, these investigations are based on the electron equation of state valid for the non-relativistic limit. Some investigations have been made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma based on the degenerate electron equation of state valid for ultra-relativistic limit [8]. To the best of our knowledge, no theoretical investigation has been developed to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits. Therefore, in our present investigation, we consider a degenerate dense plasma containing non-relativistic degenerate cold ion fluid, both non-relativistic and ultra-relativistic degenerate electrons, and negatively charged static dust (as it is possible to be some heavy element in the system) to study the basic features of the solitary waves in such degenerate dense plasma. The model is relevant to compact interstellar objects (e.g., white dwarf, neutron star, etc.).

We consider the propagation of electrostatic perturbation in a degenerate dense plasma containing non-relativistic degenerate cold ion and degenerate electron fluids. Thus, at equilibrium we have $n_{i0} = n_{e0} + Z_d n_{do}$, where $n_{i0}\left(n_{e0}\right) Z_d n_{do}$ is the ion (electron) dust number density at equilibrium with $Z_d\left(n_{do}\right)$ be the charge per dust grain (number of dust per unit volume). The nonlinear dynamics of the electrostatic waves propagating in such a degenerate plasma is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i u_i\right) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i}\frac{\partial n_i^{\alpha}}{\partial x} + \eta \frac{\partial^2 u_i}{\partial x^2} = 0, \tag{2}$$

$$n_e \frac{\partial \varphi}{\partial x} - K_2 \frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \left(1-\mu\right) n_e - n_i + \mu, \tag{4}$$

where $n_i\left(n_e\right)$ is the ion (electron) number density normalized by its equilibrium value $n_{i0}\left(n_{e0}\right)$, $u_i$ is the ion fluid speed normalized by $C_i = \left(m_e c^2/m_i\right)^{1/2}$ with $m_e$ ($m_i$) being the electron (ion) rest mass mass and $c$ being the speed of light in vacuum, $\phi$ is the electrostatic wave potential normalized by $m_e c^2/e$ with $e$ being the magnitude of the charge of an electron, the time variable ($t$) is normalized by $\omega_{pi} = \left(4\pi n_0 e^2/m_i\right)^{1/2}$, and the space variable ($x$) is normalized by $\lambda_s = \left(m_e c^2/4\pi n_0 e^2\right)^{1/2}$. The coefficient of viscosity $\eta$ is a normalized quantity given by $\omega_{pi}\lambda_s^2 m_s n_{s0}$ (with $s = i, e$) and $\mu$, the ratio the number density of charged dust and ion. The constants $K_1 = n_{i0}^{\alpha-1} K_i/m_i c^2$ and $K_2 = n_{e0}^{\gamma-1} K_e/m_i c^2$. The equations of state used here are given by

$$P_i = K_i n_i^{\alpha}, \tag{5}$$

where

$$\alpha = \frac{5}{3};\ K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{6}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2\times10^{-10}$ cm, and $\hbar$ is the Planck constant divided by $2\pi$). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{7}$$

where

$$\gamma = \alpha; K_e = K_i \text{ for non-relativistic limit, and} \tag{8}$$

$$\gamma = \frac{4}{3};\ K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{9}$$

in the ultra-relativistic limit [1,2,4,8].

To examine electrostatic perturbations propagating in the ultra-relativistic degenerate dense plasma due to the effect of dispersion by analyzing the outgoing solutions of (1)-(4), we first introduce the stretched coordinates [26]

$$\zeta = -\varepsilon^{1/2}\left(x + V_p t\right), \tag{10}$$

$$\tau = \varepsilon^{3/2} t, \tag{11}$$

where $V_p$ is the wave phase speed ($\omega/k$ with $\omega$ being angular frequency and $k$ being the wave number of the perturbation mode), and $\varepsilon$ is a smallness parameter measuring the weakness of the dispersion ($0<\varepsilon<1$). We then expand $n_i$, $n_e$, $u_i$, and $\phi$, in power series of $\varepsilon$:

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \cdots, \tag{12}$$

$$n_e = 1 + \varepsilon n_e^{(1)} + \varepsilon^2 n_e^{(2)} + \cdots, \tag{13}$$

$$u_i = \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \cdots, \tag{14}$$

$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \cdots, \tag{15}$$

and develop equations in various powers of $\varepsilon$. To the lowest order in $\varepsilon$, (1)-(15) give $u_i^{(1)} = -V_p \phi^{(1)}/\left(V_p^2 - K_1'\right)$, $n_i^{(1)} = \phi^{(1)}/\left(V_p^2 - K_1'\right)$, $n_e^{(1)} = \phi^{(1)}/K_2'$, and $V_p = \sqrt{K_2'/\left(1-\mu\right) + K_1'}$ where $K_1' = \alpha K_1/\left(\alpha - 1\right)$ and $K_2' = \gamma K_2/\left(\gamma-1\right)$. The relation $V_p = \sqrt{K_2'/\left(1-\mu\right) + K_1'}$ represents the dispersion relation for the dust ion-acoustic type electrostatic waves in the degenerate plasma under consideration.

We are interested in studying the nonlinear propagation of these dispersive dust ion-acoustic type electrostatic waves in a degenerate plasma. To the next higher order in $\varepsilon$, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}\left[u_i^{(2)} + n_i^{(1)} u_i^{(1)}\right] = 0, \tag{16}$$

$$\begin{aligned} &\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \zeta} - u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} - \frac{\partial \phi^{(2)}}{\partial \zeta} \\ &- K_1' \frac{\partial}{\partial \zeta}\left[n_i^{(2)} + \frac{\left(\alpha-2\right)}{2}\left(n_i^{(1)}\right)^2\right] = 0, \end{aligned} \tag{17}$$

 

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[ n_e^{(2)} + \frac{(\gamma-2)}{2}\left(n_e^{(1)}\right)^2 \right] = 0, \quad (18)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = (1-\mu) n_e^{(2)} - n_i^{(2)}. \quad (19)$$

Now, combining (16)-(19) we deduce a Korteweg-de Vries equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \quad (20)$$

where

$$A = \frac{\left(V_p^2 - K_1'\right)^2}{2V_p}\left[ \frac{3V_p^2 + K_1'(\alpha-2)}{\left(V_p^2 - K_1'\right)^3} + \frac{(1-\mu)(\alpha-2)}{K_1'^2} \right], \quad (21)$$

$$B = \frac{\left(V_p^2 - K_1'\right)^2}{2V_p}. \quad (22)$$

The stationary solitary wave solution of (20) is

$$\phi^{(1)} = \phi_m sech^2\left(\frac{\xi}{\Delta}\right), \quad (23)$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude, $\phi_m = 3u_0/A$, and the width, $\Delta = \left(4B/u_0\right)^{1/2}$. It is obvious from (21) and (32) that the degenerate plasma under consideration supports compressive electrostatic solitary waves which are associated with a positive potential. It is obvious from (21)-(32) that the amplitude [$\phi_m$] of these solitary structures depends on the density parameter $\mu$, *i.e.*, the ratio of electron to ion number density. The electrostatic solitary profiles are shown in **Figures 1-2**. This is obvious that the profiles are quite different from those obtained from the previous investigation [8]. And the potential for non-relativistic degenerate ion fluid and ultra-relativistic degenerate electron fluid is different from that when both the particles follow the same limit.

We now turn to (20) with the term $\phi^{(1)}$ which changes proportionally with the parameter $\mu$. We have numerically solved (20), and have studied the effects of $\mu$ on electrostatic solitary structures in both non-relativistic and ultra-relativistic degenerate electrons (ion always being non-relativistic degenerate). The results of the first case are depicted in **Figures 1** and **2**.

To examine electrostatic perturbations propagating in the relativistic degenerate dense plasma due to the effect of dissipation by analyzing the outgoing solutions of (1)-(4), we now introduce the new set of stretched coordinates [26]

$$\zeta = -\varepsilon\left(x + V_p t\right), \quad (24)$$

$$\tau = \varepsilon^2 t, \quad (25)$$

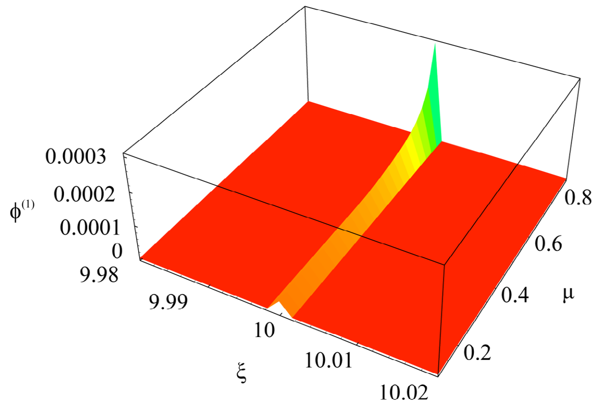


**Figure 1. The solitary profiles represented by (32) with $u_0$ = 1 and both the constituent particles non-relativistic.**

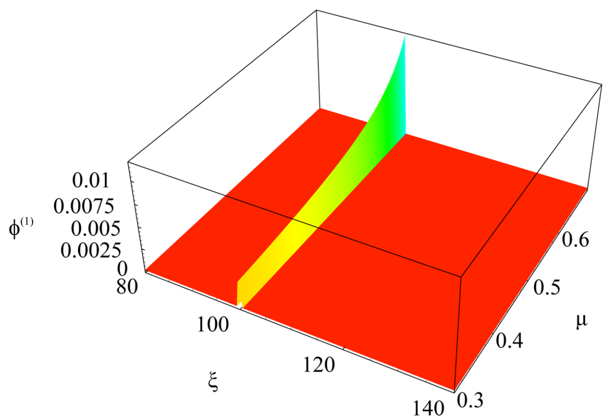


**Figure 2. (Colour online) The solitary profiles represented by (32) with $u_0$ = 0.1 and both the constituent particles non-relativistic.**

To the lowest order in $\varepsilon$, (1)-(9), (24), (25), and (12)-(15) give the same results as we have had for the solitary waves.

To the next higher order in $\varepsilon$, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}\left[ u_i^{(2)} + n_i^{(1)} u_i^{(1)} \right] = 0, \quad (26)$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \zeta} - u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} - \frac{\partial \phi^{(2)}}{\partial \zeta} + \eta \frac{\partial^2}{\partial \zeta^2} u_i^{(1)} - K_1' \frac{\partial}{\partial \zeta}\left[ n_i^{(2)} + \frac{(\alpha-2)}{2}\left(n_i^{(1)}\right)^2 \right] = 0, \quad (27)$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[ n_e^{(2)} + \frac{(\gamma-2)}{2}\left(n_e^{(1)}\right)^2 \right] = 0, \quad (28)$$

$$0 = (1-\mu) n_e^{(2)} - n_i^{(2)}. \quad (29)$$

Now, combining (26)-(29) we deduce a Burger's equation

 

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}, \tag{30}$$

where the value of $A$ is the same as before and $C$ is given by

$$C = \frac{\eta}{2m_i n_{i0}}. \tag{31}$$

The stationary shock wave solution of (30) is

$$\phi^{(1)} = \phi_0^{(1)} \left[1 - \tan h\left(\frac{\xi}{\delta}\right)\right], \tag{32}$$

where $\phi_0^{(1)} = u_0/A$ and $\delta = 2C/u_0$.

The profiles of shock wave caused by the balance between nonlinearity and dissipation are shown in **Figures 3-6**. It is observed from **Figures 3** and **4** that there is no effect of $\eta$ (the value of $\eta$ was chosen from the experimental evidences [27] and it was made coincided with our present investigation) on potential of the shock wave, but a significant effect of $u_0$ (shown in **Figures 5** and **6**). The potential of wave profile for non-relativistic degenerate ion fluid and ultra-relativistic degenerate electron fluid is very small compared to that of for both non-relativistic electron-ion fluid (from **Figures 3-6**).

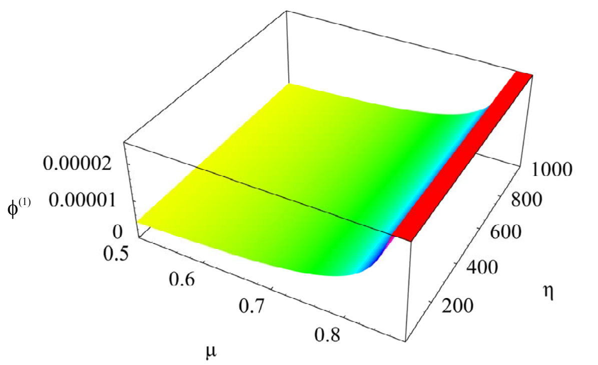


**Figure 3. The effect of the variation of $\mu$ and $\eta$ on the potential of shock wave for both electron-ion being non-relativistic degenerate with $u_0$ = 10.**

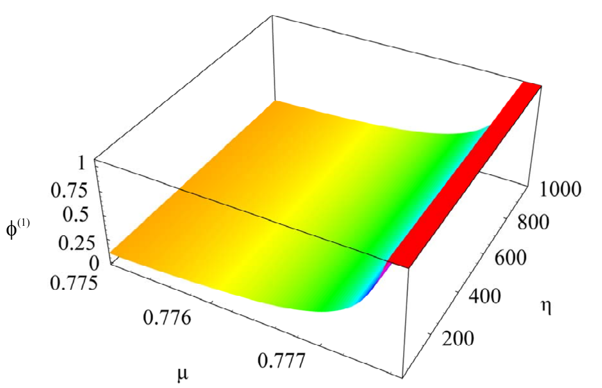


**Figure 4. The effect of the variation of $\mu$ and $\eta$ on the potential of shock wave for electron being ultra-relativistic and ion being non-relativistic degenerate with $u_0$ = 10.**

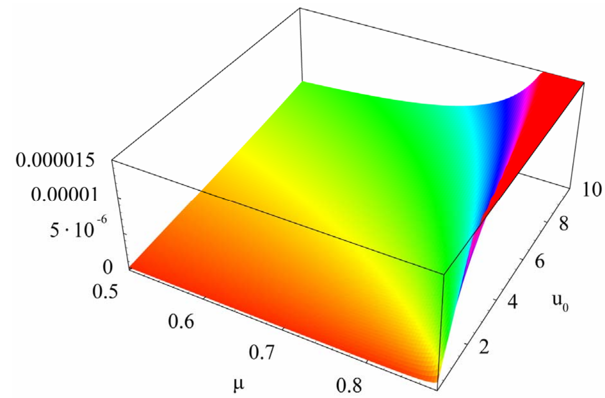


**Figure 5. The effect of the variation of $\mu$ and $u_0$ on the potential of shock wave for both electron-ion being non-relativistic degenerate.**

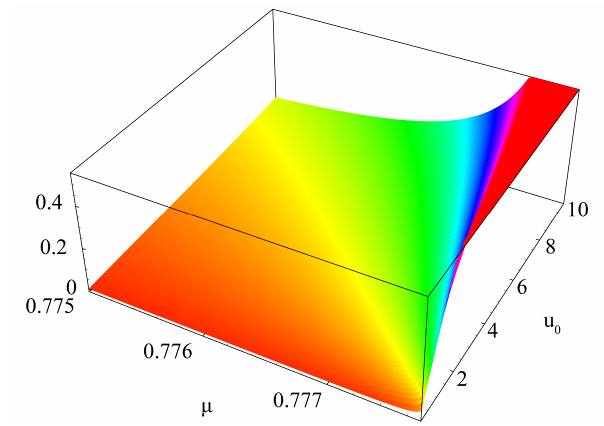


**Figure 6. The effect of the variation of $\mu$ and $u_0$ on the potential of shock wave for electron being ultra-relativistic and ion being non-relativistic degenerate.**

The existence of the nonlinear structures (both solitary and shock waves) have been verified using the standard values of different parameters related to our present situation [8]. The effect of some parameters on the plas- ma system have been studied directly from this investigation.

To summarize, we have investigated electrostatic solitary and shock waves in a degenerate dense plasma, which is relevant to interstellar compact objects [6,28-33]. The degenerate dense plasma is found to support solitary structures whose basic features (amplitude, width, speed, etc.) depend only on the plasma number density. It has been shown here that the amplitude, width, and speed increase with the increase of the plasma number density, particularly, the maximum number of the light particles (electrons). This work is very much effective and quite different from others and is more general than the relevant previous works [8]. We hope that our present investigation will be helpful for understanding the basic features of the localized electrostatic disturbances in compact astrophysical objects (e.g. white dwarf stars).

## Acknowledgements

The authors are very grateful to the TWAS research institute for their research equipment through this work.

## REFERENCES

[1] S. Chandrasekhar, "The Density of White Dwarf Stars," *Philosophical Magazine*, Vol. 11, No. 7, 1931, pp. 592-596.

[2] S. Chandrasekhar, "The Maximum Mass of Ideal White Dwarfs," *Astrophysical Journal*, Vol. 74, No. 1, 1931, pp. 81-82. doi:10.1086/143324

[3] S. Chandrasekhar, "Stellar Configurations with Degenerate Cores," *The Observatory*, Vol. 57, No. 1934, p. 373.

[4] S. Chandrasekhar, "The Highly Collapsed Configurations of a Stellar Mass (Second Paper)," *Monthly Notices of the Royal Astronomical Society*, Vol. 95, No. 1935, pp. 226-260.

[5] D. Koester and G. Chanmugam, "Physics of White Dwarf Stars," *Reports on Progress in Physics*, Vol. 53, No. 7, 1990, p. 837. doi:10.1088/0034-4885/53/7/001

[6] S. L. Shapiro and S. A. Teukolsky, "Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects," Wiley, New York, 1983.

[7] E. Garcia-Berro, S. Torres, L. G. Althaus, I. Renedo, P. Loren-Aguiltar, A. H. Corsico, R. D. Rohrmann, M. Salaris and J. Isern, "A White Dwarf Cooling Age of 8 Gyr for NGC 6791 from Physical Separation Processes," *Nature*, Vol. 465, 2010, pp. 194-196. doi:10.1038/nature09045

[8] A. A. Mamun and P. K. Shukla, "Nonplanar Dust Ion-Acoustic Solitary and Shock Waves in a Dusty Plasma with Electrons Following a Vortex-Like Distribution," *Physics Letters A*, Vol. 374, No. 3, 2010, pp. 472-475. doi:10.1016/j.physleta.2009.08.071

[9] G. Manfredi, "How to Model Quantum Plasmas," *Proceedings of the Workshop on Kinetic Theory*, The Fields Institute Communications Series, Vol. 46, 2005, p. 263.

[10] L. Stenfo, P. K. Shukla and M. Marklund, "New Low-Frequency Oscillations in Quantum Dusty Plasmas," *Europhysics Letter*, Vol. 74, 2006, p. 844. doi:10.1209/epl/i2006-10032-x

[11] P. K. Shukla, "New Dust in Quantum Plasma," *Physics Letter A*, Vol. 352, No. 3, 2006, pp. 242-243. doi:10.1016/j.physleta.2005.11.065

[12] P. K. Shukla and L. Stenfo, "Jeans Instabilities in Quantum Dusty Plasmas," *Physics Letter A*, Vol. 355, No. 4-5, 2006, pp. 378-380. doi:10.1016/j.physleta.2006.02.054

[13] P. K. Shukla and B. Eliasson, "Formation and Dynamics of Dark Solitons and Vortices in Quantum Electron Plasmas," *Physics Review Letter*, Vol. 96, No. 24, 2006, Article ID: 245001. doi:10.1103/PhysRevLett.96.245001

[14] B. Eliasson and P. K. Shukla, "Nonlinear Aspects of Quantum Plasma Physics: Nanoplasmonics and Nanostructures in Dense Plasmas," *Plasma and Fusion Research*, Vol. 4, No. 032, 2009.

[15] D. Shaikh and P. K.Shukla, "Fluid Turbulence in Quantum Plasmas," *Physics Review Letter*, Vol. 99, No. 12, 2007, Article ID: 125002. doi:10.1103/PhysRevLett.99.125002

[16] G. Brodin and M. Marklund, "Spin Magnetohydrodynamics," *New Journal of Physics*, Vol. 9, No. 8, 2007, p. 277. doi:10.1088/1367-2630/9/8/277

[17] G. Brodin and M. Marklund, "Spin Solitons in Magnetized Pair Plasmas," *Physics of Plasmas*, Vol. 14, No. 11, 2007, Article ID: 112107. doi:10.1063/1.2793744

[18] M. Marklund and G. Brodin, "Dynamics of Spin-1/2 Quantum Plasmas," *Physics Review Letter*, Vol. 98, 2007, Article ID: 025001. doi:10.1103/PhysRevLett.98.025001

[19] P. K. Shukla, "A New Spin in Quantum Plasmas," *Nature Physics*, Vol. 5, 2009, pp. 92-93. doi:10.1038/nphys1194

[20] M. Marklund, B. Eiasson and P. K. Shukla, "Magnetosonic Solitons in a Fermionic Quantum Plasma," *Physics Review E*, Vol. 76, No. 6, 2007, Article ID: 067401. doi:10.1103/PhysRevE.76.067401

[21] P. K. Shukla and B. Eliasson, "Nonlinear Aspects of Quantum Plasma Physics," *Physics Uspekhi*, Vol. 53, 2010, p. 51. doi:10.3367/UFNe.0180.201001b.0055

[22] W. Masood, B. Eiasson and P. K. Shukla, "Electromagnetic Wave Equations for Relativistically Degenerate Quantum Magnetoplasmas," *Physics Review E*, Vol. 81, No. 6, 2010, Article ID: 066401. doi:10.1103/PhysRevE.81.066401

[23] F. Hass, "Variational Approach for the Quantum Zakharov System," *Physics of Plasmas*, Vol. 14, No. 4, 2007.

[24] A. Misra and S. Samanta, "Quantum Electron-Acoustic Double Layers in a Magnetoplasma," *Physics of Plasmas*, Vol. 15, No. 12, 2008, Article ID: 122307. doi:10.1063/1.3040014

[25] A. P. Misra, S. Banerjee, F. Haas, P. K. Shukla and L. P. G. Assis, "Temporal Dynamics in the One-Dimensional Quantum Zakharov Equations for Plasmas," *Physics of Plasmas*, Vol. 17, No. 3, 2010, Article ID: 032307. doi:10.1063/1.3356059

[26] S. Maxon and J. Viecelli, "Spherical Solitons," *Physics Review Letter*, Vol. 32, No. 1, 1974, pp. 4-6. doi:10.1103/PhysRevLett.32.4

[27] A. Gavrikov, *et al.*, "32nd EPS Conference on Plasma Physics (2005)," *Tarragona*, Vol. 29C, 2005, O-2.011.

[28] F. C. Michel, "Theory of Pulsar Magnetospheres," *Review of Modern Physics*, Vol. 54, No. 1, 1982, pp. 1-66. doi:10.1103/RevModPhys.54.1

[29] H. R. Miller and P. J. Witta, "Active Galactic Nuclei," Springer, New York, 1987, p. 202.

[30] E. Tandberg-Hansen and A. G. Emslie, "The Physics of Solar Flares," Cambridge University Press, Cambridge, 1988, p. 124.

[31] J. Hoyos, A. Reisenegger and J. A. Valdivia, "Magnetic Field Evolution in Neutron Stars: One-Dimensional Multi-

 

Fluid Model," *Astronomy and Astrophysics*, Vol. 487, No. 3, 2008, pp. 789-803. doi:10.1051/0004-6361:200809466

[32] F. Asenjo, V. Munoz, J. A. Valdivia and S. M. Hahajan, "A Hydrodynamical Model for Relativistic Spin Quantum Plasmas," *Physics of Plasmas*, Vol. 18, No. 1, 2011, Article ID: 012107. doi:10.1063/1.3533448

[33] F. A. Asenjo, V. Munoz, J. A. Valdivia and T. Hada, "Circularly Polarized Wave Propagation in Magnetofluid Dynamics for Relativistic Electron-Positron Plasmas," *Physics of Plasmas*, Vol. 16, No. 12, 2009, Article ID: 122108. doi:10.1063/1.3272667

 