J. Service Science & Management, 2010, 3: 51-66
doi:10.4236/jssm.2010.31006 Published Online March 2010 (http://www.SciRP.org/journal/jssm)

Evolution or Revolution of Organizational Information Technology – Modeling Decision Makers' Perspective

Niv Ahituv[1*], Gil Greenstein[2]

[1]Faculty of Management, Tel Aviv University, Tel Aviv, Israel; [2]Faculty of Technology Management, Holon Institute of Technology, Holon, Israel; *Corresponding Author.
Email: ahituv@post.tau.ac.il, gilgr@hit.ac.il

Received December 28th, 2009; revised January 28th, 2010; accepted February 21st, 2010.

ABSTRACT

This paper suggests a new normative model that attempts to analyze why improvement of versions of existing decision support systems do not necessarily increase the effectiveness and the productivity of decision making processes. Moreover, the paper suggests some constructive ideas, formulated through a normative analytic model, how to select a strategy for the design and switching to a new version of a decision support system, without having to immediately run through a mega conversion and training process while temporarily losing productivity. The analysis employs the information structure model prevailing in Information Economics. The study analytically defines and examines a systematic informativeness ratio between two information structures. The analysis leads to a better understanding of the performances of decision support information systems during their life-cycle. Moreover, this approach explains normatively the phenomenon of "leaks of productivity", namely, the decrease in productivity of information systems, after they have been upgraded or replaced with new ones. Such an explanation may partially illuminate findings regarding the phenomenon known as the Productivity Paradox. It can be assumed that the usage of the methodology that is presented in this paper to improve or replace information structure with systematically more informative versions of information structures over time may facilitate the achievement of the following major targets: increase the expected payoffs over time, reduce the risk of failure of new versions of information systems, and reduce the need to cope with complicated and expensive training processes.

Keywords: *Decision Analysis, Decision Support Systems, Productivity and Competitiveness, Information Technology Productivity, the Productivity Paradox*

1. Introduction

A major and continuing problem in the information technology (IT) profession is the high rate of failure of new information systems (IS) or upgraded versions of them. From a rational point of view it may be assumed that IS professionals usually analyze and design IS "properly". But is it really so? Are they aware of the possibility of limits in perception among IS users, especially decision makers? Do they realize that "improvement" of decision support information systems might lead sometimes to a result opposite to what has been expected, namely degradation in the level of the productivity of the firms, since new and unfamiliar decision rules have not been fully implemented and adopted by the decision-makers?

This article suggests a new normative model that attempts to explain that improvement of versions of existing information systems do not necessarily increase the effectiveness and productivity of decision making processes. It also suggests some constructive ideas, formulated through a normative analytic model, how to select a strategy of switching to new version of a system, without having to immediately run through a mega training program, and to take a risk of losing productivity.

The methodological and theoretical foundations for the analysis presented here anchor in the literature on information economics. The earliest mathematical model presenting the relaying of information in a quantitative form was that of Shannon [1]. The model distinguished between two situations:

1) A noise-free system—a univalent fit between the transmitted input data and the received signals;

2) A noisy system—the transmitted input data (denot-

ing a state of nature) are translated into signals probabilistically.

In assigning an expected normative economic value to information, some researchers made use of Microeconomics and Decision Theory tools [2]. The combination of utility theory and the perception that information systems can be noisy led to the construction of a probabilistic statistical model that accords to an information system the property of transferring input data (states of nature) to output (signals) in a certain statistical probability [3–5]. This model, which delineates a noisy information system, is called the ***information structure*** model. It is based on the assumption that a system is noisy but it does not examine the nature of the noise. This paper expands the analysis by examining some patterns of noise. The consequences of that analysis are then demonstrated.

Over the years significant research was conducted to explore aspects of the phenomenon termed by Simon [6] as "***bounded rationality***"[1] and its main derivative—***satisficing*** behavior. Some of its aspects were presented comprehensively by Rubinstein [7]. Ahituv and Wand [8] showed that when satisficing is incorporated into the information structure model, there might be a case where none of the optimal decision rules will be pure anymore (unlike the results of optimizing behavior).

Ahituv [9] incorporated one of the aspects of bounded rationality into the information structure model: the inability of decision-makers to adapt instantaneously to a new decision rule when the technological characteristics of the information system, as expressed by the probabilities of the signals, are suddenly changed. Moreover, Ahituv [10] portrayed a methodology in which decision support systems are designed to act consistently during their lifecycle (in accordance with a constant decision rule). He suggested that this decision rule (that was an optimal decision rule in a previous version of the information system) guarantees improvement of expected outcomes, although it is not necessarily the optimal decision rule for later versions of this information system.

This study presents a conceptual methodology that combines aspects of bounded rationality [9,10] dealing with a rigid decision rule and the life-cycle of information systems, with elements of rational behavior presented in the information structure model [5].

The article raises some questions: Is it possible to improve an existing information system without adopting a new decision rule? What are the analytical conditions that enable a "smooth" (without much disturbance) upgrading or replacement of an information system? In a decision situation where two information structures are activated probabilistically, and one of them is generally more informative than the other, are there analytical conditions encouraging to enhance the percentage of usage of the superior system?

A normative framework is suggested to cope with essential processes (e.g.: implementation processes, correction of bugs, or upgrading of versions) during the life cycle of a decision support system [11]. By defining and analyzing a new informativeness relationship - "the systematic informativeness ratio", this paper demonstrates situations where decision-makers are equipped with partial information. Through these cases, it is explained how to assure a "smooth" implementation of new or upgraded information systems, as well as how to reduce the investment in implementation activities.

Moreover, It is shown that the existence of this new relationship (ratio) between two information structures enables to improve the level of informativeness without the awareness and the involvement of the users (the decision makers).

"The systematic informativeness ratio approach", which is presented and analyzed for the first time in this paper, contributes to better understanding of various aspects of the "productivity paradox" [12–14]. Furthermore, it portrays a methodology that suggests how to deal with some aspects of the "productivity paradox" which were explored in earlier studies [15,16].

The next section summarizes the information structure model and the Blackwell Theorem [5]. It describes the motivation to use convex combinations in order to describe processes during the life cycle of a decision support system. Section 3 describes, analyzes, and demonstrates a new informativeness relationship between two information systems—"***the systematic informativeness ratio***". Section 4 explores the existence of systematic informativeness ratio between un-noisy information structures. Section 5 presents some implications that could be extrapolated to noisy information structures. The last section provides a summary and conclusions, and presents the contribution of the study and the directions it opens for further research. Proofs of the theorems and lemmas appear in the appendix.

2. The Basic Models

2.1 The Information Structure Model and Blackwell Theorem

The source model employed in the forthcoming analysis is the information structure model [5]. This is a general model for comparing and rank ordering information systems based on the rules of rational behavior.[2]

The information structure model enables a comparison of information systems using a quantitative measurement reflecting their economic value. An information structure

[1]Simon termed the human decision-making process, which is affected by bounded rationality as "satisficing", and the decision-maker in accordance as a "satisficer" (aims to be satisfied with his or her decision). This is in contrast to the perception of the decision-maker under rational behavior assumptions in "classical" Utility theory who is an "optimizer" (aims to achieve the best out of his or her decision)

Q_1 is said to be more informative than an information structure Q_2 if the expected payoff of using Q_1 is not lower than the expected payoff of using Q_2. The expected payoff is trace (Π*Q*D*U)[3], where trace is an operator that sums the diagonal elements of a square matrix. The objective function for maximizing the expected compensation is $\underset{D}{Max}\left(trace(\Pi * Q * D * U)\right)$[4].

Let us examine a numerical example. Assume that an investment company serves its customers by using a web based information system. Let Q_1 be an information structure that predicts the attractiveness of investing in various alternative channels. The IS supports the decision-making of the investors. For simplicity, suppose there are three categories of states of nature: S_1 - accelerated growth (probability: 0.2), S_2 - stability (probability: 0.6), and S_3 - recession (probability: 0.2). Assume also that there are three possible decisions: A_1 - Invest in bank deposits; A_2 - Invest in stocks; A_3- Invest in foreign currency; Q_1 - The information system provides the following signals: Y_1- Accelerated growth is expected; Y_2 - Stability is expected; Y_3 - Recession is expected;

$$Q_1 = \begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.4 & 0.6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The compensations matrix U, which represents the expected percentage of profit or loss, is described as follows: $U = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 5 & 0 \\ -1 & -1 & 3 \end{pmatrix}$

$$\underset{D}{Max}\left(trace(\Pi * Q_1 * D * U)\right) = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.2 \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.4 & 0.6 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_{1,1} & D_{1,2} & D_{1,3} \\ D_{2,1} & D_{2,2} & D_{2,3} \\ D_{3,1} & D_{3,2} & D_{3,3} \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 & 1 \\ 0 & 5 & 0 \\ -1 & -1 & 3 \end{pmatrix} = 2.4$$

where $D^* = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$[5]. Invest *"$A_1$"*, while the signal is Y_1 or Y_2. Invest "A_3", while the signal is Y_3.

Given two information systems that deal with the same state of nature and are represented by the information structures Q_1 and Q_2, Q_1 is defined as generally more informative[6] Given two information systems that deal with the same state of nature and are represented by the information structures Q_1 and Q_2, Q_1 is defined as generally more informative.[7] The rank ordering is transitive.[8]

Over the years, a number of researchers developed analytical models to implement the concept of the *information structure model* in order to evaluate the value of information technology. Ahituv [10], demonstrated the life cycle of decision support information system with the model. Ahituv and Elovici [17] evaluated the value of performances of distributed information systems. Elovici

[2]According to the information structure model, four factors determine the expected value of information.

The a priori probabilities of pertinent states of nature. Let S be a finite set of n states of nature: $S=\{S_1,..,S_n\}$. Let P be the vector of a priori probabilities for each of the states of nature: $P=(p_1,..,p_n)$.

The information structure – a stochastic (Markovian) matrix that transmits signals out of states of nature. Let Y be a finite set of n signals, $Y=\{Y_1,..,Y_m\}$. An ***information structure*** Q is defined such that its elements obtain values between 0 and 1, Q: $SxY \to [0,1]$. $Q_{i,j}$ is the probability that a state of nature S_{ii} displays a signal Y_j $\sum_{j=1}^{m} Q_{i,j}=1$

The decision matrix – a stochastic matrix that links signals with the decision set of the decision-maker. Let A be a finite set of k possible decisions, $A=\{A_1,..,A_k\}$. Let D be the decision function. Similar to Q, D is a stochastic (Markovian) matrix, namely, it is assumed that the decision selected for a given signal is not necessarily always the same. $D:Y \times A \to [0,1]$

The payoff matrix – a matrix that presents the quantitative compensation to the decision-maker resulting from the combination of a decision chosen and a given state of nature. Let U be the payoff function: $U:A \times S \to \Re$ (a combination of a state of nature and a decision provides a fixed compensation that is a real number). $U_{i,j}$ – is the compensation yields when decision maker decides –"A_i", while state of nature *"S_j"* occurs.

[3]Sometimes Q represents an un-noisy (noise free) information structure. In these cases Q represents an information function f, $f{:}S \to Y$ [4]. Q is a stochastic matrix that contains elements of 0 or 1 only. This means that for each state of nature the information structure will always act identically (will produce the same signal), although it is not guaranteed that the state of nature will be exclusively recognized.

[4]When the utility function is linear, that is, the decision-maker is of the type EMV [2], a linear programming algorithm may be applied to solve the problem, where the variables being the elements of the decision matrix D. It can be proved that at least one of the optimal solutions is in a form of a decision matrix whose elements are 0 or 1 (a pure decision rule), [5]. For numerical illustrations of the model, see [8–10].

[5] D^* is a decision matrix which represent the optimal decision rules in this decision situation.

[6]It should be noted that when we deal with two information functions rather than structures we use the term "fineness" to describe the general informativeness ratio [4].

[7]In terms of the information structure model, if for every possible payoff matrix U, and for every a priori probability matrix Π $\underset{D}{Max}\left(trace(\Pi * Q_1 * D * U)\right) \geq \underset{D}{Max}\left(trace(\Pi * Q_2 * D * U)\right)$, then Q_1 is generally more informative than Q_2, Denoted: $Q1 \geq Q2$. Blackwell Theorem states that Q_1 is generally more informative than Q_2 if and only if there is a Markovian (stochastic) matrix R such that $Q_1 * R = Q_2$. R is termed the ***garbling*** matrix

[8]It should be noted that the general informativeness ratio is a partial rank ordering of information structures. There is not necessarily rank order between any two information structures.

et al [18] used this method to compare performances of Information Filtering Systems. Ahituv and Greenstein [15] used this model to assess issues of centralization vs. decentralization. Aronovich and Spiegler [19] use this model in order to assess the effectiveness of data mining processes.

The model was expanded to evaluate the value of information in several aspects: the value of a second opinion [20]; the value of information in non-linear models of the Utility Theory [21]; analyzing the situation of case dependent signals (the set of signal is dependent on the state of nature, [22]); a situation of a two-criteria utility function [23]. The model was also implemented to evaluate empirically the value of information in postal services [24], and in analysis of Quality Control methods [25,26].

2.2 The Use of *Convex Combinations*[9] of Information Structures to Represent Evolution during Their Life-Cycle

A possible reason why we should consider probabilistic combination of information systems is the existence of decision support systems that use Internet (or intranet) based search engines. These engines can retrieve information from several information sources, and produce signals accordingly. The various sources are not always available.

Information sources are essential for the proper survivability of competitive organizations. As a result, the importance of proper functioning of information systems is increasing. When a certain source in unavailable it is possible to acknowledge the users about it by alarming them with a special no-information signal [15]. Another option, which is presented in this paper, is to consider implementation of a "mixture" of information systems. For example: suppose there is "a state of the art" organizational information center that can serve, during peak times only 90% of the queries. How will the rest 10% are served? One alternative is to reject them.[10] Another one is to direct those queries to a simpler (perhaps cheaper) information system whose responses are less informative. This leads to consider probabilistic usage of information systems that can be delineated by a convex combination.

The analysis focuses on the convex combination of information structures reflecting a probabilistic employment of a variety of information systems (structures), where the activation of each one of them is set by a given probability. The various systems react to the same states of nature and produce the same set of signals.[11]

The mechanism of convex combinations[12] of information systems is employed in an earlier research by Ahituv and Greenstein [15] which analyses the effect of probabilistic availability of information systems on productivity, and illuminates some aspects of the phenomenon that are termed as "the productivity paradox" [12–14].

3. The Systematic Informativeness Ratio

3.1 Definition of the Systematic Informativeness Ratio

As mentioned in Section 2, when an information structure Q_1 is more informative than an information structure Q_2 irrespective of compensations and a priori probabilities, a general informativeness ratio exists between the two of them [5].

If an information structure Q_1 is more informative than Q_2 when the optimal decision rule of Q_2 is employed, and given some certain a priori probabilities of the states of nature, then under some assumptions on the payoffs, an informativeness ratio under a rigid decision rule is defined between them [9].[13]

---

[9]The convex combination of information structures was discussed in earlier studies. Marschak [4] notices that the level of informativeness of convex combination of two information structures (denoted Q_1 and Q_2) which produce the same set of signals is not equivalent to the level of informativeness of using Q_1 with a probability p and Q_2 with the complementary probability (1-p).

[10]Sulganik [27] indicates that a convex combination of information structure could be used to describe experimental processes (with a probability p of success and (1-p) of failure). For example, he investigates the convex combination of two information structures: one presents perfect information and the other one no-information (its rows are identical).

[11]It should be noted that in case that the two information structures do not produce the same set of signals, the non-identical signals can be represented in by columns of zeroes respectively [9].

[12]A convex combination of two information systems is defined as follows: Let Q_1 and Q_2 be two information structures describing information systems. Let $S=\{S_1,\dots,S_n\}$ be their set of the states of nature. Let $Y=\{Y_1,\dots,Y_m\}$ be their set of signals. When a decision situation is given let p the probability that Q_1 will be activated, and (1-p) that Q_2 will be activated. Since, decision makers do not aware which information structure is activated, Q_3, the weighted information structure, is represented by a convex combination of Q_1 and Q_2.

$Q_3 = p * Q_1 + (1-p) * Q_2$

[13]Given two information systems that deal with the same states of nature, produce the same set of signals, and are represented by the information structures Q_1 and Q_2 respectively, Q_1 will be considered more informative than Q_2 under a rigid decision rule if its expected payoff is not lower than that of Q_2 for the following conditions:

$\forall i, i = 1,..,n$. Let $U(k(i), i)$ the single maximum payoff when the state of nature S_i occurs.

Denote:

$\bar{u}^* = \max_i (U(k(i), i)$; $\underline{u}^* = \min_i (U(k(i), i)$; $\bar{u}0 = \max_{i,k\neq k(i)} (Uk, i)$; $\underline{u}0 = \min_{i,k\neq k(i)} (Uk, i)$;

$q_1^* = (q_{1(1,(k(1)),..,} q_{1(n,(k(n))})^t$ $q_2^* = (q_{2(1,(k(1)),..,} q_{2(n,(k(n))})^t$, respectively.

$\bar{\delta}^* = \bar{u}^* - \bar{u}_0$; $\underline{\delta}^* = \bar{u}^* - \underline{u}_0$; $\underline{\delta}_0 = \bar{u}_0 - \underline{u}_0$;

The theorem which is proved by Ahituv [9], states that:

If $\Pi^t(\underline{\delta}^* q_1^* - \bar{\delta}^* q_2^*) \geq \delta_0$ then Q_1 more informative than Q_2 with regard to U *and* Π.

The ratio will be denoted: $Q_1 \underset{R}{\geq} Q_2$

The informativeness ratio under a rigid decision rule is a partial rank ordering of information structures.

Assume those two informativeness ratios can be conceptually combined to a new informativeness ratio: Let Q_1 and Q_2 be two information structures that deal with the same state of nature and produce the same set of signals. Q_1 will be considered systematically more informative than Q_2 if for any decision situation (for any a priori probabilities vector- Π and any payoff matrix- U), its expected payoff is not lower than that of Q_2 while Q_1 operates under an optimal decision rule of Q_2. In terms of the information structure model, this is presented hereinafter by Definition 1.

Definition 1: Let Q_1 and Q_2 be two information structures representing two information systems operating on the same set of states of nature $S = \{S_1,...,S_n\}$ and producing the same set of signals $Y = \{Y_1,...,Y_m\}$. Q_1 is defined ***systematically more informative*** than Q_2, denote $Q_1 \underset{S}{\geq} Q_2$, if for any decision situation (irrespective of payoffs and a priori probabilities) Q_1 is more informative than Q_2 under an optimal decision rule of Q_2.

It means that if Q_1 is systematically more informative than Q_2, then for every decision situation[14] there exists an optimal decision rule of the inferior information structure Q_2, that can be used with the superior information structure Q_1, and guarantees at least the optimal outcomes of using Q_2.

Mathematically it looks this way:

$$\exists D_{Q2} \in \{D\max(Q_2)\}, trace(\Pi * Q_1 * D_{Q2} * U) \geq$$
$$\underset{D}{Max}(trace(\Pi * Q_2 * D * U)) = trace(\Pi * Q_2 * D_{Q2} * U))$$

where $\{D\max(Q_2)\}$ - denotes the set of optimal decision rules, when $Q2$ is activated in this specific decision situation.

In contrast to the general informativeness ratio, in the systematic informativeness ratio the information structure Q_2 can be replaced with the superior systematically information structure Q_1, without an immediate awareness of the decision makers (the users), since the decision rule does not necessarily have to be changed instantaneously. It means that when the systematic informativeness ratio exists between two information structures, at least the same level of expected payoffs is guaranteed when the superior[15] information structure is activated. Hence the decision maker does not have to adopt a new optimal decision urgently.

Let us now examine the informativeness ratio between two information systems from the point of view of "smooth" implementation. This is presented in Definition 2.

Definition 2: Let Q_1 and Q_2 be two information structures representing two information systems operating on the same set of states of nature $S=\{S_1,...,S_n\}$ and producing the same set of signals $Y=\{Y_1,...,Y_m\}$. Assume Q_1 is generally more informative than Q_2. A ***smooth implementation*** of Q_1 instead of Q_2 is defined if for any level of usage p $(0 \leq p \leq 1)$ $p * Q_1 + (1-p) * Q_2 \geq Q_2$.

The importance of this ratio is that in any probabilistic level of usage of the superior information system Q_1, the mean of the expected payoffs (compensation) that the decision-makers gain is not less than that achieved by using only the inferior information system. It contributes to a smooth implementation of the superior information structure Q_1.

In our study we argue that those definitions (1 & 2) are equivalent. Theorem 1 proves analytically the equivalence of Definition 1 and 2.

Theorem 1[16]

Let Q_1 and Q_2 be two information structures operating on the same set of states of nature $S = \{S_1,...,Sn\}$, and producing the same set of signals $Y = \{Y_1,...,Ym\}$. Then

$$Q_1 \underset{S}{\geq} Q_2 \Leftrightarrow \forall p, 0 \leq p \leq 1, \quad p * Q_1 + (1-p) * Q_2 \geq Q_2$$

This theorem shows that the two ratios which have been defined above are identical. Replacement (or improvement, or upgrade) of an information structure with a more systematically informative, information structure than it, guarantees smooth implementation, and vice versa.

Moreover, from the aforementioned equivalence it is understood that during a smooth implementation of the superior information structure Q_1 [17], we do not have to adopt a new decision rule, and we can stick to an optimal decision rule we used in the past with the inferior information structure Q_2. In fact this theorem sets a new normative perspective that defines the necessary and sufficient conditions for the ability to implement a superior information structure smoothly without immediate interference in the routine work of decision makers.

Using this method facilitates information systems professionals to plan systems under the assumption that during a certain transition period the decision-makers may act identically and stick to the same decision-rule [10]. The existence of this informativeness ratio reduces the criticality of an urgent implementation process.

3.2 A Framework to Examine the Existence of the Systematic Informativeness Ratio

In order to identify the existence of a systematic informativeness ratio between two information structures when one of them is generally more informative than the other, we would analyze a special case in which the number of signal and the number of states of nature are identical. In this case, the identity square matrix I is a complete and perfect information structure. We will try to find out whether I is systematically more informative than any other square stochastic matrix of similar

[14] A given set of a-priori probabilities - Π , and a given utility matrix - U.

[15] Systematically more informative than the other.

[16] The proof is provided in the appendix.

[17] Since $\forall p, 0 \leq p \leq 1, \quad p * Q_1 + (1-p) * Q_2 \geq Q_2$, $Q_1 \underset{S}{\geq} Q_2$.

dimensions.

The motivation to do this is provided by Lemma 1. Assume two information structures Q_1 and Q_2 act on the same set of states of nature, and respond with the same set of signals, and $Q_2 = Q_1 * R$, where R is a stochastic matrix (Blackwell Theorem's condition). In Lemma 1 it is shown that the existence of systematic informativeness ratio between I and R sets a pre-condition (sufficient condition) to the existence of the general informativeness ratio between Q_1 and Q_2.

Lemma 1[18]

Let Q_1 and Q_2 be two information structures operating on the same set of states of nature $S = \{S_1,...,S_n\}$, and producing the same set of signals $Y = \{Y_1,...,Y_m\}$**.** Assume that Q_1 is generally more informative than Q_2, implying that $Q_2= Q_1*R$, where R is a stochastic matrix [5]**.**

If $\forall p, 0 \le p \le 1,\ \ p*I + (1-p)*R \ge R$,

Then $\forall p, 0 \le p \le 1,\ \ p*Q_1 + (1-p)*Q_2 \ge Q_2$

From Lemma 1 it can be shown that if Q_1 is generally more informative than Q_2, namely $Q_2 = Q_1*R$ $\big($*R is a stochastic matrix) and* $I \underset{S}{\ge} R$ (I is systematically more informative than R) then $Q_1 \underset{S}{\ge} Q_2$ (Q_1 is systematically more informative than Q_2).

3.3 The Monotony of the Systematic Informativeness Ratio

The following lemma deals with the improvement of the accuracy level of information systems by enhancing the probability to receive perfect information.

Lemma 2[19]

Let I be an information structure that provides perfect information. Let Q be any information operating on the same set of states of nature $S = \{S_1,..,S_n\}$ and producing the same set of signals $Y = \{Y_1,...,Y_n\}$**.**

If: for $0 \le p \le 1, p*I + (1-p)*Q \ge Q$ (every convex combination of I and Q is generally more informative than Q)

Then:

$$\forall q, 0 \le q \le p \le 1, p*I + (1-p)*Q \ge q*I + (1-q)*Q$$

Conclusion:

If: $\forall p,\ 0 \le p < 1, p*I + (1-p)*Q \ge Q$

Then:

$$\forall p, \forall q,\ 0 \le q \le p < 1, p*I + (1-p)*Q \ge q*I + (1-q)*Q$$

This lemma proves the monotony of the systematic informativeness ratio. Actually it is shown that an improvement in the accuracy level of information (expressed by increasing the probability of perfect information) is positively correlated with the general informativeness ratio of a convex combination.

3.4 The Systematic Informativeness Ratio – An Example

We continue the example of Q_1, an Information system for choosing an investment option, which was first demonstrated in Section 2. $Q_1 = \begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.4 & 0.6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Suppose it is intended to replace the information system with an improved one, Q_2: $Q_2 = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.9 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Due to technological and organizational limitations, e.g.: inability to implement the system simultaneously all across the organization and the need to monitor carefully the system's performances, the system is implemented step by step.

By using some of the lemmas and theorems presented above, it can be demonstrated that the information structure Q_2 is systematically more informative than Q_1.

Let Q_0 be an information system: $Q_o = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Let Q_3 be an information structure, which represents perfect information. $Q_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Since,

$\forall p,\ \ 0 \le p \le 1,\qquad \big(p*Q_3 + (1-p)*Q_0\big)*Q_0 = Q_0$ [20]

$$\left(p*\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (1-p)*\begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

$$*\begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

then, from Theorem 1 it is clear that Q_3 is systematically more informative than Q_0.

Let us present Q_1 and Q_2 as convex combination of Q_3 and Q_0.

$$Q_2 = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.9 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0.8*Q_3 + 0.2*Q_0$$

[18]The proof and an example are provided in the Appendix.
[19]The proof is provided in the Appendix.
[20]We use the information structure Q_0, as a garbling (stochastic) matrix, either.

$$= 0.8 * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 0.2 * \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.4 & 0.6 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0.2 * Q_3 + 0.8 * Q_0$$

$$= 0.2 * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 0.8 * \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

According to Lemma 2 Q_2 is systematically more informative than Q_1. **Table 1** demonstrates the implication of the existence of the systematic informativeness ratio between Q_2 and Q_1.

This example illustrates that if an upgrading of an information system is based on the implementation of later versions of it which are systematically more informative than the earlier versions, then sticking to the old and familiar decision rule will not harm productivity.

The principle of developing information systems to be systematically more informative provides the luxury of training and on-site implementation which is "life-cycle independence". It facilitates the implementation of a new version of information system or insertion of minor changes, without the immediate awareness of the decision makers. Therefore, organizations can schedule the optimal timing of wide training processes. That is in contrary to the usual situation, when the scheduling of training and on-site implementation might interfere with other organizational considerations and requirement (e.g.: periodically tasks). Moreover, development of new decision support systems without adopting this principle may explain, normatively, "leaks of productivity". In other words it may explain the decrease in user performance of information systems, although they have been improved. This degradation in the expected outcomes while using improved information systems can be attributed to the inability of users to adapt immediately to new decision rules.

4. The Systematic Informativeness Ratio – A Noise Free Scenario.

4.1 Conditions for Existence of the Systematic Informativeness Ratio

Historically, the starting point for analyzing the value of information in noisy information structures was the analysis of the value of information in noise free information structures. These are also termed **information functions** [4,5]. Following this approach, we will start with a simple presentation of the informativeness ratio between in formation functions, which could be classified as unnoisy information structures.

In order to identify the existence of a systematic informativeness ratio between two information functions, a new aggregation ratio (a fineness ratio that keeps orders of signals) between information functions is defined, hereinafter.

Definition 3

Let f_I be the identity information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature and $Y = \{Y_1,..,Y_n\}$ the set of signals f_I produces. Hence, $f_I(S_i) = Y_i$.

Assume for simplicity that $S = \{1,..,n\}$, $Y = \{1,..,n\}$ then $f_I(i) = i$

Table 1. Comparison of two information structures, one of them is systematically more informative than the other

	The current information structure	The improved information structure	
Information structure	$Q_1 = \begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.4 & 0.6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$Q_2 = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.9 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
The matrix of a priori probabilities for the states of nature + The matrix of compensation (percentage): $\Pi = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.2 \end{pmatrix}$ $U = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 5 & 0 \\ -1 & -1 & 3 \end{pmatrix}$			
The Decision rules: *A*1 – Invest in Bank Deposits *A2* – Invest in stocks *A3* – Invest in foreign currency	$D = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$D = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Expected compensation (percentage)	2.4	2.4	3.12

Let F be a **set** of information functions that acts on the same decision environment of fi .

$$g \in F \Leftrightarrow \big(g(i) = i \quad \text{or if} \quad g(i) = k \neq i, \text{ then } g(k) = k\big)$$

Since there is an isomorphism between the representation of information functions and a set of information structures representing them, F can be defined analogically as a set of un-noisy information structures.

$$F1 \in F \Leftrightarrow (F1_{i,i} = 1 \quad \text{or if} \quad F1_{i,k} = 1, k \neq i, \text{ then } F1_{k,k} = 1)$$

Following that fi is equivalent (for example) to I, an information structure that produces perfect information I is represented by the identity Matrix of the order nxn.

In fact F is a complete set of information functions that could be termed as aggregations of fi . If $g \in F$ its way of transforming of states of nature into signals does not contradict the way fi transforms states of nature into signals. In other words it can be said that fi is a higher resolution of every information function belonging to the set F .

Theorem 2 sets the necessary and sufficient conditions for the existence of systematic informativeness ratio between I (the identity information structure), and any non-noisy information structure:

Theorem 2[21]

Let $F1$ be an information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be its set of signals. Let I be the identity information function, which represent perfect information, then $I \underset{S}{\geq} F1$ **if and only if** $F_1 \in F$.

It is shown in Theorem 2 that for every information function (non-noisy information structure) $F1$, the necessary and sufficient condition that I is systematically more informative than $F1$, is that $F1^2 = F1$. In fact, four equivalent conditions are found as we will show below.

Let $F1$ be an information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be its set of signals. The following conditions are equivalent.[22]

1) $I \underset{S}{\geq} F_1$

2) $\forall p,\ 0 \leq p \leq 1 \quad p{*}I + (1\text{-}p){*}F_1 \geq F_1$

3) $F_1 \in F$

4) $F_1^2 = F_1$

The equivalence of the first and second conditions (which was demonstrated earlier by Definition 1 and 2 respectively) was proven by Theorem 1. Conditions 1 and 2 are not specific to un-noisy information structures, and can hold for any type of structure. In contrast to Conditions 1 and 2, the third and fourth conditions are relevant only to un-noisy information structures. By using those two latter conditions we can explicitly classify un-noisy and diagonal information structures into two separate classes:

1) Structures that the identity information structure is systematically more informative than them,

2) Structures that the identity information structure is not systematically more informative than them.

4.2 The Implications of the Systematic Informativeness Ratio – An Example

In the example that follows, two scenarios are presented, analyzed, and compared. The first scenario: upgrading an un-noisy information structure F_1 to I—the identity information structure, while I is systematically more informative than F_1.

The second scenario: upgrading an un-noisy information structure F_2 to I—the identity information structure, while I is **not** systematically more informative than F_2.

We use the situation of choosing an investment alternative, that was shown earlier in Section 3, except the fact that the current information structures are F_1 or F_2 respectively.

Suppose an un-noisy information structure that predicts the attractiveness of an investment in various channels is installed. This information structure does not distinguish between S_1 – accelerated growth, and S_2 – stability. It is intended to replace the information system with I - an information function that provides perfect information: $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Due to technological and organizational limitations, e.g.: inability to implement the system simultaneously all across the organization and the need to monitor carefully the system's performance, the system is implemented step by step.

First Scenario: The existing information structure is $F1$.

$$F_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow F_1^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = F_1 \Rightarrow F_1 \in F$$

Since $F_1 \in F$, it can be shown from Theorem 2 that $I \underset{S}{\geq} F_1$. **Table 2** demonstrates that while the probability

[21]The proof is provided in the Appendix.

[22]While, Theorem 1 proves the equivalence between expressions 1&2, Theorem 2 proves the equivalence between expressions 3&4, and then proves the equivalence between expressions 1&3.

for perfect information increases, the expected compensation increases too.

Second Scenario: Suppose the decision situation is identical to the previous one, but instead of F_1 the existing information structure is F_2 where:

$$F_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow F_2^2$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \neq F_2 \Rightarrow F_2 \notin F$$

Since $F_2 \notin F$, according to Theorem 2 I is not systematically more informative than F_2.

Table 3 demonstrates that although the probability of perfect information increases the level of informativeness declines.

The comparison between those two scenarios is demonstrated in **Figure 1**:

By observing the aforementioned example it can be concluded that, when a new (improved) information system is systematically more informative than the current information system two important goals are achieved:

1) "Decision situation independence" -The ability to implement the information system step by step and to improve the level of informativeness is guaranteed.

2) "Life-cycle independence" -The ability to implement the information system without interfering the users (the decision makers) and while existing expected outcomes are guaranteed (without the necessity to start training and testing processes).

5. Towards Assessing the Systematic Informativeness Ratio between Noisy Information Structures – the Dominancy of Trace

A characteristic of F is that its diagonal elements are (weakly) dominant (in accordance with Definition 4). From Theorem 3 it can be shown that this characteristic is a necessary condition for existence of the systematic informativeness ratio between I and Q:

Theorem 3[34]

Let I (the identity matrix), and Q be two information structures. $S = \{S_1,..,S_n\}$ is the common set of states of nature, and $Y = \{Y_1,..,Y_n\}$ is their same set of signals they produce.

$I \underset{S}{\geq} Q \Rightarrow \forall i, i = 1,..,n,\ Q_{i,i} \geq Q_{j,i}\ i \neq j$ (The diagonal elements are weakly dominant in each and every column).

This theorem implies that the dominancy of the diagonal elements in each and every column of an information structure is a necessary condition for the existence of a systematic informativeness ratio between the identity information structure (which represents complete information) and the non-identity information structure. This casts a preliminary condition for the existence of the informativeness ratio.

The following example demonstrates, by using Theorem 3, that the systematic informativeness ratio is not always transitive.

$$F_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, F_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Table 2. Expected compensation in various levels of prob. for perfect information (1st scenario)

Characteristics of the Decision situation	The probability to receive I.	The probability to receive F_1	Expected compensation
A-priory probabilities: (0.2,0.6,0.2)	0	1	2.6
	0.2	0.8	2.76
Perfect information $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	0.4	0.6	2.92
	0.6	0.4	3.08
	0.8	0.2	3.24
Partial information $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	1	0	3.4

Table 3. Expected compensation in various levels of probability of perfect inf. (2nd scenario)

Characteristics of the Decision situation	The probability to receive I	The probability to receive F_2	Expected compensation
A-priory probabilities: (0.2,0.6, 0.2)	0	1	2.6
	0.2	0.8	2.52
Perfect information $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	0.333 (1/3)	0.666 (2/3)	2.4666
	0.4	0.6	2.56
	0.6	0.4	2.84
	0.8	0.2	3.12
Partial information $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	1	0	3.4

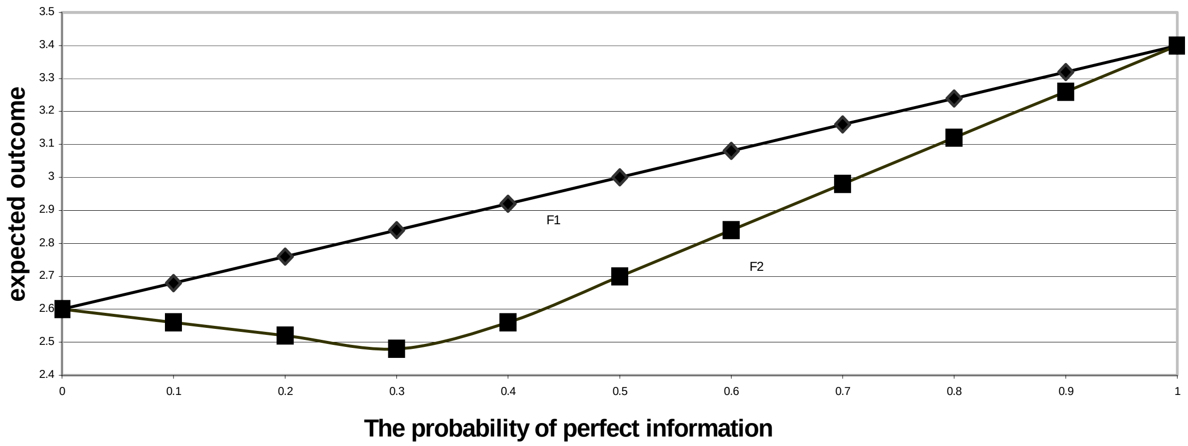

Figure 1. A comparison between the two scenarios

$$Q_1 = 0.25 * I + 0.75 * F_1 = 0.25 * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$+0.75 * \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.75 & 0.25 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = Q_1 * \left(0.4 * I + 0.6 * F_2\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0.75 & 0.25 & 0 \\ 0 & 0 & 1 \end{pmatrix} *$$

$$\left(0.4 * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 0.6 * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \right) =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0.75 & 0.25 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.6 & 0.4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.75 & 0.25 & 0 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

Since $F_1 \in F$, from Theorem 2 it is concluded that $I \underset{S}{\geq} Q_1$. Moreover, since $F_2 \in F$, from Lemma 1 it is concluded that $Q_1 \underset{S}{\geq} Q_2$. However, from Theorem 3, it is concluded that since $Q2_{3,2} > Q2_{2,2}$, I is not systematically more informative than Q_2.

Since the systematic informativeness ratio is not always transitive, when there is a multi stage implementation and improvement program during the life-cycle of a decision-support information system and the informativeness ratio of this information system can be improved systematically, the preservation of systematically informativeness ratio is not automatically guaranteed during the whole life-cycle of information system. Hence, the importance of a long-range perspective arises. This can be achieved in one of two ways, depending on the ability to guarantee whether the last version of information structure can be systematically more informative than any previous version, or only superior to its predecessor version:

When the systematic informativeness ratio can be obtained between each and every two sequential versions of an information system during its lifecycle, then a long-range plan of the versioning mechanism is required. This could guarantee that the latest version of an information system will be systematically more informative than any of the previous versions. Moreover, it will guarantee a growth (or at least stability) in expected outcomes during the lifecycle of the decision support system, without alerting the decision makers. Hence, implementation and training processes between versions of the information system become less critical.

When the systematic informativeness ratio can be achieved only between the last version of an information system and its predecessor version, then a training and implementation plan is required. However, the existence of systematic informativeness ratio between consequent versions reduces the costs and lowers the criticality of the implementation and training processes.

6. A Summary and Conclusions

This paper analytically examines and identifies the systematic informativeness ratio between two information structures. The methodological approach presented here may lead to a better understanding of the performances of decision support information systems during their life-cycle.

This approach may explain, normatively, the phenomenon of "leaks of productivity". In other words it may explain the decrease in productivity of information systems, after they have been improved or upgraded. This degradation in the expected outcomes can be explained by the inability of the users to adapt immediately to new decision rules.

It can be assumed that the usage of the methodology that was presented in this paper to improve or replace information structure with systematically more informative versions of information structures over time may facilitate the achievement of the following major targets:

1) Increase the expected payoffs over time.

2) Reduce the risk of failure of new information systems as well as new versions of information systems.

3) Reduce the need to cope with complicated and expensive training processes during the implementation stages of information systems (as well as the implementation of new versions of the systems). Moreover, sometimes this process can be completely skipped during the installation of a new version of an information system.

The paper analyzes the conditions for the existence of a systematic informativeness ratio between I -the identity information structure which represents complete information, and another information structure. In the case of non-noisy information structures the necessarily and sufficient conditions for existence of the systematic informativeness ratio between I and a second information structure are set and proved comprehensively. As a result, some necessary and sufficient conditions are set, proved and demonstrated for the noisy information environment as well.

Further research can be carried out in some directions:

1) Exploration of additional analytical conditions for the existence of the systematic informativeness ratio between I, the identity information structure and noisy information structures.

2) Classification of cases where the systematic informativeness ratio inheritably exists by using the conditions those are set so far.

3) Devising empirical methods to examine the impact of using the principle of developing decision support information systems is systematically more informative over time, on the performance of decision-makers, as well as on their perceived satisfaction from using those systems.

4) Designing empirical studies (experiments, case studies and surveys) to validate the theoretical analysis provided here.

REFERENCES

[1] C. E. Shannon and W. Weaver, "The mathematical theory of communication," University of Illinois Press, Urbana, Illinois, 1949.

[2] H. Raiffa, "Decision analysis," Reading, Addison-Wesley, Massachusetts, 1968.

[3] G. A. Feltham, "The value of information," The Accounting Review, Vol. 43, No. 4, pp. 684–696, 1968.

[4] J. Marschak, "Economic of information systems," Journal of the American Statistician Association, Vol. 66, pp. 192–219, 1971.

[5] C. B. McGuire and R. Radner, (editors), "Decision and organization (ch. 5)," University of Minnesota Press, 2nd edition, Minneapolis, Minnesota, 1986.

[6] H. A. Simon, "The new science of management decisions," Harper and Row, New-York, 1960.

[7] A. Rubinstein, "Modeling bounded rationality," MIT. Press, Cambridge, Massachusetts, 1998.

[8] N. Ahituv and Y. Wand, "Comparative evaluation of information under two business objectives," Decision Sciences, Vol. 15, No. 1, pp. 31–51, 1984.

[9] N. Ahituv, "A comparison of information structure for a 'Rigid Decision Rule' case," Decision Science, Vol. 12, No. 3, pp. 399–416, 1981.

[10] N. Ahituv, "Describing the information system life cycle as an adjustment process between information and decisions," International Journal of Policy Analysis and Information Systems, Vol. 6, No. 2, pp. 133–145, 1982.

[11] G. Ariav and M. J. Ginzberg, "DSS design: A systemic view of decision support," Communications of the ACM, Vol. 28, No. 10, pp. 1045–1052, 1985.

[12] E. Brynjolfsson, "The productivity paradox of information technology," Communications of the ACM, Vol. 36, No. 12, pp. 67–77, 1993.

[13] E. Brynjolfsson, "Paradox lost," CIO, Vol. 7, No. 14, pp. 26–28, 1994.

[14] E. Brynjolfsson and L. M. Hitt, "Beyond the productivity paradox," Communications of the ACM, Vol. 41, No. 8, pp. 49–55, 1998.

[15] N. Ahituv and G. Greenstein, "Systems inaccessibility and the productivity paradox," European Journal of Operational Research, Vol. 161, pp. 505–524, 2005.

[16] M. C. Anderson, R. D. Banker, and S. Ravindran, "The new productivity paradox," Communications of the ACM, Vol. 46, No. 3, pp. 91–94, 2003.

[17] N. Ahituv and Y. Elovici, "Evaluating the performance of an application running on a distributed system," Journal of the Operational Research Society, Vol. 52, pp. 916-927, 2001.

[18] Y. Elovici, B. Shapira, and P. B. Kantor, "Using the information structure model to compare profile-based information filtering," Information Retrieval, Vol. 6, No. 1,

pp. 75–97, 2003.

[19] L. Aronovich and I. Spiegler, "CM-tree: A dynamic clustered index for similarity search in metric databases," Data & Knowledge Engineering, Vol. 3, pp. 919–946, 2007.

[20] N. Ahituv and B. Ronen, "Orthogonal information structure—a model to evaluate the information provided by a second opinion," Decision Sciences, Vol. 19, No. 2, pp. 255–268, 1988.

[21] Z. Safra and E. Sulganik, "On the nonexistence of blackwell's theorem-type, results with general preference relations," Journal of Risk and Uncertainty, Vol. 10, pp. 187–201, 1995.

[22] E. Sulganik and I. Zilcha, "The value of information the case of signal dependent opportunity sets," Journal of Economic Dynamics and Control, Vol. 21, pp. 1615–1625, 1997.

[23] B. Ronen and Y. Spector, "Theory and methodology evaluating sampling strategy under two criteria," European Journal of Operational Research, Vol. 80, pp. 59–67, 1995.

[24] N. Carmi and B. Ronen, "An empirical application of the information-structures model: The postal authority case," European Journal of Operational Research, Vol. 92, pp. 615–627, 1996.

[25] N. Margaliot, "Selecting a quality control attribute sample: An information-economics method," Annals of Operations Research, Vol. 91, pp. 83–104, 1999.

[26] B. Ronen, "An information-economics approach to quality control attribute sampling," European Journal of Operational Research, Vol. 73, pp. 430–442, 1994.

[27] E. Sulganik, "The value of information under diverse economic environments," Ph. D. Dissertation, Tel Aviv University, Faculty of Management (Hebrew), Tel Aviv, 1993.

Appendix

Theorem 1:

Let Q_1 and Q_2 be two information structures operating on the same set of states of nature $S = \{S_1,...,S_n\}$, and producing the same set of signals $Y = \{Y_1,...,Y_m\}$. Then

$$Q_1 \underset{S}{\geq} Q_2 \iff \forall p, 0 \leq p \leq 1,\quad p*Q_1+(1-p)*Q_2 \geq Q_2$$

First, Lemma 1.1 is proven.

Lemma 1.1:

Let Q_1 and Q_2 be two information structures describing information systems. Let $S = \{S_1,...,S_n\}$ be their set of the states of nature of Q_1 and Q_2. Let $Y=\{Y_1,...,Y_m\}$ be their set of signals. Then for any given decision situation described by Π (a matrix of a-priori probabilities of states of nature), U (a matrix of utilities or compensations), A (a set of decisions), where $\{D_{pure\text{-}\max}(Q_2)\}$ is the set of optimal decision rules when Q_2 is used, there exists ε>0, such that if 0<p≤ε, and Dp is an optimal decision rule of the Information structure $p*Q_1+(1-p)*Q_2$

Then $D_p \in \{D_{pure\text{-}\max}(Q_2)\}$.

Proof (of Lemma 1.1):

1) It can be assumed that every optimal decision rule is a convex combination of pure decision rules [10]. So we try to find the optimal decision rule of $p*Q_1+(1-p)*Q_2$ in the set of the optimal pure decision rules of Q_2, $D_p \in \{D_{pure\text{-}\max}(Q_2)\}$.

2) Let k be the number of possible decisions in this given decision situation. This means that there are k^m pure decision rules, denoted $D_1,..,D_{k^m}$.

Let $\{D_{pure}\}$ the full set of the possible pure decision rules for this given decision situation

3) If $\{D_{pure\text{-}\max}(Q_2)\} = \{D_{pure}\}$, that means that every pure decision rule is an optimal decision rule it is obvious that $D_p \in \{D_{pure\text{-}\max}(Q_2)\}$.

4) So, assume that $\{D_{pure\text{-}\max}(Q_2)\} \underset{\neq}{\subset} \{D_{pure}\}$.

5) Hence: $\{D_{pure}\} \setminus \{D_{pure\text{-}\max}(Q_2)\} \neq \phi$

6) Let's calculate for every pure strategy D_i the following values: $V_{1i} = trace(\Pi * Q_1 * D_i * U)$,

$V_{2i} = trace(\Pi * Q_2 * D_i * U)$

7) $trace(\Pi*(p*Q_1+(1-p)*Q_2)*D_i*U)) =$

8) $= p*trace(\Pi*Q_1*D_i*U))$
$+(1-p)*(trace(\Pi*Q_2*D_i*U)) =$

9) $= p*V_{1i}+(1-p)*V_{2i}$

10) Let's define in this specific decision situation:

- $V_1\max = \underset{D}{Max}\{V_{1i}\}$ - The (optimal) expected value when using the information structure Q_1.
- $V_1 = \underset{Di\in Dpure\text{-}\max(Q2)}{Max}\{V_{1i}\}$ - The (optimal) expected value when using the information structure Q_1, when the set of decision rule is limited to the optimal set of pure decision rules when using the information structure Q_2.
- $V_{2\max} = \underset{Di\in Dpure\text{-}\max(Q2)}{Max}\{V_{2i}\}$ - The (optimal) expected value when using the information structure Q_2.
- $V_2 = \underset{Di\notin Dpure\text{-}\max(Q2)}{Max}\{V_{2i}\}$ - The (optimal) expected value when using the information structure Q_2, when the set of decision rule is limited to the non-optimal set of pure decision rules when using the information structure Q_2.

11) According to expression (4) $\{D_{pure\text{-}\max}(Q_2)\} \underset{\neq}{\subset} \{D_{pure}\}$

Hence: $\Delta V_2 = (V_{2\max} - V_2) > 0$

12) Moreover: $\Delta V_1 = (V_{1\max} - V_1) \geq 0$

13) Let's examine for $Di \in \{Dpure\} \setminus \{Dpure\text{-}\max(Q2)\}$ when it is not an optimal decision rule of $p*Q_1+(1-p)*Q_2$. We try to identify a small value of p that will always give $p*V_{1i}+(1-p)*V_{2i} < p*V_1+(1-p)*V_{2\max}$. In fact, the purpose is to find an "environment" of Q_2 where an optimal decision rule of Q_2 is also an optimal decision rule of $p*Q_1+(1-p)*Q_2$.

14) From (9) it is concludes that

$$p*V_{1i}+(1-p)*V_{2i} \leq p*V_{1\max}+(1-p)*V_2$$

15) Let's examine weather exists:

$$p*V_{1\max}+(1-p)*V_2 < p*V_1+(1-p)*V_{2\max}$$

16) $\iff p*(V_{1\max}-V_1)+p*(V_{2\max}-V_2) < V_{2\max}-V_2$
$\iff p*(\Delta V_1+\Delta V_2) < \Delta V_2$

$$\iff 0 < p < \frac{\Delta V_2}{\Delta V_1+\Delta V_2}$$

That's according to (10), (11)

$\Delta V_1 \geq 0,\quad \Delta V_2 > 0,\quad \Delta V_1+\Delta V_2 > 0$

17) Let's pick: $0 < \varepsilon < \dfrac{\Delta V_2}{\Delta V_1+\Delta V_2} \leq 1$

18) And in this environment (for every $0 \leq p \leq \varepsilon$) at least one optimal decision rule of $Q2$ is an optimal decision rule of $p*Q_1+(1-p)*Q_2$

Q.E.D (Lemma 1.1)

Proof (of the theorem itself):

First direction: assume that for every decision situation:

1) $\exists D_{Q2} \in \{D_{\max}(Q_2)\}, trace(\Pi*Q_1*D_{Q2}*U)$
$\geq \underset{D}{Max}(trace(\Pi*Q_2*D*U))$

where $\{D_{\max}(Q_2)\}$ is the set of optimal decision rules when Q_2 is used in this specific decision situation.

Then implies:

2) $\underset{D}{Max}(trace(\Pi * (p * Q_1 + (1-p) * Q_2) * D * U)) =$

3) $= \underset{D}{Max}(p * trace(\Pi * Q_1 * D * U) + (1-p) * (trace(\Pi * Q_2 * D * U))$

4) $\geq p * trace(\Pi * Q_1 * D_{Q2} * U) + (1-p) * trace(\Pi * Q_2 * D_{Q2} * U)$

5) $\geq p * trace(\Pi * Q_2 * D_{Q2} * U) + (1-p) * trace(\Pi * Q_2 * D_{Q2} * U) =$

6) $= \underset{D}{Max}(trace(\Pi * Q_2 * D * U))$ (first direction is proven)

Second Direction:

7) $\forall p, 0 \leq p \leq 1, p * Q_1 + (1-p) * Q_2 \geq Q_2$

8) According to the lemma there exists ε>0, such that if 0<p1≤ε<1, then exists *Dp1 an* optimal decision rule of the IS: $p_1 * Q_1 + (1-p_1) * Q_2$ that implies

$Dp_1 \in \{D_{pure - \max}(Q_2)\}$.

9) Let D_{Q_2} this optimal decision rule

$D_{Q2} \in \{D_{pure\text{-}\max}(Q2)\}$

10) $\underset{D}{Max}(trace(\Pi * (p_1 * Q_1 + (1-p_1) * Q_2) * D * U)) =$

11) $= (trace(\Pi * (p_1 * Q_1 + (1-p_1) * Q_2) * D_{Q2} * U)) =$

12) $p_1 * trace(\Pi * Q_1 * D_{Q_2} * U) + (1-p_1) trace(\Pi * Q_2 * D_{Q_2} * U) \geq$

13) $\geq trace(\Pi * Q_2 * D_{Q2} * U)$ (According to (7))

14) $= p_1 * trace(\Pi * Q_2 * D_{Q_2} * U) + (1-p_1) trace(\Pi * Q_2 * D_{Q_2} * U)$

15) From (12), (13), (14) $\Rightarrow$ $trace(\Pi * Q_1 * D_{Q_2} * U)$

$\geq trace(\Pi * Q_2 * D_{Q_2} * U)$

That is correct for every decision situation (any given Π and U)

Q.E.D

Lemma 1

Let Q_1 and Q_2 be two information structures operating on the same set of states of nature $S=\{S_1,\ldots,S_n\}$, and producing the same set of signals $Y=\{Y_1,\ldots,Y_m\}$. Assume that Q_1 is generally more informative than Q_2, implying that $Q_2 = Q_1 * R$, where R is a stochastic matrix [5].

If $\forall p, 0 \leq p \leq 1,\ \ p * I + (1-p) * R \geq R$,

then $\forall p, 0 \leq p \leq 1,\ \ p * Q_1 + (1-p) * Q_2 \geq Q_2$

Proof:

1) According to the second condition of Blackwell's theorem [5] for every p there exists *Rp*, where *Rp* is a stochastic matrix of the order nxn.

$(p * I + (1\text{-}p) * R) * Rp = R$

2) Therefore: $Q_1 * (p * I + (1\text{-}p) * R) * Rp = Q_1 * R$

3) Hence: $(p * Q_1 + (1-p) * Q_2) * R_p = Q_2$

Q.E.D.

Lemma 2:

Let I be an information structure that provides perfect information. Let Q be any information operating on the same set of states of nature $S = \{S_1,..,S_n\}$ and producing the same set of signals $Y = \{Y_1,..,Y_n\}$.

If: for $0 \leq p \leq 1, p * I + (1-p) * Q \geq Q$ (every convex combination of I and Q is generally more informative than Q)

Then:

$\forall q, 0 \leq q \leq p \leq 1, p * I + (1-p) * Q \geq q * I + (1-q) * Q$

Proof:

1) According to the 2nd condition of Blackwell's theorem, $\exists R_p$, R_p is a stochastic matrix, such that: $(p * I + (1\text{-}p) * Q) * Rp = Q$

2) Since, $p \geq q$, $\left[\frac{q}{p} * I + \frac{p-q}{p} * R_p\right]$ is a stochastic matrix.

3) Let's examine:

$[p * I + (1\text{-}p) * Q] * [\frac{q}{p} * I + \frac{p-q}{p} * R_p] =$

4) $= [p * \frac{q}{p} * I + (1\text{-}p) * Q * \frac{q}{p} * I] + \frac{p-q}{p} * [p * I + (1\text{-}p) * Q] * R_p =$

5) $= q * I + \frac{q\text{-}p*q}{p} * Q + \frac{p-q}{p} * Q = q * I + \frac{q\text{-}p*q + p\text{-}q}{p} * Q = q * I + (1-q) * Q$

6) According to the 2nd condition of Blackwell's theorem (5 $\Rightarrow$)

$\forall q, 0 \leq q \leq p \leq 1, p * I + (1-p) * Q \geq q * I + (1-q) * Q$

Q.E.D

Theorem 2:

Let F_1 be an information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be its set of signals. Let I be the identity information function, which represent perfect information, then $I \underset{S}{\geq} F_1$ if and only if $F_1 \in F$

First 3 lemmas are demonstrated and proven:

Lemma 2.1

Let fI be the identity information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be its sets of signals. $f_I(S_i) = Y_i$.

Without loosing generality (for the sake of simplicity)

Assume $S = \{1,..,n\}$, $Y = \{1,..,n\}$.

Let F be the set of information functions (without garbling of signals) that f_I is systematically more informative than each one of them:

$g \in F \Leftrightarrow (g(i) = i \quad or \quad (if \ \ g(i) = k \neq i, \ then \ \ g(k) = k)$

Let $g_1, g_2 \in F$ and g_1 is finer than g_2 then:

$\forall i, i = 1,..,n, \ g_2(g_1(i)) = g_2(i)$

Proof (of Lemma 2.1):

1) Let's check all the possible situations, given:

$g \in F \Leftrightarrow (g(i) = i \quad or \quad (if \ \ g(i) = k \neq i, \ then \ \ g(k) = k)$

2) 1st Case: $g_1(i) = g_2(i) = i \Rightarrow g_2(g_1(i)) = g_2(i) = i$

3) 2nd Case: $g_1(i) = g_2(i) = k \neq i \Rightarrow g_2(k) = k = g_2(g_1(i)) = g_2(k) = k$

4) 3rd Case: $g_1(i) = i, \ g_2(i) = k \neq i \Rightarrow g_2(g_1(i)) = g_2(i) = k$

5) 4th Case: $g_1(i) = k \neq i, g_2(i) = i \Rightarrow g_1(k) = k$

6) Since g_1 is finer than g_2: $g_2(k) = i$

7) Hence: $(5),(6) \Rightarrow g_2(g_1(i)) = g_2(k) = i = g_2(i)$

8) 5th Case:

$g_1(i) = k \neq i, g_2(i) = j \neq i \Rightarrow g_1(k) = k, \ g_2(j) = j$

9) Moreover, since g_1 is finer than g_2: $g_2(k) = j$

10) Hence: $(8),(9) \Rightarrow g_2(g_1(i)) = g_2(k) = j = g_2(i)$

11) It is proved for any possible situation that: $\forall i, i = 1,..,n, \ g_2(g_1(i)) = g_2(i)$

Q.E.D (Lemma 2.1)

Following that an Adaptation to the information structure model is concluded straight forward: Let f_I be the identity information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be its sets of signals. $f_I(S_i) = Y_i$.

Without loosing generality (for the sake of simplicity)

Assume $S = \{1,..,n\}$, $Y = \{1,..,n\}$. Let F be the set of information functions (without garbling of signals) that f_I is systematically more informative than each one of them:

Let $g_1, g_2 \in F$ and g_1 is finer than g_2 .g_1 is equivalent to G_1, and g_2 is equivalent to G_2.

Then $G_1 * G_2 = G_2$

Lemma 2.2:

Let F_1, F_2 and F_3 be information structures. F_1 represents information functions accordingly. Let $S = \{S_1,..,S_n\}$ be their set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be their set of signals.

$$F_1 = F_2 = F_3 \Leftrightarrow \forall p, 0 < p < 1, \\ F_1 = p * F_2 + (1 - p) * F_3$$

Proof (of Lemma 2.2):

1) First direction - Assume: $F_1 = F_2 = F_3$ then necessarily: $\forall p, 0 < p < 1, \ F_1 = p * F_2 + (1 - p) * F_3$.

2) Second direction - Assume: $\forall p, 0 < p < 1, \ F_1 = p * F_2 + (1 - p) * F_3$. without loosing generality suppose (on the negative form) there exists an index $i, F_{1_{i,j}} < F_{2_{i,j}}$.

3) Then (by calculating):

$F_{3_{i,j}} = \frac{1}{1-p} * (F_{1_{i,j}} - p * F_{2_{i,j}}) < 0$. It is a contradiction.

Q.E.D (Lemma 2.2)

Lemma 2.3

Let F_1 be an information structure, which represents information function. Let $S = \{S_1,..,S_n\}$ be its set of states of nature. Let $Y = \{Y_1,..,Y_n\}$ be its set of signals.

Then: $F_1 \in F \Leftrightarrow F_1^2 = F_1$

Proof (of Lemma 2.3):

1) 1st direction: From the definition of F, and Lemma 2.1 it is obvious that $F_1 \in F \Leftrightarrow F_1 = F_1^2$

2) 2nd direction: Assume (on the negative form) $F_1^2 = F_1$ and $F_1 \notin F$

3) Let's examine f_I, which is described by $F1$. $f_1 : \{1,..,n\} \to \{1,..,n\}$

4) $F_1 \notin F$, Hence there exist an index $i, f_1(i)=j, f_1(j) \neq j$

5) $f_1(i) = j = f_1(f_1(i)) \neq j$, a contradiction.

6) Hence, $F_1 \in F$

Q.E.D (Lemma 2.3)

Proof of the theorem:

1) 1st direction: it is clear from Lemma 2.3 that

$F_1 \in F \Rightarrow (q * I + (1-q) * F_1) * F_1 = q * F_1 + (1-q) * F_1 * F_1 = F_1$

2) Hence: $\forall q, 0 \le q \le 1 \ \ q * I + (1-q) * F_1 \ge F_1$

3) From Theorem 1 it is proven that,

$\forall q, 0 \le q \le 1 \ \ q * I + (1-q) * F_1 \ge F_1 \Leftrightarrow I \underset{S}{\ge} F_1$

4) 2nd direction: From Theorem 1 it is proven that,

$\forall q, 0 \le q \le 1 \ \ q * I + (1-q) * F_1 \ge F_1 \Leftrightarrow I \underset{S}{\ge} F_1$.

5) According to the 2nd condition of Blackwell's theorem [5], there exists a stochastic matrix R,

$$F_1 = (q * I + (1-q) * F_1) * R .$$

6) $F_1 = q * R + (1-q) * F_1 * R$

7) From Lemma 2.2 it is obvious that: $R = F_1$

8) Moreover, from Lemma 2.2 it is understood that $F_1 * R = F_1$.

9) Hence: $F_1 = F_1^2$.

Q.E.D

Theorem 3

Let I (the identity matrix), and Q be two information structures. $S = \{S_1,..,S_n\}$ is the common set of states of nature, and $Y = \{Y_1,..,Y_n\}$ is their same set of signals they respond with.

$I \underset{S}{\ge} Q \Rightarrow \forall i, i = 1,..,n, \ Q_{i,i} \ge Q_{j,i} \ i \neq j$ (The diagonal elements are weakly dominant in each and every column).

Proof:

1) Suppose (on the negative way): There exists an index j , $Q_{i,i} < Q_{j,i}$ $i \neq j$ then $1 \geq Q_{j,i} - Q_{i,i} = \Delta > 0$ where (without loosing generality) $Q_{j,i}$ is the maximal element in the i column.

2) Let's examine a specific decision situation: suppose there are n possible decisions, and

$$\forall k, k = 1, \ldots, n, \quad \Pi_{k,k} = \frac{1}{n}.$$

Let's define U (the utility matrix) as follows:

$$\forall r, r = 1, \ldots, n, \quad \forall s, s = 1, \ldots, n$$

$$U_{r,s} = \begin{cases} A, \quad (A > \Delta), & r = j, s = i \\ A - \Delta, & r = i, s = i \\ \Delta, & r = i, s = j \\ 0, & else \end{cases}$$

3)

$$\max_D \left(trace\left(\Pi * Q * D * U \right) \right) = \frac{1}{n} * \max_D \left(trace\left(U * Q * D \right) \right)$$

$$\forall r, r = 1, \ldots, n, \quad \forall s, s = 1, \ldots, n \quad \left(Q * U \right)_{r,s} =$$

4)

$$= \sum_{m=1}^{n} Q_{r,m} * U_{m,s} = \begin{cases} A * Q_{i,s}, & r = j \\ \Delta * Q_{j,s} + (A - \Delta) * Q_{i,s}, & r = i \\ 0, & else \end{cases}$$

5)

$$\left(Q * U \right)_{j,i} = A * Q_{i,i} < \left(Q * U \right)_{i,i} = \Delta * Q_{j,i} + (A - \Delta) * Q_{i,i} =$$

$$= \Delta * \left(Q_{i,i} + \Delta \right) + (A - \Delta) * Q_{i,i} = A * Q_{i,i} + \Delta^2$$

6) Suppose D_1 represents the optimal decision rule. $(5) \Rightarrow D1_{i,i} = 1$.

7) Moreover, $\sum_{\substack{m=1 \\ m \neq i}}^{n} \left(Q * U \right)_{j,m} = A * \left(1 - Q_{i,i} \right)$

.

8) From (5), (7) it is derived that:

$$trace\left(\Pi * Q * D1 * U \right) \geq \frac{1}{n} * \left(A * \left(1 - Q_{i,i} \right) + A * Q_{i,i} + \Delta^2 \right)$$

$$= \frac{1}{n} * \left(A + \Delta^2 \right)$$

9) Moreover, $D1_{i,i} = 1 \Rightarrow D1_{i,j} = 0$

10) Hence,

$$trace\left(\Pi * I * D1 * U \right) \leq \frac{1}{n} * \left(U_{i,j} + U_{i,i} \right)$$

$$= \frac{1}{n} * \left(\Delta + A - \Delta \right) = \frac{1}{n} * A$$

11) It means that:

$$trace\left(\Pi * I * D1 * U \right) < trace\left(\Pi * Q * D1 * U \right)$$

12) Under this decision situation Q is precisely more informative than *I*. Hence I is not systematically more informative than *Q*.

Q.E.D

