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The Construction Method for Solving Radial Flow Problem through the Homogeneous Reservoir

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ABSTRACT

On the basis of similar structure of solutions of ordinary differential equation (ODE) boundary value problem, the similar construction method was put forward by solving problems of fluid flow in porous media through the homogeneous reservoir. It is indicate that the pressure distribution of dimensionless reservoir and bottom hole in Laplace space, which take on the radial flow, also shows similar structure, and the internal relationship between the above solutions were illustrated in detail.

Keywords: Differential Equation; Fluid Flow in Porous Media; Boundary Value Problem; Construction Method; Similar Structure; Radial Flow; Homogeneous Reservoir

1. Introduction

Due to the permeability of porous media, reservoir engineers could simulate fluid flow by using media such as ground rock, filters and catalyst beds as well, it is useful to those researches who are interested in the behavior of porous media in different engineering applications. Mechanics of porous media flow plays an important role in many branches of engineering, including geotechnical engineering, material science, biomechanics and petroleum industry.

In 2004, S. C. Li [1,2] proposed a important conjecture that the solution formula of some differential equations under different conditions have the similarity, which is similar to that real numbers can be expressed as continued fraction and geometric graphics have certain similarity, we call it the similar structure of solutions. Over the past six years, for some second-order homogeneous ODE [1-14], and some second-order homogeneous linear partial differential equations (PDE) in Laplace space [15-21] as well as some mathematical models of fluids flow in porous media [22-34], the right smart evolution had been obtained on the study of the similar structure of their solutions.

The previous studies are separate, since the similar structure of solutions merely deduced from one certain model of the differential equation boundary value problem, and that was apparently gone against the analysis of

the internal relationship between solutions of different models. This paper is intended to reveal the internal relationships between different mathematical models. First of all, in section two, we provide the theoretical background materials of the similar structure of solutions for solving the modified Bessel equation boundary value problem. Secondly, in section tree and four, we devote to solve the problem of fluid flow in porous media through the homogeneous reservoir. At last, in the fifth section, we reveal the inherent laws between above solutions.

2. The Similar Structure of Solutions of the Modified Bessel Equation Boundary Value Problem

In [1] gives the boundary value problem of the modified Bessel equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \frac{\mathrm{d}y}{\mathrm{d}x} - \left(x^2 + \upsilon^2\right) y = 0 \tag{1}$$

$$\left[ay + \left(1 + ab\right) \frac{\mathrm{d}y}{\mathrm{d}x} \right]_{x=1} = Q \tag{2}$$

$$y\left(\infty\right) = 0 \tag{3a}$$

$$y\left(R\right) = 0 \tag{3b}$$

$$\left. \frac{\mathrm{d}y}{\mathrm{d}x} \right|_{x=R} = 0 \tag{3c}$$

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where a, b, υ, R and Q are real constants, and $R > 1$, its solution can be expressed as the following similar structure form

$$y = \frac{Q}{a + \dfrac{1}{b + \dfrac{1}{\upsilon - \dfrac{1}{\Psi(1)}}}} \cdot \frac{1}{b + \dfrac{1}{\upsilon - \dfrac{1}{\Psi(1)}}} \cdot \frac{1}{\upsilon - \dfrac{1}{\Psi(1)}} \cdot \frac{\Psi(x)}{\Psi(1)}$$

(4)

where $\Psi(x)$ is called the similar kernel function and defined as

$$\Psi(x) = \frac{K_\upsilon(x)}{K_{\upsilon+1}(1)}, \quad (y(\infty) = 0)$$

(5)

$$\Psi(x) = \frac{\Psi_{\upsilon,\upsilon}(x,R)}{\Psi_{\upsilon+1,\upsilon}(1,R)}, \quad (y(R) = 0)$$

(6)

$$\Psi(x) = \frac{\upsilon \Psi_{\upsilon,\upsilon}(x,R) + R \Psi_{\upsilon,\upsilon+1}(x,R)}{\upsilon \Psi_{\upsilon+1,\upsilon}(1,R) + R \Psi_{\upsilon+1,\upsilon+1}(1,R)}, \quad \left(\left.\frac{dy}{dx}\right|_{x=R} = 0\right)$$

(7)

where

$$\Psi_{m,n}(t,s) = K_m(t) I_n(s) + (-1)^{m-n+1} I_m(t) K_n(s),$$

$I_\upsilon(\cdot)$ denotes the first kind modified Bessel function with order υ, and $K_\upsilon(\cdot)$ denotes the second kind modified Bessel function with order υ. Furthermore, we have

$$y_w = \left[y + b\frac{dy}{dx}\right]_{x=1} = \frac{Q}{a + \dfrac{1}{b + \dfrac{1}{\upsilon - \dfrac{1}{\Psi(1)}}}}$$

(8)

When $\upsilon = 0$ in the modified Bessel Equation (1), if we setting $\sqrt{\lambda}\xi = x$ (where λ is real constant, and $\lambda > 0$), then y satisfies the ODE

$$y\xi^2 \frac{d^2 y}{d\xi^2} + \xi \frac{dy}{d\xi} - \lambda \xi^2 y = 0$$

(9)

Similarly as Equations (2) and (3) the boundary value conditions are

$$\left[ay + (1 + ab)\frac{dy}{d\xi}\right]_{\xi=1} = Q$$

(10)

$$y(\infty) = 0$$

(11a)

$$y(R) = 0$$

(11b)

$$\left.\frac{dy}{dx}\right|_{\xi=R} = 0$$

(11c)

Quite similar to solve the ODE boundary value prob-

lem, by Equations (9) and (11) have the following similar structure form of solution

$$y = \frac{Q}{a + \dfrac{1}{b + \Phi(1,\lambda)}} \cdot \frac{1}{b + \Phi(1,\lambda)} \cdot \Phi(\xi,\lambda)$$

(12)

where $\Phi(\xi,\lambda)$ is also called the similar kernel function and defined as

$$\Phi(\xi,\lambda) = -\frac{K_0(\sqrt{\lambda}\xi)}{\sqrt{\lambda} K_1(\sqrt{\lambda})}, \quad (y(\infty) = 0)$$

(13)

$$\Phi(\xi,\lambda) = -\frac{\Psi_{0,0}(\xi,R,\sqrt{\lambda})}{\sqrt{\lambda} \Psi_{1,0}(1,R,\sqrt{\lambda})}, \quad (y(R) = 0)$$

(14)

$$\Phi(\xi,\lambda) = -\frac{\Psi_{0,1}(\xi,R,\sqrt{\lambda})}{\sqrt{\lambda} \Psi_{1,1}(1,R,\sqrt{\lambda})}, \quad \left(\left.\frac{dy}{d\xi}\right|_{\xi=R} = 0\right)$$

(15)

where

$$\Psi_{m,n}(\alpha,\beta,s) = K_m(\alpha s) I_n(\beta s) + (-1)^{m-n+1} I_m(\alpha s) K_n(\beta s)$$

(16)

In particular, we also have

$$y_w = \left[y(\xi) + b\frac{dy}{d\xi}\right]_{\xi=1} = \frac{Q}{a + \dfrac{1}{b + \Phi(1,\lambda)}}$$

(17)

3. The Radial Flow Problem through the Homogeneous Reservoir

According to [22], the mathematical model of radial flow through the homogeneous reservoir has been described as follows

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}, \quad (r_D > 1, t_D > 0)$$

(18)

$$p_D(r_D, 0) = 0$$

(19)

$$p_{wD}(t_D) = \left[p_D - S r_D \frac{\partial p_D}{\partial r_D}\right]_{r_D=1}$$

(20)

$$\left(r_D \frac{\partial p_D}{\partial r_D}\right)\bigg|_{r_D=1} = -\left[q_D(t_D) - C_D \frac{dp_{wD}}{dt_D}\right]$$

(21)

$$y(\infty, t_D) = 0$$

(22a)

$$y(R, t_D) = 0$$

(22b)

$$\left.\frac{\partial p_D}{\partial r_D}\right|_{r_D=R_D} = 0$$

(22c)

where p_D, r_D, t_D, C_D, R_D, and q_D are dimensionless variables respectively represent the pressure of any point in reservoir, the distance of reservoir from any point to the wellbore, time, the wellbore storage coefficient, the outer boundary radius of circular reservoir and the liquid flow output of bottom hole (which may depend on time t), and S is the skin factor (well bottom pollution factor). The outer boundary conditions

$$p_D(\infty, t_D) = 0, \quad p_D(R_D, t_D) = 0$$

and

$$\left. \frac{\partial p_D}{\partial r_D} \right|_{r_D = R_D} = 0$$

denote that the outer boundary of circular reservoir are infinite, constant pressure and closed respectively.

Taking the Laplace transform of $p_D(r_D, t_D)$ with respect to t_D, we obtain the ODE with parameter z (where z is variable in Laplace space)

$$\frac{d^2 \overline{p}_D}{dr_D^2} + \frac{1}{r_D} \frac{d \overline{p}_D}{dr_D} = z \overline{p}_D, \left(r_D \geq 1 \right) \quad (23)$$

and with the boundary conditions

$$\overline{p}_{wD}(z) = \left[\overline{p}_D - S r_D \frac{d \overline{p}_D}{dr_D} \right]_{r_D = 1} \quad (24)$$

$$\left(r_D \frac{d \overline{p}_D}{dr_D} \right)\bigg|_{r_D = 1} = - \left[\overline{q}_D(z) - C_D z \overline{p}_{wD} \right] \quad (25)$$

$$\overline{p}_D(\infty, z) = 0 \quad (26a)$$

$$\overline{p}_D(R_D, z) = 0 \quad (26b)$$

$$\left. \frac{d \overline{p}_D}{dr_D} \right|_{r_D = R_D} = 0 \quad (26c)$$

If let

$$y = \overline{p}_D, \xi = r_D, \lambda = z, a = -C_D z, \\ b = -S, Q = -\overline{q}_D, R = R_D \quad (27)$$

then the boundary value problem, Equations (23)-(26) becomes the boundary value problem of Equations (9)-(11). Consequently, according to Equation (12), we know that the similar structure formula of dimensionless reservoir pressure distribution at any point for homogeneous reservoir radial flow model in Laplace space is

$$\overline{p}_D(r_D, z) = \overline{q}_D(z) \cdot \frac{1}{C_D z + \dfrac{1}{S + \Psi(1, z)}} \\ \cdot \frac{1}{S + \Psi(1, z)} \cdot \Psi(r_D, z) \quad (28)$$

where $\Psi(r_D, z)$ is called the similar kernel function and

defined as

$$\Psi(r_D, z) = \frac{K_0\left(\sqrt{z} r_D\right)}{\sqrt{z} K_1\left(\sqrt{z}\right)}, \left(p_D(\infty, t_D) = 0 \right) \quad (29)$$

$$\Psi(r_D, z) = \frac{\Psi_{0,0}\left(r_D, R_D, \sqrt{z}\right)}{\sqrt{z} \Psi_{1,0}\left(1, R_D, \sqrt{z}\right)}, \left(p_D(R_D, t_D) = 0 \right) \quad (30)$$

$$\Psi(r_D, z) = \frac{\Psi_{0,1}\left(r_D, R_D, \sqrt{z}\right)}{\sqrt{z} \Psi_{1,1}\left(1, R_D, \sqrt{z}\right)}, \left(\left. \frac{\partial p_D}{\partial r_D} \right|_{r_D = R_D} = 0 \right) \quad (31)$$

Similarly, according to Equation (17), we see that the similar structure formula of dimensionless well bottom hole pressure distribution for homogeneous reservoir radial flow model in Laplace space is

$$\overline{p}_{wD}(z) = \left[\overline{p}_D - S r_D \frac{d \overline{p}_D}{dr_D} \right]_{r_D = 1} \\ = \overline{q}_D(z) \cdot \frac{1}{C_D z + \dfrac{1}{S + \Psi(1, z)}} \quad (32)$$

In particular, when the liquid flow output of bottom hole is constant, set $\overline{q}_D(z) = 1/z$, then Equations (28) and (32) still hold.

4. The Radial Flow Problem through the Homogeneous Reservoir Considering the Effective Well Radius

According to [35], the mathematical model of radial flow through the homogeneous reservoir has been described as follows

$$\frac{\partial^2 p_D}{\partial r_{eD}^2} + \frac{1}{r_{eD}} \frac{\partial p_D}{\partial r_{eD}} = \frac{1}{C_D e^{2S}} \frac{\partial p_D}{\partial T_D}, \left(r_{eD} > 1, T_D > 0 \right) \quad (33)$$

$$p_D \big|_{T_D = 0} = 0 \quad (34)$$

$$\left(r_{eD} \frac{\partial p_D}{\partial r_{eD}} \right)\bigg|_{r_{eD} = 1} = -q_D + \frac{\partial p_{wD}}{\partial T_D} \quad (35)$$

$$p_D(\infty, T_D) = 0 \quad (36a)$$

$$p_D(R_{eD}, T_D) = 0 \quad (36b)$$

$$\left. \frac{\partial p_D}{\partial r_{eD}} \right|_{r_{eD} = R_{eD}} = 0 \quad (36c)$$

where r_{eD}, T_D and R_{eD} are given as follows, respectively

$$r_{eD} = r_D e^S, \quad T_D = \frac{t_D}{C_D}, \quad R_{eD} = R_D e^S \quad (37)$$

The outer boundary conditions

$$p_D(\infty, T_D) = 0, \quad p_D(R_{eD}, T_D) = 0$$

and

$$\left. \frac{\partial p_D}{\partial r_{eD}} \right|_{r_{eD} = R_{eD}} = 0$$

denote that the outer boundary of circular reservoir are infinite, constant pressure and closed, respectively.

Taking the Laplace transform of $p_D(r_{eD}, T_D)$ with respect to T_D, we obtain the ODE with parameter z (where z is variable in Laplace space)

$$\frac{d^2 \bar{p}_D}{dr_{eD}^2} + \frac{1}{r_{eD}} \frac{d\bar{p}_D}{dr_{eD}} = \frac{z}{C_D e^{2S}} \bar{p}_D, \quad (r_{eD} \geq 1) \qquad (38)$$

and with the boundary value conditions

$$\left(r_{eD} \frac{d\bar{p}_D}{dr_{eD}} \right) \bigg|_{r_{eD} = 1} = -\left[\bar{q}_D(z) - z\bar{p}_D \big|_{r_{eD} = 1} \right] \qquad (39)$$

$$\bar{p}_D(\infty, z) = 0 \qquad (40a)$$

$$\bar{p}_D(R_{eD}, z) = 0 \qquad (40b)$$

$$\left. \frac{d\bar{p}_D}{dr_{eD}} \right|_{r_{eD} = R_{eD}} = 0 \qquad (40c)$$

If let

$$y = \bar{p}_D, \xi = r_{eD}, \lambda = \sqrt{z/C_D} e^{-S},$$
$$a = -z, b = 0, Q = -\bar{q}_D, R = R_{eD} \qquad (41)$$

then the boundary value problem Equations (38)-(40) becomes the boundary value problem Equations (9)-(11). Consequently, according to Equation (12), we know that the similar structure formula of dimensionless reservoir pressure distribution at any point for homogeneous reservoir radial flow model in Laplace space is

$$\bar{p}_D(r_{eD}, z) = \bar{q}_D(z) \cdot \frac{1}{z + \frac{1}{\Psi(1, z)}} \cdot \frac{\Psi(r_{eD}, z)}{\Psi(1, z)} \qquad (42)$$

where $\Psi(r_{eD}, z)$ is called the similar kernel function and defined as

$$\Psi(r_{eD}, z) = \frac{K_0(\sqrt{z/C_D} e^{-S} r_{eD})}{\sqrt{z/C_D} e^{-S} K_1\left(\sqrt{z/C_D} e^{-S}\right)}, \qquad (43)$$
$$\left(p_D(\infty, T_D) = 0 \right)$$

$$\Psi(r_{eD}, z) = \frac{\Psi_{0,0}\left(r_{eD}, R_{eD}, \sqrt{z/C_D} e^{-S}\right)}{\sqrt{z/C_D} e^{-S} \Psi_{1,0}\left(1, R_{eD}, \sqrt{z/C_D} e^{-S}\right)}, \qquad (44)$$
$$\left(p_D(R_{eD}, T_D) = 0 \right)$$

$$\Psi(r_{eD}, z) = \frac{\Psi_{0,1}\left(r_{eD}, R_{eD}, \sqrt{z/C_D} e^{-S}\right)}{\sqrt{z/C_D} e^{-S} \Psi_{1,1}\left(1, R_{eD}, \sqrt{z/C_D} e^{-S}\right)},$$
$$\left(\left. \frac{\partial p_D}{\partial r_{eD}} \right|_{r_{eD} = R_{eD}} = 0 \right) \qquad (45)$$

Similarly, according to Equation (17), we know that the similar structure formula of dimensionless bottom hole pressure distribution for homogeneous reservoir radial flow model in Laplace space is

$$\bar{p}_{wD}(z) = \bar{p}_D(1, z) = \bar{q}_D(z) \cdot \frac{1}{z + \frac{1}{\Psi(1, z)}} \qquad (46)$$

In particular, when the liquid flow output of bottom hole is constant, set $\bar{q}_D(z) = 1/z$, then Equations (42) and (46) still hold.

5. Conclusions

Form above discussion, the conclusion can be reached that

1) In section three and four, we use the property of similar structure of ODE boundary value problem, and in the process of solving problem of radial flow through homogeneous reservoir in Laplace space, we can get the expecting outcome by means of algebra constructive theory in Laplace space, and avoid the procedure of solving Bessel equation and the complicated partial derivative operation, but making simple change of variables (such as Equation (27) or Equation (41)) only.

We find this is a simple, convenient and effective method, and an innovation idea for solving problem of fluid flow in porous media.

2) Different similar kernel functions (see Equations (5)-(7), (13)-(15), (29)-(31) and (43)-(45)) corresponding to different right (outer) boundary value conditions (see Equations (3), (11), (22) and (36)), respectively. Actually, the similar kernel functions are the basic solutions which corresponding to the especial left (inner) boundary value conditions (see $\left. \frac{dy}{dx} \right|_{x=1} = 1$, $\left. \frac{dy}{d\xi} \right|_{\xi=1} = 1$, $\left. \frac{dp_D}{dr_D} \right|_{r_D=1} = 1$

and $\left. \frac{dp_D}{dr_{eD}} \right|_{r_{eD}=1} = 1$) respectively.

3) The similar structure formula of solutions (see Equations (4) or (8), (12) or (17), (28) or (32) and (42) or (46)) depend on the left (inner) boundary conditions (see Equations (2), (10), (24), (25) and (39)), instead of the determine equations (see Equations (1), (9), (23) and (38)) and the right (outer) boundary value conditions (see Equations (3), (11), (22) and (36)). From the unified ex-

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pression in Equation (28) and (32), it is easy to see how the wellbore storage effects, skin factors, as well as the outer boundaries influence the reservoir pressure and bottom hole pressure. This expression brought great convenience for programming well test analysis software, and simplified the program algorithm.

4) If take the Laplace transform to the measured pressure data, well test analysis (dynamic analysis of pressure) can be conducted directly in Laplace space, this also can reflect the advantage of the similar structure of solutions in this paper. In general, using Stehfest numerical inversion formula, the solutions we got in this paper can be transformed into numerical solution in real space, which fully meat the need of the application of well test analysis.

5) This arouses our interest in studying the similar structure of solutions. On the one hand, it is clearly that we can get direct solutions satisfying more especial left (inner) boundary conditions (such as the real constants and in Equation (2) satisfying restricted conditions) from the similar structure expression. On the other hand, using the similar structure expression and the basic solutions (*i.e.* the similar kernel functions), we can construct some more complicated practical problems. Therefore, we put forward the construction method for solving the boundary value problem of ODE, and it is called the similar construction method.

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