A Modified Algorithm for the Generalized Eigen-space-Based Beamformer

Shuyan Ni [1], Naiping Cheng [1], Tongling Liu [2]
1. Academy of Equipment Command and Technology, Beijing, 101416, China
2. China Department of Satellite Marine Track and Control, Jiangyin, 214413, China

Abstract: The generalized eigenspace-based beamformer (GEIB) has demonstrated more robust capabilities than the linearly constrained minimum variance beamformer and the eigenspace-based beamformer. However, it still couldn't deal with large pointing errors near the mainlobe edge. To cure the problem, the paper presents a modified algorithm, which obtains the weight vector by firstly calibrating the presumed steering vector with a rotated vector, and then utilizing the calibrated steering vector for the GEIB. Several computer simulations are provided for illustrating the advantages of the proposed algorithm over the GEIB.

Keywords: array signal processing; beamforming; linear constraint; eigenspace

1 Introduction

Linear constraints are used in beamforming to achieve various purposes. Ideally the linearly constrained minimum variance beamformer (LCMV) can achieve the maximum output SINR. But in practical circumstances, there are various mismatches which cause the desired signal being suppressed as an interferer. The eigenspace-based algorithm (ESB) has been widely investigated to improve the robust capabilities of the adaptive beamformers [1]. Combining the features of the ESB with the LCMV, the generalized eigenspace-based beamformer (GEIB) is proposed in [2]. It is realized by projecting the LCMV weight vector onto a modified signal subspace and provides more robust capabilities than the ESB and the LCMV.

However, the GEIB couldn't deal with large pointing errors near the mainlobe edge. Especially in the case of a large array size, a small pointing error can cause severe performance degradation. To cure this problem, several modified algorithm for the ESB are proposed in [3~6]. The essence of these beamformers is to enhance the robust capabilities by calibrating the presumed steering vector. In [3] the actual steering vector is found by angle searching, and in [4] it is estimated using a subarray partition scheme. In [5] a modified GEIB is presented, which estimates the actual steering vector by using the full array data. Though all the three algorithms mentioned above provide more robust capabilities to large pointing errors, the processes of searching and estimating greatly increase the complexity of calculating.

In [6], a robust algorithm with low complexity is proposed based on the vector-rotating method. Combining the GEIB with the vector-rotating method, we present a modified algorithm with less sensitivity to large pointing errors. The algorithm firstly obtains two rotated vectors in two opposite directions and by utilizing the average of them calibrates the presumed steering vector. The calibrated steering vector is then be used for the GEIB to obtain the weight vector. By the calibration, the new algorithm provides much more robust capabilities than the conventional GEIB.

2 The GEIB

Consider P+1 narrowband uncorrelated sources impinging on an M elements uniform linear array with half-wavelength interelement spacing. One is the desired signal and the others are interferers. The received signal can be expressed as

$$\begin{aligned} &\boldsymbol{x}(t)=\boldsymbol{A}\boldsymbol{s}(t)+\boldsymbol{n}(t)\\ &\boldsymbol{A}=[\boldsymbol{a}(\theta_d),\boldsymbol{a}(\theta_1),\ldots,\boldsymbol{a}(\theta_P)]^T\\ &\boldsymbol{s}(t)=[s_d(t),s_1(t),\ldots,s_P(t)] \end{aligned} \tag{1}$$

where $s_d(t)$ and $\boldsymbol{a}(\theta_d)$ represent the waveform and the steering vector of the desired signal, $s_p(t)$ and $\boldsymbol{a}(\theta_p)$ for p =1, 2, …, P are those of the interferers, $\boldsymbol{n}(t)$ is the background noise. The correlation matrix of the array input is

$$\boldsymbol{R}_x=E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]=\boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H+\sigma_n^2\boldsymbol{I} \tag{2}$$

where $\boldsymbol{R}_s$ is correlation matrix of $\mathbf{s}(t)$ and

978-1-935068-01-3

$\boldsymbol{R}_s = E[\boldsymbol{s}(t)\boldsymbol{s}^H(t)]$, σ_n^2 is the noise power, $\boldsymbol{I}$ is the identity matrix with size $M \times M$.

With the linear constraint $\mathbf{C}^H\mathbf{w} = \mathbf{g}$, where $\boldsymbol{C} = [\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_L]$ is the constraint matrix with size $L \times M$, $\boldsymbol{g}$ is the response vector with size $L \times 1$. The weight vector of the LCMV can be described as

$$\boldsymbol{w}_{LCMV} = \boldsymbol{R}_x^{-1}[\boldsymbol{a}(\theta_d)\ \boldsymbol{C}]([\boldsymbol{a}(\theta_d)\ \boldsymbol{C}]^H \boldsymbol{R}_x^{-1}[\boldsymbol{a}(\theta_d)\ \boldsymbol{C}])^{-1}\begin{pmatrix}1\\ \boldsymbol{g}\end{pmatrix} \tag{3}$$

The GEIB calculates the weight vector by projecting the LCMV weight vector onto a modified signal subspace. With the assumption that the received source number is less than the array element number ($P+1<M$), the correlation matrix $\boldsymbol{R}_x$ can be eigendecomposed as

$$\boldsymbol{R}_x = \sum_{i=1}^{M} \lambda_i \boldsymbol{e}_i \boldsymbol{e}_i^H = \boldsymbol{E}_s \boldsymbol{\Lambda}_s \boldsymbol{E}_s^H + \boldsymbol{E}_n \boldsymbol{\Lambda}_n \boldsymbol{E}_n^H \tag{4}$$

where $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_{P+1} \ge \lambda_{P+2} = \cdots = \lambda_M = \sigma_n^2$ are eigenvalues in the descending order, $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, …, $\boldsymbol{e}_M$ are the corresponding eigenvectors, $\boldsymbol{\Lambda}_n = diag\left[\lambda_{P+2}, ..., \lambda_M\right]$, $\boldsymbol{E}_s = [\boldsymbol{e}_1, \boldsymbol{e}_2, ..., \boldsymbol{e}_{P+1}]$, $\boldsymbol{E}_n = [\boldsymbol{e}_{P+2}, ..., \boldsymbol{e}_M]$. The column vectors of $\boldsymbol{E}_s$ and $\boldsymbol{E}_n$ span the signal subspace and the noise subspace respectively.

Performing the Gram-Schmidt decomposition on the column vectors of $\boldsymbol{E}_s$ and $\boldsymbol{C}$, we can obtain the matrix that contains the orthonormal basis of the modified signal subspace

$$\hat{\boldsymbol{E}}_s = [\hat{\boldsymbol{e}}_1, ..., \hat{\boldsymbol{e}}_{P+1}, \hat{\boldsymbol{c}}_1, ..., \hat{\boldsymbol{c}}_{P+1}] \tag{5}$$

Generally, the null constraints can be preserved by the GEIB for suppressing nonstationary interferes, and other constraints for model parameter mismatches would not be preserved. Therefore, for simplicity, we assume there are only null constraints. Propose the presumed steering vector is $\boldsymbol{a}_s = \boldsymbol{a}(\theta_s)$. Projecting the LCMV weight vector onto the modified signal subspace, the weight vector of the GEIB can be expressed as

$$\boldsymbol{w}_{GEIB} = \hat{E}_s\hat{E}_s^H \boldsymbol{R}_x^{-1}[\boldsymbol{a}_s\ \boldsymbol{C}]([\boldsymbol{a}_s\ \boldsymbol{C}]^H \boldsymbol{R}_x^{-1}[\boldsymbol{a}_s\ \boldsymbol{C}])^{-1}\begin{pmatrix}1\\ \boldsymbol{g}\end{pmatrix} \tag{6}$$

Though the GEIB has much more robust capabilities than the LCMV and the ESB, it can't deal with large pointing errors. According to the result in [7], with a pointing error present, the separation of the presumed steering vector $\boldsymbol{a}_s$ and the actual steering vector $\boldsymbol{a}_d$ can be represented by

$$\cos\phi = \frac{\boldsymbol{a}_d \boldsymbol{a}_s^H}{M} \tag{7}$$

which determines the array performance. The smallzer the $\cos\phi$, the lower the output SINR. When the desired signal impinges from the direction near the edge of the mainlobe, $\boldsymbol{a}_d\boldsymbol{a}_s^H \approx 0$, the output SINR approaches to zero. And the larger the array size, the narrower the error scope in which the GEIB beamformer can perform well.

3 The Proposed Algorithm

To cure the sensitivity of the GEIB to pointing errors near the mainlobe edge, a modified algorithm based on the vector-rotating method (RGEIB) is presented in the paper. The algorithm obtains the weight vector by calibrating the presumed steering vector with a rotated vector and utilizing the new one for the GEIB beamforming. The calibration process is as follows.

Rotating the presumed direction to the right and to the left responsively with an angle $\Delta\theta$, we can obtain two rotated vectors

$$\begin{aligned}\boldsymbol{a}(\theta_s + \Delta\theta) &= \left[1, \exp(j\varphi_1), ..., \exp(j\pi(M-1)\varphi_1)\right]^T \\ \boldsymbol{a}(\theta_s - \Delta\theta) &= \left[1, \exp(j\varphi_2), ..., \exp(j\pi(M-1)\varphi_2)\right]^T\end{aligned} \tag{8}$$

where $\varphi_1 = \pi\sin(\theta_s + \Delta\theta)$ and $\varphi_2 = \pi\sin(\theta_s - \Delta\theta)$. When $\Delta\theta$ is very small, we can get the approximations of φ_1 and φ_2

$$\begin{aligned}\varphi_1 &\approx \pi(\sin\theta_s + \Delta\theta\cos\theta_s) = \varphi + \Delta\varphi \\ \varphi_2 &\approx \pi(\sin\theta_s - \Delta\theta\cos\theta_s) = \varphi - \Delta\varphi\end{aligned} \tag{9}$$

where $\varphi = \pi\sin\theta_s$ and $\Delta\varphi \approx \pi\Delta\theta\cos\theta_s$. The rotated vectors can be approximately expressed as

$$\begin{aligned}\boldsymbol{a}(\theta_s + \Delta\theta) &= \left[1, ..., \exp(j\pi(M-1)\varphi)\exp(j\pi(M-1)\Delta\varphi)\right]^T \\ \boldsymbol{a}(\theta_s - \Delta\theta) &= \left[1, ..., \exp(-j\pi(M-1)\varphi)\exp(-j\pi(M-1)\Delta\varphi)\right]^T\end{aligned} \tag{10}$$

Assuming $\boldsymbol{B} = diag\left\{1, \exp(j\Delta\varphi), ..., \exp(j(M-1)\Delta\varphi)\right\}$, we can get

$$a(\theta_s + \Delta\theta) \approx Ba(\theta_s)$$
$$a(\theta_s - \Delta\theta) \approx B^H a(\theta_s) \tag{11}$$

Now we obtain the approximations of two rotated vectors. Utilizing the average of them, the calibrated steering vector is given by

$$b(\theta_s) = \frac{1}{2}\left[a(\theta_s + \Delta\theta) + a(\theta_s - \Delta\theta)\right] \approx \frac{1}{2}(B + B^H)a(\theta_s) \tag{12}$$

We assume

$$C = \frac{1}{2}(B + B^H) = diag\left[1, \cos(\Delta\varphi), ..., \cos((M-1)\Delta\varphi)\right] \tag{13}$$

The calibrated steering vector can be expressed as

$$b(\theta_s) = Ca(\theta_s) \tag{14}$$

From the process above, we can see that the calibrated vector equals to the original steering vector multiplying a diagonal matrix. By utilizing the calibrated steering vector $b_s = b(\theta_s)$ instead of a_s, we can get the modified weight vector of the LCMV beamformer. Then by projecting it onto the modified signal subspace, the weight vector of the proposed RGEIB algorithm can be expressed as

$$w_{RGEIB} = \hat{E}_s \hat{E}_s^H R_x^{-1}[b_s\ C]([b_s\ C]^H R_x^{-1}[b_s\ C])^{-1}\begin{pmatrix}1\\ g\end{pmatrix} \tag{15}$$

The value of the diagonal matrix C is related to the presumed direction θ_s and the rotated angle $\Delta\theta$. The half-power bandwidth of a M-element uniform linear with half-wavelength interelement spacing can be expressed as $BW_{0.5} = 1.772\sec\theta_s / M$. Avoiding the rotated angle going beyond the mainlobe, $\Delta\theta$ can not exceed half of the mainlobe width [6]. So $\Delta\theta < 0.886\sec\theta_s / M$, and $\Delta\varphi < 0.886\pi / M$.

4 Computing Simulation

Several simulation examples are presented in this section for illustrating the effectiveness of the proposed algorithm. Suppose a desired signal and two uncorrelated interferers from the directions of 40º and -30º impinging on a uniform linear array with half-wavelength interelement spacing. The interfere-noise ratios of the interferers are 10dB and 15dB respectively. There is a null constraint of 60º. Each of the simulation results is obtained by taking the average of 100 independent runs.

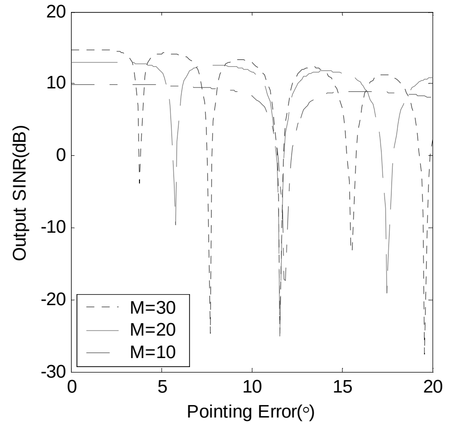


Figure 1. Ou tput SINR versus the pointing error f or the GEIB in different array sizes

Example 1: Suppose the presumed direction is 0º, the element number of the array is assumed to be 10, 20 and 30 respectively. Observe the sensitivity for the GEIB to pointing errors. Figure 1 shows the curves of its output SINR versus the pointing error in different array sizes. From the figure we can see that, the output SINR of the GEIB drops periodically along with the pointing error. The drop curves are close to the beam patterns of the presumed direction. So when the pointing error is near the mainlobe edge, the performance of the GEIB suffers from severe degradation. As the array element increases, the mainlobe width becomes narrow, the scope of the pointing error in which the GEIB could maintain good performance becomes smaller, and then a very small pointing error can severely degrade its performance.

Example 2: Suppose the presumed direction is 0º and a 10-element array is used. Investigate the performance of the proposed RGEIB algorithm and compare it with the LCMV algorithm and the GEIB algorithm. Assume the rotated angle $\Delta\theta$ = 0.6/M. Figure 2 shows the curves of the output SINR versus the pointing error for different algorithms. From the figure we can see that, both of the RGEIB algorithm and the GEIB algorithm have better output performance than the LCMV algorithm. However, as same as the LCMV algorithm, the output SINR is dropped sharply when the pointing error is about 12º. The proposed algorithm overcomes their shortcomings

978-1-935068-01-3 © 2009 SciRes.

and have robust capabilities in a wide scope of pointing errors.

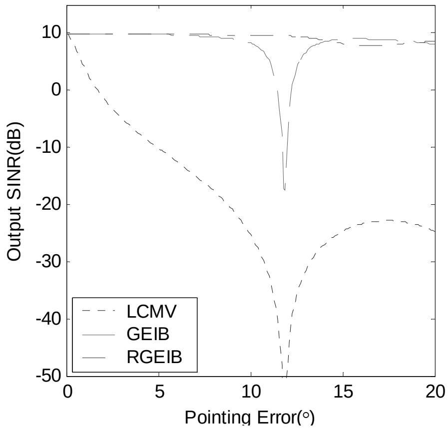
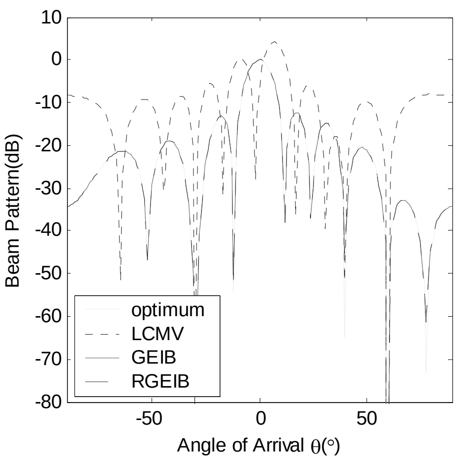
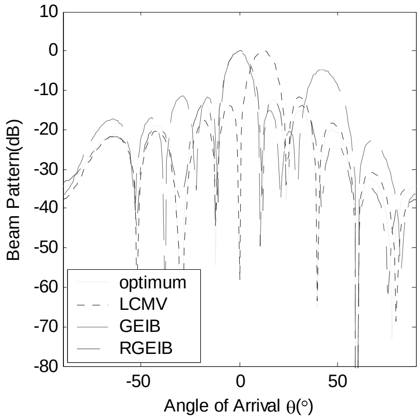


Figure 2. Output SINR versus the pointing error for different algorithms

Example 3: The desired signal is supposed to be 0º and the other supposes are the same with example 2. Compare the beam patterns of different beamformers in figure 3. From the figure we can see that, all of the three aforementioned algorithms form deep hollows in the direction of the null constraint. Due to the pointing error, the LCMV suppressed the desired signal as an interferer, while the other two methods form mainlobes in the direction of the desired signal. In the presence of the small pointing error, the beam patterns of the later two algorithms are almost the same. When the pointing error is 12º, which goes near the mainlobe edge, the sidelobe level of the GEIB becomes high and it can not suppress the interferers efficiently. The RGEIB improves the sidelobe level obviously and forms deep hollow in the directions of the two interferers. Its beam pattern approaches to the optimum one.



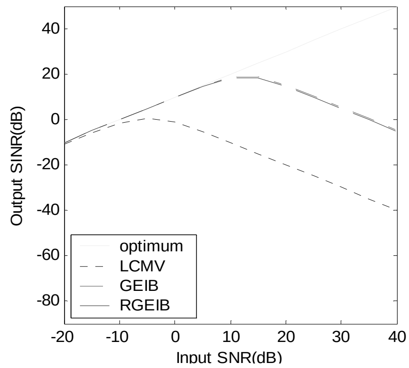
(a) The pointing error is 2º



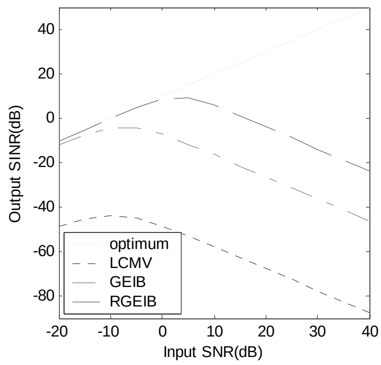
(b) The pointing error is 12º

Figure 3. The beam patterns of different beamformers

Example 4: Suppose the condition is the same as example 3. Investigate the curves of the output SINR versus the input SNR for different algorithms in figure 4. From the figure we also can see that, the performances of the RGEIB and the GEIB are almost the same and both are better than the LCMV when the pointing error is 2º. When the pointing error is near the mainlobe edge, the output SINR of the RGEIB suffers from a very little loss only in high input SNR, while those of the other two algorithms drop sharply.



(a) The pointing error is 2º



(b) The pointing error is 12º

Figure 4. Output SINR versus the input SNR for different beamformers

5 Conclusion

To overcome the drawback of the GEIB beamformer to large pointing errors near the mainlobe edge, the paper presents a modified GEIB beamformer based on the vector-rotating algorithm. It firstly obtains two rotated vectors in two opposite directions and by utilizing the average of them calibrates the proposed steering vector. The calibrated steering vector is then be used for GEIB beamforming to obtain the weight vector. Several computer simulations demonstrated the proposed algorithm provides much more robust capabilities than the former algorithms and it can perform very well even in very large pointing errors.

References

[1] Chang L, Yeh C C. Performance of DMI and eigenspace-based beamformers[J]. IEEE Trans. on Antennas and Propagation, 1992, 40(11): 1336~1347

[2] Yu J L, Yeh C C. Generalized eigenspaced-based beamformers[J]. IEEE Trans. on Signal Processing[J], 1995, 43(11): 2453~2461

[3] Zhao Yongbo，Zhang Shouhong. A modified eigenspace-based algorithm for adaptive beamforming[J]. Signal Processing Proceeding of ICSP, 2000, 1(8): 468~471

[4] Cheng-Chou Lee, Ju-Hong Lee. Eigenspace-based adaptive array beamforming with robust capabilities[J]. IEEE Trans on Antennas and Propagation, 1997, 45 (12): 313~316

[5] Ann-Chen Chang, Ching-Tai Chiang, Yuan-Hwang Chen. A generalized eigenspace-based beamformer with robust capabilities[J]. IEEE Proc. Phased Array Systems and Technology, 2000(5): 553~556

[6] Wu Sijun, Zhang Jinzhong, Zhang Shu. Robust adaptive beamforming against pointing error[J]. Journal of Harbin Engineering University, 2005, 26(4): 531~535(in Chinese)

[7] Lena Chang, Chien-Chung Yeh. Effect of pointing errors on the performance of the projection beamformer[J]. IEEE Trans. on Antennas and Propagation, 1993, 41(8): 1045~1052

978-1-935068-01-3 © 2009 SciRes.