

Calculation of Reliability for Emergency Response System Based on Minimum Path

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Abstract: In order to analyze and calculate the reliability of the emergency system, first of all, it discussed composition and structure of the emergency system and showed emergency system were made up of hardware subsystem and software subsystem, person subsystem, the relationship subsystem between person and hardware and the relationship subsystem between person and software. Then, basing on the minimum path of its emergency response system, it got analytical expression of reliability for emergency response system and analyzed all kinds of subsystems' reliability. It makes a conclusion that analytical expression of the reliability of emergency response system can be gotten directly from the minimum path, which can be used in other complex system.

Keywords: Emergency System; Minimum Path; Response; Reliability

1 Introduction

Public emergency refers to events that happen suddenly, they have caused or will cause casualties, property losses damage to the ecological environment and social hazards. According to the process, nature and mechanism of public emergency, they can be divided into natural disasters, accidents disasters, public health incidents and social safety. In recent years, public emergencies took place frequently, such as the SARS epidemic in the spring and summer of 2003, Indian Ocean tsunami on December 26, 2004, the snowstorm in mid-January 2008 in southern China, the earthquake in WenChuan of SiChuan province in May 12, 2008, the earthquake in YuShu of QingHai province in April 14, 2010, and so on. In order to prevent the occurrence of public emergency effectively or control the loss caused by public emergency, it has become very necessary to establish an effective emergency response system. Whether established emergency response systems are effective or not is directly relevant with the reliability of emergency response system^[1-3]

2 Composition and Structure of Emergency System

Public emergency response system S is mainly made up of hardware subsystem S_1 , software subsystem S_2 and people subsystem S_3 . Hardware subsystem includes material resources such as equipments and facilities allocated for sudden emergency. Software subsystem includes public emergency policies, formulated rules and regulations, organizational structure, etc. People subsystem includes all the personnel who deal with emergencies, make decision, make operation and grasp information and make supervision in public emergency. People subsystem is the most critical in the three subsystems,

because it manages the hardware subsystem and formulates software subsystem. Today about 85% of various types of accidents occurred in industrial enterprises directly or indirectly derived from the human factor. In China, about 70% of various types of accidents that occurred in the nuclear industry, chemical industry, aviation, metallurgy and mining industries were relevant with people.^[4] Therefore, in the course of the emergency system, the system also includes the relationship subsystem between person and hardware S_4 and the relationship subsystem between person and software S_5 . Emergency response system is comprised of five subsystems S_1 , S_2 , S_3 , S_4 , S_5 . The composition and structure of the emergency response system for public events shows as Figure 1.

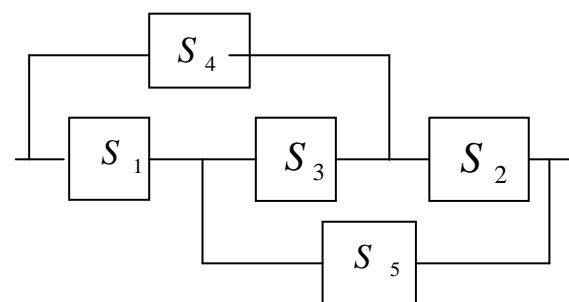


Figure 1. Emergency system composition chart

Emergency system has prevention and response two important features. Response is the last barriers for emergency system to mitigate and control the events. Its reliable function plays a crucial role. Fast, accurate and effective emergency response will improve the reaction speed and coordination level, the reliability level is the

basis of successful emergency.

3 Calculation of Reliability for Emergency Response System

The response reliability of emergency system is defined as the ability to complete the respond function in the required conditions and time when sudden public incidents occur. When using the probability value indicates this ability, it is known as the emergency response system reliability. Hardware, software reliability refers to the ability of hardware subsystem, software subsystem to complete the function in the required period of time and the required conditions. When using the probability value indicates this ability, it is known respectively as the reliability of hardware subsystems reliability, software subsystems reliability. The people reliability in general is defined as: people's ability to complete task without error in the required period of time and the required conditions, When using the probability of value to indicate this ability, it is called as the people reliability. The reliability of relationship subsystem between person and hardware or software in general is defined as: people's ability to complete the function on the condition that hardware or software failed or missed. When using the probability value to indicate, it is known as the reliability of relationship subsystems between person and hardware or software.

Suppose Y , Y_1 , Y_2 , Y_3 , Y_4 and Y_5 as the life variables for system and subsystems above. $F_s(t)$, $F_1(t)$, $F_2(t)$, $F_3(t)$, $F_4(t)$ and $F_5(t)$ are their distribution functions. Their reliability functions are $R_s(t)$, $R_1(t)$, $R_2(t)$, $R_3(t)$, $R_4(t)$, $R_5(t)$. Suppose $R_3(t)$ as the reliability that the person acts the state of their own safety, $R_4(t)$, $R_5(t)$ shows the reliability that people deal with the failure of hardware and software missing. So it can be thought that the five subsystems are independent. People subsystem must respond firstly to make emergency system response normal. After people subsystem has responded rightly, if the hardware subsystem and software subsystem have one or two subsystems not respond, the corresponding relationship subsystem must respond correctly in order to guarantee the normal response to emergency system. For example, when hardware subsystem has no normal response, only the relationship subsystem between person and hardware has normal response can make system operate normally. Take the fire emergency system as an example, when the fire occurs, if the hardware of emergency response system such as automatic fire extinguisher system does not respond, at that time the company would need relevant personnel take timely measures to repair hardware quickly or take other means to offset the impact of hardware failure. Therefore the emergency response system has three minimal paths^[5]:

$\{S_1, S_2, S_3\}, \{S_2, S_3, S_4\}, \{S_1, S_3, S_5\}$.

Let

$$Z_1 = \min\{Y_1, Y_2, Y_3\}, \quad Z_2 = \min\{Y_2, Y_3, Y_4\}, \quad Z_3 = \min\{Y_1, Y_3, Y_5\}.$$

Then their distribution functions are:

$$\begin{aligned} F_{Z_1}(t) &= P\{Z_1 \leq t\} = P\{\min\{Y_1, Y_2, Y_3\} \leq t\} \\ &= 1 - P\{\min\{Y_1, Y_2, Y_3\} > t\} \\ &= 1 - P\{Y_1 > t\}P\{Y_2 > t\}P\{Y_3 > t\} \\ &= 1 - R_1(t)R_2(t)R_3(t) \end{aligned} \quad (1)$$

$$F_{Z_2}(t) = P\{Z_2 \leq t\} = 1 - R_2(t)R_3(t)R_4(t) \quad (2)$$

$$F_{Z_3}(t) = P\{Z_3 \leq t\} = 1 - R_1(t)R_3(t)R_5(t) \quad (3)$$

So

$$R_{Z_1}(t) = R_1(t)R_2(t)R_3(t) \quad (4)$$

$$R_{Z_2}(t) = R_2(t)R_3(t)R_4(t) \quad (5)$$

$$R_{Z_3}(t) = R_1(t)R_3(t)R_5(t) \quad (6)$$

From $Y = \max\{Z_1, Z_2, Z_3\}$ gets

$$\begin{aligned} F_s(t) &= P\{Y \leq t\} = P\{\max\{Z_1, Z_2, Z_3\} \leq t\} \\ &= P\{Z_1 \leq t, Z_2 \leq t, Z_3 \leq t\} \end{aligned} \quad (7)$$

Furthermore gets

$$\begin{aligned} R_s(t) &= P\{Y > t\} = P\{Z_1 > t \text{ or } Z_2 > t \text{ or } Z_3 > t\} \\ &= P\{Z_1 > t\} + P\{Z_2 > t\} + P\{Z_3 > t\} \\ &\quad - P\{Z_1 > t, Z_2 > t\} - P\{Z_2 > t, Z_3 > t\} \\ &\quad - P\{Z_1 > t, Z_3 > t\} + P\{Z_1 > t, Z_2 > t, Z_3 > t\} \\ &= 1 - F_{Z_1}(t) + 1 - F_{Z_2}(t) + 1 - F_{Z_3}(t) \\ &\quad - P\{Y_1 > t, Y_2 > t, Y_3 > t, Y_4 > t\} \\ &\quad - P\{Y_1 > t, Y_2 > t, Y_3 > t, Y_4 > t, Y_5 > t\} \\ &\quad - P\{Y_1 > t, Y_2 > t, Y_3 > t, Y_5 > t\} \\ &\quad + P\{Y_1 > t, Y_2 > t, Y_3 > t, Y_4 > t, Y_5 > t\} \\ &= R_{Z_1}(t) + R_{Z_2}(t) + R_{Z_3}(t) - R_1(t)R_2(t)R_3(t)R_4(t) \\ &\quad - R_1(t)R_2(t)R_3(t)R_5(t) \\ &= R_1(t)R_2(t)R_3(t) + R_2(t)R_3(t)R_4(t) + R_1(t)R_3(t)R_5(t) \\ &\quad - R_1(t)R_2(t)R_3(t)R_4(t) - R_1(t)R_2(t)R_3(t)R_5(t) \end{aligned} \quad (8)$$

(8) can be written:

$$\begin{aligned} R_s(t) &= R_3(t)[R_1(t)(R_2(t) + R_5(t) - R_2(t)R_5(t)) \\ &\quad + (1 - R_1(t))R_2(t)R_4(t)] \end{aligned} \quad (9)$$

Or

$$\begin{aligned} R_s(t) &= R_3(t)[R_2(t)(R_1(t) + R_4(t) - R_1(t)R_4(t)) \\ &\quad + (1 - R_2(t))R_1(t)R_5(t)] \end{aligned} \quad (10)$$

(9) and (10) corresponds with the emergency systems of

the three minimal paths: $\{S_1, S_2, S_3\}, \{S_2, S_3, S_4\}, \{S_1, S_3, S_5\}$. Three minimal paths have S_3 , then each of $R_s(t)$ has $R_3(t)$. If S_3 is removed by three paths, it will get $\{S_1, S_2\}, \{S_2, S_4\}, \{S_1, S_5\}$. These three sets can be divided into two categories. If they are divided by whether they contain S_1 , the set with S_1 needs S_2 or S_5 , the set without S_1 needs S_2 and S_4 , which corresponds to type (9). If they are divided by whether they contain S_2 , it corresponds to type (10).

4 Reliability Calculation of Subsystem

Assume that there are n kinds of equipments and facilities allocated for public emergency and k kinds of mutual policies, laws and regulations. Suppose the reliability of the i th emergency equipment and facilities is $R_{1_i}(t) (i=1,2,\dots,n)$, the reliability of the i th policy rules and regulations is $R_{2_i}(t) (i=1,2,\dots,k)$, therefore, the reliability of hardware subsystem $R_1(t)$ and the reliability of software subsystem $R_2(t)$ are as follows:

$$R_1(t) = \prod_{i=1}^n R_{1_i}(t) \quad (11)$$

$$R_2(t) = \prod_{i=1}^k R_{2_i}(t) \quad (12)$$

Suppose people subsystem of emergence system is made up of m person, the reliability of the i th person is $R_{3_i}(t) (i=1,2,\dots,m)$, $\lambda_i(t)$ is instantaneous error rate of the i th person, t is response time, people subsystem reliability $R_3(t)$ shows as follows:

$$R_3(t) = \prod_{i=1}^m R_{3_i}(t) = \prod_{i=1}^m e^{-\int_0^t \lambda_i(t) dt} = e^{-\int_0^t (\sum_{i=1}^m \lambda_i(t)) dt} \quad (13)$$

Let $\lambda(t) = \sum_{i=1}^m \lambda_i(t)$, calls it people subsystem cumulative error rate, therefore:

$$R_3(t) = e^{-\int_0^t \lambda(t) dt} \quad (14)$$

Suppose $R_{4_1}(t), R_{4_2}(t), R_{4_3}(t)$ as the people perception reliability, judgment reliability and reaction reliability of failure or missing of hardware, $R_{5_1}(t), R_{5_2}(t), R_{5_3}(t)$ as the people perception reliability, judgment reliability and reaction reliability of failure or missing of software. The relationship subsystem

tem between person and hardware is series system made up of perception, judgment and reaction. So the reliability of relationship subsystem between person and hardware $R_4(t)$ is as follows:

$$R_4(t) = R_{4_1}(t) R_{4_2}(t) R_{4_3}(t) \quad (15)$$

Similarly, the reliability of relationship subsystem between people and software $R_5(t)$ is as follows:

$$R_5(t) = R_{5_1}(t) R_{5_2}(t) R_{5_3}(t) \quad (16)$$

Suppose $F_{S_2}, F_{O_2}, F_{R_2}$ as the basic error rate of perception, judgment and reaction as the people face failure or missing of hardware, $F_{S_3}, F_{O_3}, F_{R_3}$ as the basic error rate of perception, judgment and reaction as the people face failure or missing of software, $a_{s_i}, a_{o_i}, a_{r_i} (i=1,2,\dots,l)$ as the quantitative value of the main factor which actions influence, C_H as the human capacity for error correction, K_S, K_O, K_R as the error-correcting capacity correction factor of perception, judgment and reaction respectively, according to the literature [6], [7], [8], [9], it gets

$$R_{4_1}(t) = 1 - (\prod_{i=1}^l a_{s_i}) (1 - K_S C_H) F_{S_2} \quad (17)$$

$$R_{4_2}(t) = 1 - (\prod_{i=1}^l a_{o_i})^2 (1 - K_O C_H)^2 [1 - R_{4_1}(1 - F_{O_2})] F_{O_2} \quad (18)$$

$$R_{4_3}(t) = 1 - (\prod_{i=1}^l a_{r_i})^3 (1 - K_R C_H)^3 [1 - R_{4_1} R_{4_2} (1 - F_{R_2})] [1 - R_{4_2} (1 - F_{R_2})] F_{R_2} \quad (19)$$

$$R_{5_1}(t) = 1 - (\prod_{i=1}^l a_{s_i}) (1 - K_S C_H) F_{S_3} \quad (20)$$

$$R_{5_2}(t) = 1 - (\prod_{i=1}^l a_{o_i})^2 (1 - K_O C_H)^2 [1 - R_{5_1}(1 - F_{O_3})] F_{O_3} \quad (21)$$

$$R_{5_3}(t) = 1 - (\prod_{i=1}^l a_{r_i})^3 (1 - K_R C_H)^3 [1 - R_{5_1} R_{5_2} (1 - F_{R_3})] [1 - R_{5_2} (1 - F_{R_3})] F_{R_3} \quad (22)$$

Let
 $l = 5$

$$\begin{aligned} a_{o_j} &= 2.2 (j=1,2,\dots,5) & a_{s_j} &= 1.6 (j=1,2,\dots,5) \\ a_{r_j} &= 1.1 (j=1,2,\dots,5) & F_{S_2} &= F_{S_3} = 0.002633, \\ F_{O_2} &= F_{O_3} = 0.00524, & F_{R_2} &= F_{R_3} = 0.00715, \end{aligned}$$

$$K_S = 0.3846, \quad K_o = 0.6923, \quad K_R = 0.9923,$$

$$C_H = 0.74$$

Thus,

$$R_4(t) = R_5(t) = 0.8586$$

5 Conclusion

The whole process of analysis shows that:

(1) As a particular emergency system, first of all, it should analyze the specific components of the system, find the minimum system path, you can get emergency system reliability according to the minimum system path.

(2) When emergency system have three minimum paths, each path contains three separate subsystems, the analytical reliability of this system can be obtained directly from three minimum paths, which can be served as a reference to other complex system.

(3) Emergency response system is a complex system, In order to discuss the reliability of emergency system and analyze the specific components of the system, it should start with the large-scale system, from the surface to point, then discuss small subsystems.

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