

The law of genetic inheritance

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ABSTRACT

In this paper, based on binomial formula, a law of genetic inheritance is proposed, and it would have certain reference value for future genetic inheritance research. Besides, a recursive algorithm for recessive inheritance is also proposed. Given any two of the male parent, female parent and filial generation, the genomic information of the third one can be calculated uniquely.

Keywords: Gene; Inheritance Law; Inheritability Ratio

1. INTRODUCTION

Human beings has experienced tens of thousands of generations since its born 5 million years ago. The role of genome has been acknowledged worldwide. This paper will present a complete set of genetic inheritance law, and a meaningful recursive algorithm in the delivery of genome information.

2. BASIC ASSUMPTION

Genetic Behavior is divided into dominant inheritance and recessive inheritance. Assume that D represents for dominant inheritance factor in offspring, and d represents for recessive inheritance factor in offspring. Two inheritance factors can be dependent or independent. Dependent means there is cross effect between D and d , that is, (Dd) is also an inheritance result.

The possible result is D or d when the inheritance factors of male parent and female parent are independent, and the possible result can be D , d , $(Dd)^+$, $(Dd)^-$, $D(Dd)^+$, $d(Dd)^+$, $D(Dd)^-$, or $(Dd)^-d$ when they are dependent.

Binomial formula: $(D+d)^n = \sum_{i=0}^n C_n^i D^i d^{n-i}$, where

$$C_n^i = \frac{n!}{i!(n-i)!}$$

Coefficients of the odd and even items in the expansion are equal.

The expansion $(D+d)^n = \sum_{i=0}^n C_n^i D^i d^{n-i}$ is just all possible outcomes of the $(n-1)$ -th generation of female and male parents hybrid.

3. TWO LEMMAS

Lemma 1: When the female parent and male parent are dominant inheritance, the inheritance factors will fit the following addition principle 1:

$$D \oplus D = D;$$

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$$d \oplus d = d.$$

Lemma 2: When the female parent and male parent are recessive inheritance, the inheritance factors will fit the following addition principle 2:

$$D \oplus D = d;$$

$$D \oplus d = D;$$

$$d \oplus D = D;$$

$$d \oplus d = d.$$

4. GENETIC INHERTANCE LAW

Genetic inheritance law:

1) All possible inheritance results of the $(n-1)$ -th generation of female and male parents hybrid will fit the following expansion:

$$\begin{aligned} (D+d)^n &= \sum_{i=0}^n C_n^i D^i d^{n-i} \\ &= C_n^0 D^n + C_n^1 D^{n-1} d + C_n^2 D^{n-2} d^2 + \dots + C_n^n d^n, \end{aligned}$$

where $C_n^i = \frac{n!}{i!(n-i)!}$;

2) Inheritance factors meet distributive law (separate regularity);

3) Inheritance factors meet associative law (law of independent assortment);

- 4) Dominant inheritance meets addition principle 1;
 5) Recessive inheritance meets addition principle 2;
 6) When D and d are independent, the possible result is D or d ;

7) When D and d are dependent, there will be cross effect between them, (Dd) will be regarded as inheritance factor which also have dominant inheritance $(Dd)^+$ and recessive inheritance $(Dd)^-$ (linkage inheritance rule), and possible result is: D , d , $(Dd)^+$, $(Dd)^-$, $D(Dd)^+$, $d(Dd)^+$, $D(Dd)^-$, or $(Dd)^-d$.

Note: when $n = 2$, this inheritance law is just Mendel's law [1].

5. INFERENCES OF THE INHERITANCE LAW

Inference 1: When meets dominant inheritance and D and d are independent, the 1st generation (1st) will be:

$$(D+d)^2 = D^2 + 2Dd + d^2 = 3D + d$$

the 2nd generation (2nd) will be:

$$(D+d)^3 = D^3 + 3D^2d + 3Dd^2 + d^3 = 7D + d$$

the 3rd generation (3rd) will be:

$$(D+d)^4 = D^4 + 4D^3d + 6D^2d^2 + 4D^1d^3 + d^4 = 15D + d$$

the $(n-1)$ -th generation $((n-1)$ -th) will be:

$$(D+d)^n = C_n^0 D^n + C_n^1 D^{n-1}d + C_n^2 D^{n-2}d^2 + \dots + C_n^n d^n \\ = (2^n - 1)D + d \text{ (see Table 1).}$$

Inference 2: When meets recessive inheritance and D and d are independent, the 1st generation (1st) will be:

$$(D+d)^2 = D^2 + 2Dd + d^2 = 2D + 2d ;$$

the 2nd generation (2nd) will be:

$$(D+d)^3 = D^3 + 3D^2d + 3Dd^2 + d^3 = 4D + 4d ;$$

the 3rd generation (3rd) will be:

$$(D+d)^4 = D^4 + 4D^3d + 6D^2d^2 + 4D^1d^3 + d^4 = 8D + 8d$$

the $(n-1)$ -th generation $((n-1)$ -th) will be:

$$(D+d)^n = C_n^0 D^n + C_n^1 D^{n-1}d + C_n^2 D^{n-2}d^2 + \dots + C_n^n d^n \\ = 2^{n-1}D + 2^{n-1}d \text{ (see Table 2).}$$

Inference 3: When meets dominant inheritance, D and d are dependent, and $(Dd)^+$ is dominant inheritance, the 1st generation (1st) will be:

$$(D+d)^2 = D^2 + 2(Dd)^+ + d^2 = D + 2(Dd)^+ + d$$

the 2nd generation (2nd) will be:

$$(D+d)^3 = D^3 + 3D^2d + 3Dd^2 + d^3 \\ = D + 3D(Dd)^+ + 3(Dd)^+d + d$$

the 3rd generation (3rd) will be:

$$(D+d)^4 = D^4 + 4D^3d + 6D^2d^2 + 4D^1d^3 + d^4 \\ = D + 4D(Dd)^+ + 6(Dd)^+ + 4(Dd)^+d + d$$

the 4th generation (4th) will be:

$$(D+d)^5 = D^5 + 5D^4d + 10D^3d^2 + 10D^2d^3 + 5Dd^4 + d^5 \\ = D + 15D(Dd)^+ + 15(Dd)^+d + d \text{ (see Table 3).}$$

Inference 4: When meets dominant inheritance, D and d are dependent, and $(Dd)^-$ is recessive inheritance, the 1st generation (1st) will be:

$$(D+d)^2 = D^2 + 2(Dd)^- + d^2 = D + 2(Dd)^- + d$$

the 2nd generation (2nd) will be:

$$(D+d)^3 = D^3 + 3D^2d + 3Dd^2 + d^3 \\ = D + 3D(Dd)^- + 3(Dd)^-d + d$$

the 3rd generation (3rd) will be:

$$(D+d)^4 = D^4 + 4D^3d + 6D^2d^2 + 4D^1d^3 + d^4 \\ = D + 4D(Dd)^- + 6(Dd)^- + 4(Dd)^-d + d$$

the 4th generation (4th) will be:

$$(D+d)^5 = D^5 + 5D^4d + 10D^3d^2 + 10D^2d^3 + 5Dd^4 + d^5 \\ = D + 15D(Dd)^- + 15(Dd)^-d + d \text{ (see Table 4).}$$

Inference 5: When meets recessive inheritance, D and d are dependent, and $(Dd)^+$ is dominant inheritance, the 1st generation (1st) will be:

$$(D+d)^2 = D^2 + 2Dd + d^2 = 2(Dd)^+ + 2d ;$$

the 2nd generation (2nd) will be:

$$(D+d)^3 = D^3 + 3D^2d + 3Dd^2 + d^3 \\ = (Dd)^+ + 3D(Dd)^+ + 3(Dd)^+d + d ;$$

the 3rd generation (3rd) will be:

$$(D+d)^4 = D^4 + 4D^3d + 6D^2d^2 + 4D^1d^3 + d^4 \\ = 7(Dd)^+ + 8(Dd)^+d + d ;$$

the 4th generation (4th) will be:

$$(D+d)^5 = D^5 + 5D^4d + 10D^3d^2 + 10D^2d^3 + 5Dd^4 + d^5 \\ = 5(Dd)^+ + 10D(Dd)^+ + 16(Dd)^+d + d \text{ (see Table 5).}$$

Inference 6: When meets recessive inheritance, D and d are dependent, and $(Dd)^-$ is recessive inheritance, the 1st generation (1st) will be:

Table 1. Inheritability ratio.

Generation \ Inheritance factor	<i>D</i>	<i>d</i>	$(Dd)^+$	$(Dd)^-$	$D(Dd)^+$	$d(Dd)^+$	$D(Dd)^-$	$(Dd)^- d$
1st	3/4	1/4						
2nd	7/8	1/8						
3rd	15/16	1/16						
$(n-1)$ -th	$(2^n - 1)/2^n$	$1/2^n$						

Table 2. Inheritability ratio.

Generation \ Inheritance factor	<i>D</i>	<i>d</i>	$(Dd)^+$	$(Dd)^-$	$D(Dd)^+$	$d(Dd)^+$	$D(Dd)^-$	$(Dd)^- d$
1st	1/2	1/2						
2nd	1/2	1/2						
3rd	1/2	1/2						
$(n-1)$ -th	1/2	1/2						

Table 3. Inheritability ratio.

Generation \ Inheritance factor	<i>D</i>	<i>d</i>	$(Dd)^+$	$(Dd)^-$	$D(Dd)^+$	$d(Dd)^+$	$D(Dd)^-$	$(Dd)^- d$
1st	1/4	1/4	1/2					
2nd	1/2	1/2	3/4		3/8 When interactive	3/8 When interactive		
3rd	5/16	5/16	3/8		1/4 When interactive	1/4 When interactive		
4th	1/2	1/2	15/16		15/32 When interactive	15/32 When interactive		

Table 4. Inheritability ratio.

Generation \ Inheritance factor	<i>D</i>	<i>d</i>	$(Dd)^+$	$(Dd)^-$	$D(Dd)^+$	$d(Dd)^+$	$D(Dd)^-$	$(Dd)^- d$
1st	1/4	1/4		1/2				
2nd	1/2	1/2					3/8 When interactive	3/8 When interactive
3rd	5/16	5/16		3/8			1/4 When interactive	1/4 When interactive
4th	1/2	1/2					15/32 When interactive	15/32 When interactive

Table 5. Inheritability ratio.

Generation \ Inheritance factor	<i>D</i>	<i>d</i>	$(Dd)^+$	$(Dd)^-$	$D(Dd)^+$	$d(Dd)^+$	$D(Dd)^-$	$(Dd)^- d$
1st		1/2	1/2					
2nd	3/8	1/2	7/8		3/8 When interactive	3/8 When interactive		
3rd		9/16	15/16			1/2 When interactive		
4th	5/16	17/32	31/32		5/16 When interactive	1/2 When interactive		

Table 6. Inheritability ratio.

Generation \ Inheritance factor	D	d	$(Dd)^+$	$(Dd)^-$	$D(Dd)^+$	$d(Dd)^+$	$D(Dd)^-$	$(Dd)^- d$
1st		1/2		1/2				
2nd	3/8	1/2		7/8			3/8 When interactive	3/8 When interactive
3rd		9/16		15/16				1/2 When interactive
4th	5/16	17/32		31/32			5/16 When interactive	1/2 When interactive

$$(D + d)^2 = D^2 + 2Dd + d^2 = 2(Dd)^- + 2d ;$$

the 2nd generation (2nd) will be:

$$(D + d)^3 = D^3 + 3D^2d + 3Dd^2 + d^3 \\ = (Dd)^- + 3D(Dd)^- + 3(Dd)^- d + d$$

the 3rd generation (3rd) will be:

$$(D + d)^4 = D^4 + 4D^3d + 6D^2d^2 + 4D^1d^3 + d^4 \\ = 7(Dd)^- + 8(Dd)^- d + d;$$

the 4th generation (4th) will be:

$$(D + d)^5 = D^5 + 5D^4d + 10D^3d^2 + 10D^2d^3 + 5Dd^4 + d^5 \\ = 5(Dd)^- + 10D(Dd)^- + 16(Dd)^- d + d. \text{ (see Table 6).}$$

6. RECURSIVE NATURE OF GENETIC ALGORITHM FOR RECESSIVE INHERITANCE

From lemma 2, we can know such addition principle:

$$D \oplus D = d , \\ D \oplus d = D , \\ d \oplus D = D ,$$

and

$$d \oplus d = d .$$

These addition principles meet orthogonality, independence and uniqueness.

This principle gives a closure algorithm, that is, the genome information of filial generation won't change when compared with male parent and female parent; all genome information of filial generation will be influenced by the parents'; and, given any two of the male parent, female parent and filial generation, the genomic information of the third one can be calculated uniquely. Above is the recursive genetic algorithm for recessive inheritance.

7. CONCLUSIONS

We think that the genetic inheritance law proposed in this paper would have certain reference value for future genetic inheritance research. Using clinical inheritance statistical information can easily get similar conclusions. We have used clinical genetic disease statistics, such as schizophreniform psychosis, verified the rationality of the law. Besides, in mathematics, the binomial theorem has been proved to be unquestionable axiom. The inheritance rule of a variety of parents can be obtained easily by $(D + d)^m (Y + y)^n$.

Gene is under the influence of the living environment, not set in stone. How to quantify the specific changes and how to determine the timing of gene mutation need further study in future.

REFERENCE

[1] Plomin, R. (2008) Behavior genetics, East China Normal University Press, Shanghai.