

Time Domain Signal Analysis Using Wavelet Packet Decomposition Approach

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Abstract

This paper explains a study conducted based on wavelet packet transform techniques. In this paper the key idea underlying the construction of wavelet packet analysis (WPA) with various wavelet basis sets is elaborated. Since wavelet packet decomposition can provide more precise frequency resolution than wavelet decomposition the implementation of one dimensional wavelet packet transform and their usefulness in time signal analysis and synthesis is illustrated. A mother or basis wavelet is first chosen for five wavelet filter families such as Haar, Daubechies (Db4), Coiflet, Symlet and dmey. The signal is then decomposed to a set of scaled and translated versions of the mother wavelet also known as time and frequency parameters. Analysis and synthesis of the time signal is performed around 8 seconds to 25 seconds. This was conducted to determine the effect of the choice of mother wavelet on the time signals. Results are also prepared for the comparison of the signal at each decomposition level. The physical changes that are occurred during each decomposition level can be observed from the results. The results show that wavelet filter with WPA are useful for analysis and synthesis purpose. In terms of signal quality and the time required for the analysis and synthesis, the Haar wavelet has been seen to be the best mother wavelet. This is taken from the analysis of the signal to noise ratio (SNR) value which is around 300 dB to 315 dB for the four decomposition levels.

Keywords: WPA, Wavelet Packet Decomposition (WPD), SNR, Haar

1. Introduction

Over the last decade much work has been done in applying time frequency transforms to the problem of signal representation and classification. Signals possessing non-stationary character are not well suited for detection and classification by traditional Fourier methods. It has been shown that wavelets can approximate time varying non-stationary signals in a better way than the Fourier transform representing the signal on both time and frequency domains [1]. Hence they can easily detect local features in a signal. Furthermore, wavelet decomposition allows analyzing a signal at different resolution levels. The discrete wavelet transform (DWT) provides a very efficient representation for a broad range of real-world signals. This property has been exploited to develop powerful signal de-noising and estimation methods [2] and extremely low-bit-rate compression algorithms [3]. The discrete wavelet transform (DWT) is usually implemented using an octave-band tree structure. This is ac-

complished by dividing each sequence into a component containing its approximated version (low-frequency part) and a component with the residual details (high-frequency part) and then iterating this procedure at each stage only on the low-pass branch of the tree [4,5]. The main drawback of the octave-band tree structure is that it does not provide a good approximation of the critical subband decomposition [6]. An alternate means of analysis is sought, so that valuable time-frequency information is not lost. The Wavelet Packet Transform (WPT) is one such time frequency analysis tools. It is a transform that brings the signal into a domain that contains both time and frequency information (Wickerhauser, 1991). Thus, analysis of the signal can be done simultaneously in frequency and time. The most basic way to do time frequency analysis is by making FFT analysis in short windows. That has the drawback that the window needs to be short to find out fast changes in the signal and long to determine low frequency components. The wavelet packet transform (WPT) offers a great deal of freedom in dealing

with different types of transient signals. Indeed the development of the wavelet transform (WT) [7–9] and wavelet packets [10–12] has sparked considerable activity in signal representation and in transient and non stationary signal analysis.[13–15].

Wavelet packet decomposition (WPD) (sometimes known as just wavelet packets) is a wavelet transform where the signal is passed through more filters than the DWT. Wavelet packets are the particular linear combination of wavelets. They form bases which retain many of the orthogonality, smoothness, and localization properties of their parent wavelets. The coefficients in the linear combinations are computed by a recursive algorithm making each newly computed wavelet packet coefficient with the result that expansions in wavelet packet bases have low computational complexity. In the DWT, each level is calculated by passing the previous approximation coefficients through high and low pass filters. However, in the WPD, both the detail and approximation coefficients are decomposed. For n levels of decomposition the WPD produces 2^n different sets of coefficients (or nodes) as opposed to $(n+1)$ sets for the DWT. However, due to the down sampling process the overall number of coefficients is still the same and there is no redundancy.

The work presented in this paper contributes a new era in wavelet packet analysis and synthesis of time domain signals. Wavelet packet transform techniques have been used to extract feature from time domain signals. Feature extraction involves information retrieval from the time signal [16]. The wavelet packet transform has more important benefits than the discrete wavelet transform. Wavelet packet functions comprise a rich family of building block functions. Wavelet packet functions are still localized in time, but offer more flexibility than wavelets in representing different types of signals. In particular, wavelet packets are better at representing signals that exhibit oscillatory or periodic behavior. Wavelet packets are organized naturally into collections, and each collection is an orthogonal basis for $L^2(R)$. It is a simple, but very powerful extension of wavelets and multiresolution analysis (MRA). The wavelet packets allow more flexibility in adapting the basis to the frequency contents of a signal and it is easy to develop a fast wavelet packet transform. The power of wavelet packet lies in the fact that we have much more freedom in deciding which basis function is to be used to represent the given function. It can be computed very fast, it demands only $O(M \log M)$ time, where M is the number of data points which is important in particular in real time applications. It also has compact support in time as well as in frequency domain and adapts its support locally to the signal which is important in time varying signals. With wavelet packets we have a much finer resolution of the

signal and a greater variety of options for decomposing it.

The paper is organized as follows. In Section 2, brief background information on Discrete Wavelet transform and wavelet packet decomposition is discussed. In Section 3 the present work is explained. The results are given in Section 4 and Section 5 gives the conclusions.

2. Background

2.1. Discrete Wavelet Transform

The DWT, which is based on subband coding, is found to yield a fast computation of Wavelet Transform. It is easy to implement and reduces the computation time and resources required. In continuous wavelet transform (CWT), the signals are analyzed using a set of basis functions which relate to each other by simple scaling and translation. In the case of DWT, a time scale representation of the digital signal is obtained using digital filtering techniques. The signal to be analyzed is passed through filters with different cutoff frequencies at different scales. In the discrete wavelet transform, a signal can be analyzed by passing it through an analysis filter bank followed by a decimation operation. When a signal passes through these filters, it is split into two bands. The low pass filter, which corresponds to an averaging operation, extracts the coarse information of the signal. The high pass filter, which corresponds to a differencing operation, extracts the detail information of the signal. The output of the filtering operations is then decimated by two. Filters are one of the most widely used signal processing functions. Wavelets can be realized by iteration of filters with rescaling. The DWT is computed by successive low pass and high pass filtering of the discrete time-domain signal as shown in **Figure 1**. This is called the Mallat algorithm or Mallat-tree decomposition.

At each decomposition level, the half band filters produce signals spanning only half the frequency band. This doubles the frequency resolution as the uncertainty in frequency is reduced by half. In accordance with Nyquist's rule if the original signal has a highest frequency of ω , which requires a sampling frequency of 2ω radians, then it now has a highest frequency of $\omega/2$ radians. It can now be sampled at a frequency of ω radians thus discarding half the samples with no loss of information. This decimation by 2 halves the time resolution as the entire signal is now represented by only half the number of samples. Thus, while the half band low pass filtering removes half of the frequencies and thus halves the resolution, the decimation by 2 doubles the scale. The filtering and decimation process is continued until the desired level is reached. The maximum number of levels depends on the length of the signal. The DWT of the original signal is then obtained by concatenating all the

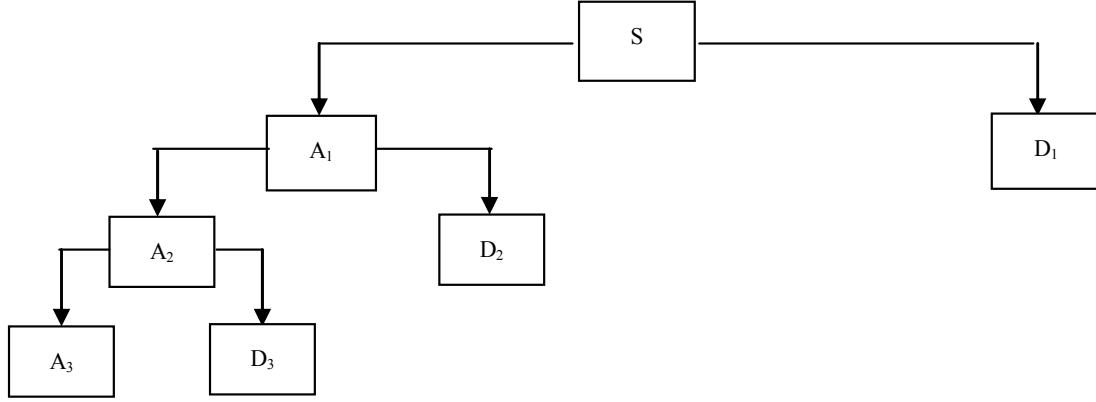


Figure 1. Level 3 decomposition using wavelet transform.

coefficients, approximation and details, starting from the last level of decomposition.

2.2. Multiresolution Analysis

An orthogonal wavelet decomposition of a signal $x(t) \in L^2(R)$ to coefficients $\{W_j^k(x)\}_{k,j \in Z^2}$ such that

$$\begin{aligned} W_j^k(x) &\equiv \left\langle x(t), 2^{-j/2} \psi(2^{-j}t - k) \right\rangle \\ &\equiv 2^{-j/2} \int_{-\infty}^{\infty} x(t) \psi^*(2^{-j}t - k) dt \end{aligned} \quad (1)$$

where the function ψ is usually referred to as a mother wavelet and $*$ stands for the complex conjugation. The orthonormal wavelet basis $\left\{ 2^{j/2} \psi(2^{-j}t - k), (k, j) \in Z^2 \right\}$ may be built from a multiresolution analysis of $L^2(R)$. In this case, the approximation of the signal at resolution 2^{-j} can be described by the coefficients

$$A_j^k(x) \equiv \left\langle x(t), 2^{-j/2} \phi(2^{-j}t - k) \right\rangle, \quad k \in Z \quad (2)$$

where the function ϕ is the scaling function. The mother wavelet and the scaling function then satisfy the so called two-scale equations:

$$2^{-1/2} \phi\left(\frac{t}{2} - k\right) = \sum_{l=-\infty}^{\infty} h_{l-2k} \phi(t-l) \quad (3)$$

$$2^{-1/2} \psi\left(\frac{t}{2} - k\right) = \sum_{l=-\infty}^{\infty} g_{l-2k} \psi(t-l) \quad (4)$$

Where $\{h_k\}_{k \in Z}$ and $\{g_k\}_{k \in Z}$ are respectively the impulse response of lowpass and highpass paraunitary Quadrature mirror filters (QMF) [17]. If we denote the vector spaces

$$\begin{aligned} V_j &\equiv \text{span}\{\phi(2^{-j}t - k), k \in Z\} \quad \text{and} \\ O_j &\equiv \text{span}\{\psi(2^{-j}t - k), k \in Z\} \end{aligned}$$

it results from Equations (3) and (4) that $V_{j+1} = V_j \oplus O_j^{\perp}$. We then find that for every $j_m \in Z$,

$$\left\{ 2^{-j/2} \psi(2^{-j}t - k), k \in Z, j \leq j_m \right\} \cup \left\{ 2^{-j_m/2} \phi(2^{-j_m}t - k), k \in Z \right\}$$

is an orthonormal basis of $L^2(R)$.

The interest in the QMF filters lies in the efficient computation of the orthogonal wavelet decomposition via a two channel filter bank structure. The decomposition which is useful in emphasizing the local features of a signal, presents however a limitation, namely its non-variance in time (or space). This implies that the wavelet coefficients of $T_{\tau}[x(t)] \equiv x(t - \tau)$, $\tau \in R$ are generally not delayed versions of $\{W_j^k(x)\}_{k \in Z}$.

2.3. Wavelet Packet Decomposition

The wavelet packet method is a generalization of wavelet decomposition that offers a richer range of possibilities for signal analysis and which allows the best matched analysis to a signal. It provides level by level transformation of a signal from the time domain into the frequency domain. It is calculated using a recursion of filter-decimation operations leading to the decrease in time resolution and increase in frequency resolution. The frequency bins, unlike in wavelet transform, are of equal width, since the WPT divides not only the low, but also the high frequency subband. In wavelet analysis, a signal is split into an approximation and a detail coefficient. The approximation coefficient is then itself split into a second-level approximation coefficients and detail coefficients, and the process is repeated. In wavelet packet

analysis, the details as well as the approximations can be split. This yields more than $2^{2^{n-1}}$ different ways to encode the signal. When the WT is generalized to the WPT, not only can the lowpass filter output be iterated through further filtering, but the highpass filter can be iterated as well. This ability to iterate the highpass filter outputs means that the WPT allows for more than one basis function (or *wavelet packet*) at a given scale, versus the WT which has one basis function at each scale other than the deepest level, where it has two. The set of *wavelet packets* collectively make up the complete family of possible bases, and many potential bases can be constructed from them. If only the lowpass filter is iterated, the result is the wavelet basis. If all lowpass and highpass filters are iterated, the *complete tree* basis results. The top level of the WPD tree is the time representation of the signal. As each level of the tree is traversed there is an increase in the trade off between time and frequency resolution. The bottom level of a fully decomposed tree is the frequency representation of the signal. **Figure 2** shows the level 3 decomposition using wavelet packet transform.

Based on the above analysis, **Figure 1** and **Figure 2** give the comparison of a three-level wavelet decomposition and wavelet packet decomposition. It can be seen in **Figure 1** that in wavelet analysis only the approximations (represented by capital A in the figure) at each resolution level are decomposed to yield approximation and detail information (represented by capital D in the figure) at a higher level. However, in the wavelet packet analysis [**Figure 2**], both the approximation and details at a certain level are further decomposed into the next level, which means the wavelet packet analysis can provide a more precise frequency resolution than the wavelet analysis.

The wavelet packet decomposition [4,5,18] is an extension of the wavelet representation which allows the best matched analysis to a signal. To define wavelet packets, we introduce functions of $L^2(R)$, $W_m(t)$, $m \in N$ such that

$$\int_{-\infty}^{\infty} W_0(t) dt = 1 \quad (5)$$

and for all $k \in Z$,

$$2^{-j/2} W_{2^j m} \left(\frac{t}{2} - k \right) = \sum_{l=-\infty}^{\infty} h_{l-2k} W_m(t-k) \quad (6)$$

$$2^{-j/2} W_{2^j m+1} \left(\frac{t}{2} - k \right) = \sum_{l=-\infty}^{\infty} g_{l-2k} W_m(t-k) \quad (7)$$

where $\{h_k\}_{k \in Z}$ and $\{g_k\}_{k \in Z}$ are the previously defined impulse responses of the QMF filters. If for every $j \in Z$, we define the vector space $\Omega_{j,m} \cong$

$\text{span}\{W_m(2^{-j}t-k), k \in Z\}$ then it can be shown that

$\Omega_{j,m} = \Omega_{j+1,2m} \oplus \Omega_{j+1,2m+1}$. As a result, if we denote by P a partition of R^+ into intervals $I_{j,m} = [2^j m, \dots, 2^j(m+1)]$, $j \in Z$ and $m \in \{0, 1, \dots, 2^j - 1\}$, then

$$L^2(R) = \bigoplus_{(j,m)/I_{j,m} \in P} \Omega_{j,m} \quad (8)$$

In an equivalent way, $\{2^{-j/2} W_m(2^{-j}t-k), k \in Z, (j,m)/I_{j,m} \in P\}$ is an orthonormal basis of $L^2(R)$. Such a basis is called a wavelet packet. The coefficients resulting from the decomposition of a signal $x(t)$ in this basis are

$$C_{j,m}^k(x) \cong \langle x(t), 2^{-j/2} W_m(2^{-j}t-k) \rangle \quad (9)$$

By varying the partition P , different choices of wavelet packets are possible. While a fixed and dyadic partitioning of time frequency domain is imposed in the case of the wavelet transform, the idea of wavelet packets is to introduce more flexibility making this partitioning adaptive to spectral content of the signal.

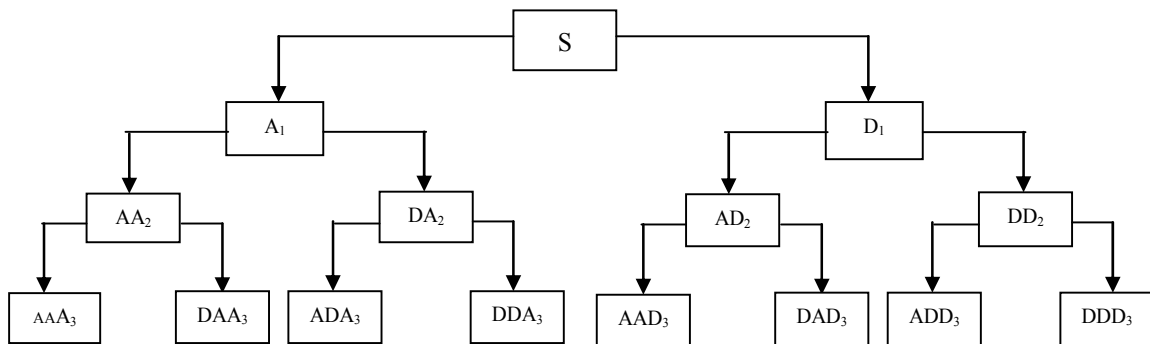


Figure 2. Level 3 decomposition using wavelet packet transform.

In wavelet packet analysis, an entropy-based criterion is used to select the most suitable decomposition of a given signal. This means we look at each node of decomposition tree and quantify the information to be gained by performing each split [19]. The wavelets have several families. The most important wavelets families are Haar, Daubechies, Symlets, Coiflets, and biorthogonal.

3. Proposed Work

The block diagram shown in **Figure 3** gives the actual implementation of the method proposed in this paper.

1) Input time domain signal

The system described above was simulated on a computer with floating point numbering system. The input time domain wave is pre-emphasized, low pass filtered with sampling frequency 8 K to 44.1 KHz range and 1–10 sec duration samples. The time domain is digitized to 16 bits.

Preprocessing:

In this block unwanted noise is removed when the signal gets recorded with the help of various digital filter such as Low pass filter (LPF) with 3.4 KHz, Notch filter to remove the line frequency effect i.e. 50 Hz.

Wavelet transforms:

In this block we applied the wavelet filter coefficients as low and high band and filter the input signal to enhance the band energies.

Decimation:

As deals with the multi-rate analysis system the decimation factor helps to enhance the band level information from wavelet transform. We consider decimation by 2, 4 etc for our experiments.

Analysis Subband Feature Vectors:

From the filtering results obtained with the 2 band system, we have extracted the various features from low pass and high pass band and then re-arranged with the desired format for further analysis part.

Interpolation:

Interpolation deals with up sampling the inverse wavelet filtering to reconstruct the original input signal.

Inverse Wavelet transforms:

Analysis of interpolated features is reordered with respect to the 2 bands system to extract the original contents from the feature vector for input signal synthesis purpose.

Noise Removal:

Input noisy signal is filtered with band stop filter with desired cut off frequency. Filtered output is further applied to post processing.

Post Processing:

Final tuning is done in the post processing block.

2) Testing Setup

Matlab programs were written to implement the structure shown in **Figure 3**. Time domain signals in a *.wav format sampled at 8 KHz were used for all simulations. Results for different setups are given in the next section. Several examples of time domain signals of different sampling frequencies, with five wavelet filter families are given below.

3) Performance Evaluation

Performance evaluation tests can be done by subjective quality measures and objective quality measures. Objective measures provide a measure that can be easily implemented and reliably reproduced. Objective measures are based on mathematical comparison of the original and processed time domain signals. The majority of objective quality measures quantify time domain quality of the signal in terms of a numerical distance measure. The signal to noise ratio is the most widely used method to measure time domain signal quality. It is calculated as the ratio of the signal to noise power in decibels.

$$SNR = 10 \log_{10} \left(\frac{\sum_n s^2(n)}{\sum_n [s(n) - \hat{s}(n)]^2} \right) \quad (10)$$

where $s(n)$ is the clean time domain signal and $\hat{s}(n)$ is the processed time domain signal.

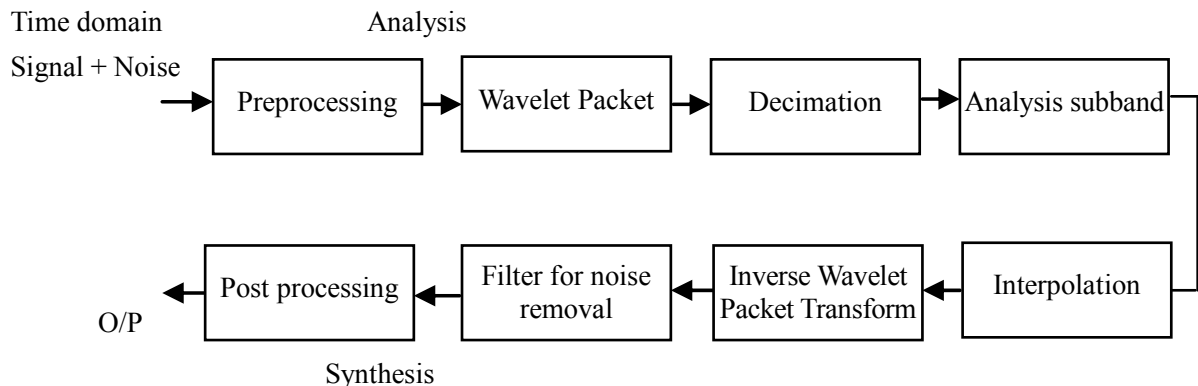


Figure 3. Block diagram of time domain signal using wavelet packet transform.

4. Results

We have tested the 10 input samples with sampling frequency of 8 K on various wavelet filtering bank i.e. Haar, Db4, Symlet, dmey, etc. as shown in **Tables 1, 3, 4, 7**. We observe that SNR for wavelet packet analysis and synthesis after filtering is around 120 dB to 320 dB for level 1 decomposition, level 2 decomposition 120 dB to 310 dB, level 3 decomposition 115 dB to 310 dB and level 4 decomposition 115 dB to 305 dB.

From **Tables 2, 4, 6, 8** as per timing consideration the total time for analysis and synthesis is tabulated. We observe that total time for analysis and synthesis using wavelet filtering is around 8 sec to 25 sec.

Table 1. SNR calculation for various mother wavelet level 1 decomposition.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	315.64	244.16	247.86	122.32	165.94
T2	315.58	244.16	247.86	122.28	165.95
T3	315.53	244.16	247.86	122.35	165.94
T4	315.55	244.16	247.86	122.34	165.94
T5	315.55	244.16	247.86	122.34	165.94
T6	315.53	244.16	247.86	122.35	165.94
T7	315.53	244.16	247.86	122.30	165.95
T8	315.53	244.16	247.86	122.35	165.94
T9	315.55	244.17	247.87	122.25	165.95
T10	315.54	244.16	247.86	122.32	165.95

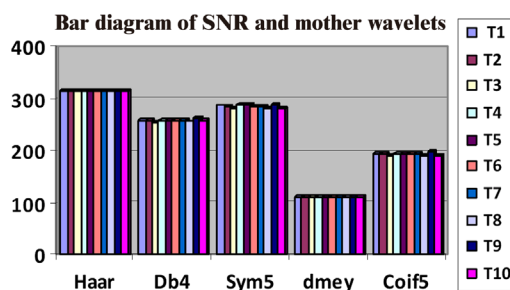


Figure 4. Graphical presentation of SNR for various mother wavelets.

Table 2. Total time calculation (in seconds) for analysis and synthesis using wavelet filtering for level 1.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	8.10	8.31	9.15	9.40	8.96
T2	8.90	10.56	9.17	11.01	9.28
T3	8.53	9.43	9.32	13.45	10.40
T4	8.85	9.34	9.06	12.31	10.20
T5	8.39	9.01	8.59	11.90	9.29
T6	10.73	11.37	12.45	13.10	11.90
T7	11.48	12.03	13.07	13.28	13.18
T8	8.65	9.23	10.46	12.57	9.62
T9	9.59	10.65	10.20	11.40	11.26
T10	9.32	9.64	9.67	13.29	10.11

Table 3. SNR calculation of various mother wavelets for level 2 decomposition.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	310.41	232.64	247.31	121.37	165.81
T2	310.41	232.64	247.31	121.34	165.82
T3	310.40	232.64	247.31	121.36	165.81
T4	310.41	232.64	247.31	121.39	165.81
T5	310.38	232.64	247.31	121.36	165.81
T6	310.40	232.64	247.31	121.42	165.81
T7	310.40	232.64	247.31	121.33	165.82
T8	310.39	232.64	247.31	121.42	165.81
T9	310.40	232.65	247.31	121.25	165.82
T10	310.41	232.64	247.31	121.34	165.82

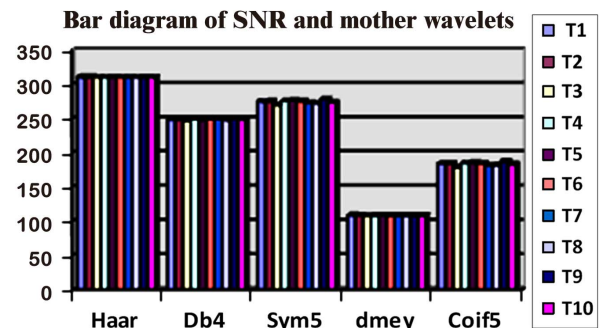


Figure 5. Graphical presentation of SNR for various mother wavelets.

Table 4. Total time calculation (in seconds) for analysis and synthesis using wavelet filtering for level 2.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	8.29	8.73	10.11	12.85	9.20
T2	10.10	11.06	11.57	14.64	11.21
T3	9.60	9.56	10.06	12.51	10.43
T4	8.93	10.43	9.48	13.95	11.67
T5	8.42	10.18	8.89	11.92	10.85
T6	11.82	13.51	12.48	15.62	13.00
T7	10.70	12.64	13.11	16.84	13.40
T8	9.45	10.53	9.82	13.28	10.56
T9	9.21	10.09	9.81	12.18	11.04
T10	10.75	10.98	11.09	14.25	10.71

Table 5. SNR calculation of various mother wavelets for level 3 decomposition.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	307.15	230.59	241.56	115.85	159.86
T2	307.15	230.60	241.56	115.84	159.86
T3	307.13	230.59	241.56	115.84	159.86
T4	307.12	230.59	241.56	115.86	159.86
T5	307.14	230.59	241.56	115.85	159.86
T6	307.14	230.59	241.56	115.88	159.86
T7	307.16	230.60	241.56	115.83	159.87
T8	307.15	230.59	241.56	115.87	159.87
T9	307.15	230.60	241.57	115.77	159.87
T10	307.14	230.60	241.56	115.83	159.86

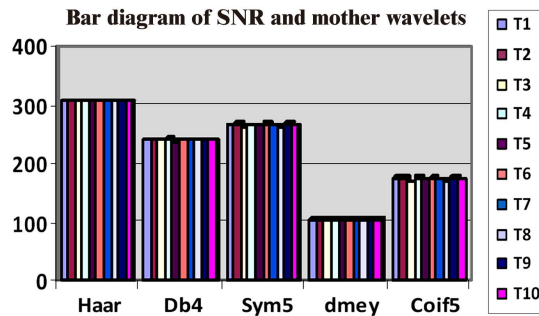


Figure 6. Graphical presentation of SNR for various Mother wavelets.

Table 6. Total time calculation (in seconds) for analysis and synthesis using wavelet filtering for level 3.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	9.45	9.42	9.76	12.46	9.40
T2	10.35	11.70	10.57	15.64	11.01
T3	10.17	11.53	13.28	14.98	12.07
T4	10.11	11.48	12.21	16.54	12.40
T5	10.00	10.89	10.04	12.89	11.57
T6	11.84	13.73	13.21	18.04	14.68
T7	12.65	12.12	12.76	20.37	13.60
T8	10.53	11.20	10.87	15.43	11.86
T9	9.56	10.00	11.59	14.32	11.96
T10	10.32	10.48	11.96	16.68	13.42

Table 7. SNR calculation of various mother wavelets for level 4 decomposition.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	304.74	226.62	241.29	115.31	159.79
T2	304.74	226.62	241.29	115.29	159.79
T3	304.74	226.62	241.29	115.33	159.79
T4	304.73	226.62	241.29	115.37	159.79
T5	304.73	226.62	241.29	115.32	159.79
T6	304.73	226.62	241.29	115.37	159.79
T7	304.74	226.62	241.29	115.27	159.80
T8	304.75	226.62	241.29	115.36	159.79
T9	304.74	226.62	241.29	115.21	159.80
T10	304.74	226.62	241.29	115.30	159.80

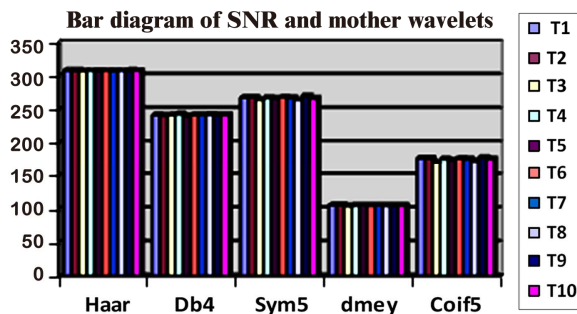


Figure 7. Graphical presentation of SNR for various Mother wavelets.

Table 8. Total time calculation (in seconds) for analysis and synthesis using wavelet filtering for level 4.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	9.95	10.90	10.76	14.92	12.26
T2	12.01	12.59	13.48	19.46	14.84
T3	12.12	12.81	14.79	22.09	12.53
T4	11.03	12.79	11.98	20.34	14.73
T5	11.34	12.37	12.54	16.79	13.09
T6	13.25	15.59	16.37	25.25	19.20
T7	14.51	15.40	17.39	25.89	17.98
T8	11.90	13.14	11.96	19.57	13.15
T9	12.09	13.17	12.51	20.01	15.48
T10	13.09	13.28	14.57	21.75	15.35

From Table 9, entropy which is a common measure of the efficiency of a signal transform is calculated using wavelet packet analysis is matched before decomposition and reconstruction.

In Table 10, Band stop filtered signal is tabulated which removes the line frequency noise from the signal, it shows a reasonable SNR of 40–46 dB.

Figure 8 shows the tree diagram associated with a *depth-3* WPT. It reflects the structure of its corresponding hierarchical filter bank, such as the structure shown in Figure 2. Moving from top to bottom in the diagram

Table 9. Entropy for 4 levels.

Sample	Haar/Db4/Sym5/dmey/Coif5
T1	21937
T2	32511
T3	29809
T4	34655
T5	29145
T6	49058
T7	51218
T8	32971
T9	34857
T10	37040

Table 10. SNR after removing noise.

Sample	Haar/Db4/Sym5/dmey/Coif5
T1	44.16
T2	44.41
T3	41.29
T4	44.52
T5	44.47
T6	44.85
T7	44.58
T8	40.39
T9	46.36
T10	41.89

of **Figure 8**, frequency is divided into ever smaller segments. Each line that emanates down and to the left of a node represents a lowpass filtering operation (h_0), and each line emanating down and to the right a highpass filtering operation (h_1). The nodes that have no further nodes emanating down from them are referred to as *terminal nodes*, *leaves*, or *subbands*. We refer to the other nodes as *non-terminal*, or *internal* nodes. As such, the tree node labeling scheme provides a simple mechanism for indicating the nodes in the tree that we can work with when imparting modifications on the signal.

For the node (j,k) , j denotes the depth within the transform (tree) and k the position. For example, at node $(0,0)$ no filtering has taken place, and we simply have the original sequence of time samples. Lowpass filtering this will produce node $(1,0)$ and highpass filtering with produces $(1,1)$. These filtering operations are equivalent to finding the correlation of the signal with the scaling function for node $(1,0)$ and the correlation of the signal with the wavelet function for node $(1,1)$. Going down the tree to the next depth, we see that $(2,0)$ and $(2,1)$ emanate from $(1,0)$. From the filter perspective, the samples at $(1,0)$ are applied to the filters and. Multiresolution is achieved because the coefficients at $(1,0)$ have been down sampled by two to achieve *critical sampling*. From the wavelet and scaling function perspective, the correlations between both and the samples of $(1,0)$ are determined through this operation.

Figure 9 shows how the block diagram wise analysis and synthesis is carried out.

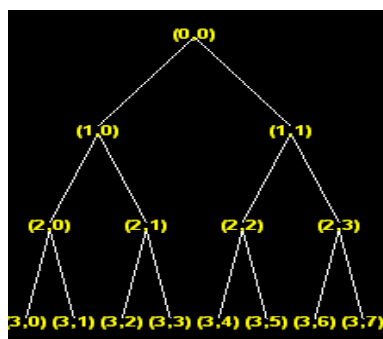
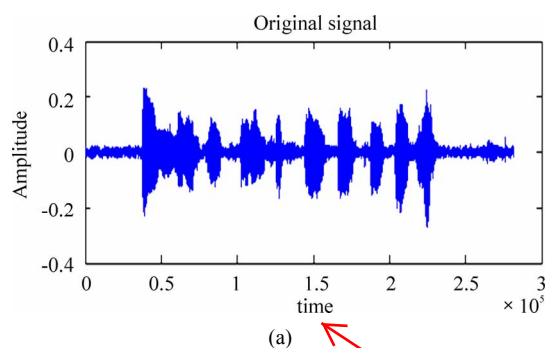
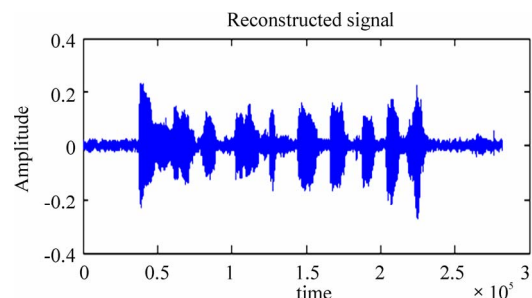


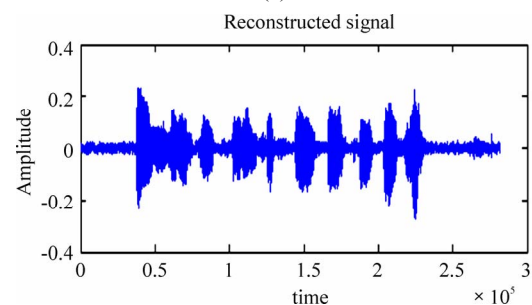
Figure 8. Wavelet packet tree for level 3 of decomposition.



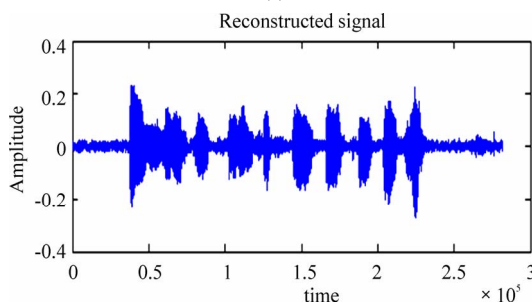
(a)



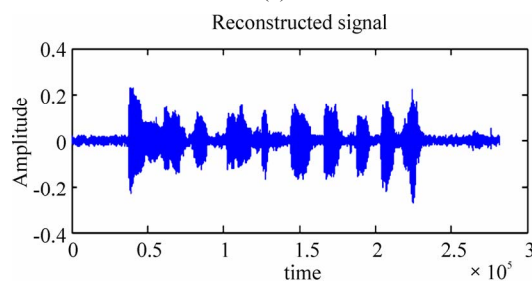
(b)



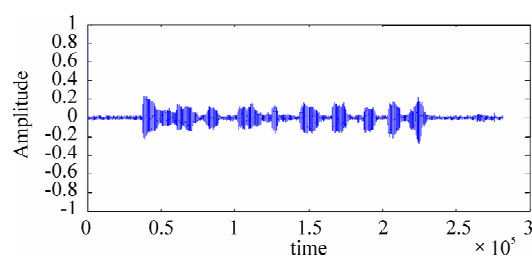
(c)



(d)



(e)



(f)

Figure 9. Input sample: (a) t7.wav; (b) For Haar wavelet; (c) For Db4 wavelet; (d) For Sym5 wavelet; (e) For dmey wavelet; (f) line freq filter.

5. Conclusions

We have presented a method for analysis and synthesis of time signals using wavelet packet filtering techniques. From this study we could understand and experience the effectiveness of wavelet packet transform in time signal analysis and synthesis. The performance of wavelet packet is appreciable while comparing with the discrete wavelet transform decomposition technique since wavelet packet analysis can provide a more precise frequency resolution than the wavelet analysis. It also has compact support in time as well as in frequency domain and adapts its support locally to the signal which is important in time varying signal. With wavelet packets we have a greater variety of options for decomposing the signal. The method presented is used for time as well as frequency analysis of time varying signals. From the results we conclude that the wavelet filtering find applications in the time domain analysis and synthesis era. In terms of signal quality, Haar wavelet has been seen to be the best mother wavelet. This is taken from the analysis of the signal to noise ratio (SNR) value around which is quite satisfactory for time varying signals. The system has been tested with various sampling frequencies for time domain samples which gave satisfactory output. Taking into consideration the signal quality and the time for analysis and synthesis it can be concluded that Haar wavelet is the best mother wavelet. Hence we conclude that the system will behave stable with wavelet packet filter and can be used for time signal analysis and synthesis purpose.

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