

# Applications of Improved Matrix Pencil Method for UHV Power System and Excitation Controller

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**Abstract:** In modern large-scale power systems, low frequency oscillation is becoming a serious problem for secure system operation. Generator excitation controller is the most direct and most effective method for suppressing low-frequency oscillations. In this paper, an improved matrix pencil method based on the idea of the size of variance contributive in principal components analysis is proposed. And the improved matrix pencil method is used to analyze the damping that excitation controller can provide in the transient process, which is a method to evaluate the effect of different excitation controllers suppressing low-frequency oscillations. The parameters of oscillation mode are picked up at the time of occurrence of low frequency oscillation in UHV trans-regional power grid which is simulated in Power System Analysis Software Package (PSASP). The effectiveness of the improved matrix pencil method is firstly tested in a typical single-machine to infinite bus system, and then proved in the UHV trans-regional power grid in PSASP.

**Keywords:** matrix pencil method; low frequency oscillation; Excitation controller; damping; principal components analysis

## 1. Introduction

With the development of modern large-scale power systems and Electric Power market, the construction and development of economic in China has been strongly supported, at the mean time, it also brings new challenges to the safe, stable and reliable operating in power systems. During the process of trans-regional power systems construction, power-angle stability, frequency stability problems are existed from the past to the present, voltage stability and inter-regional low frequency oscillations problems are more and more serious with the development of power grids.

The inhibition methods of low frequency oscillations are mainly primary and secondary control strategies [1]. In which the strategy of controlling generator excitation system is the most effective and direct way, for it's a method which focused on the source of low frequency oscillations. Optimization excitation control strategy provides enough positive damp to compensate the lack of damp in system, which can suppress the generating of low frequency oscillations. The available engineering applications of excitation control are AVR, AVR+PSS, LOEC, self-adapting excitation control, NOEC.

The effectiveness in suppressing low frequency of Excitation controller is not decided by the "swings" of the oscillation [3], but decided by the value of the damp provided to the system. The effectiveness can be judged by comparing the value of their damps in identifying low frequency oscillations. Prony method is sensitive to noises, because of force-filtering, the signals expressed in complex exponential function, there are a lot of false modes in the results. As for this problem, there appears a

lot of improved methods; these methods are mostly using iteration or singular value decomposition of matrix to identify the order, using iteration to reduce the effect of noises. Recently, Hilbert-Huang transform which is a kind of non-stationary signal processing method has been used in identifying the mode of low frequency [6], on the other side, it has high computation complexity.

Matrix pencil method is a way of extracting the oscillation mode, this paper applies improved matrix pencil method to analyze the damp which provided by different excitation controllers, so as to judge the effectiveness of suppressing oscillations with each excitation controller. This study can provide the orientation of suppressing low frequency oscillations by optimizing excitation controller in UHV trans-regional power systems.

## 2. Suppression Of Low Frequency Oscillations Achieved By Excitation Controllers

### A. Introductions of Recent Controllers

Mainly excitation controllers applied in applications with engineering operation experiences recently are: AVR, AVR+PSS (Power System Stabilizer), LOEC, NOEC.

Four different excitation controllers for comparing the effectiveness of inhibiting low frequency oscillations achieved by them are analyzed. AVR uses generator terminal voltage deviation proportional expression or PID expression excitation control; AVR+PSS are on the basis of AVR with the addition of angular velocity deviation or power deviation signals. LOEC is designed on the basis of the systems' linear modulation which modeled by partial linearization of systems on balance point. NOEC

is based on the standard model which is obtained by the precise feedback linearization of systems, concluded non-linear excitation control law from the linear optimal control designed on the basis of the standard bruffus model.

### B. Control Law of Excitation Controller

Considering the transient state of excitation winding in single machine infinite system, generator's third-orders model is enough [2]. AVR uses proportional control, composite enlarge coefficient is  $K_e$ . PSS use power deviation as additional control signals, the transfer function is:

$$G(s) = \frac{Ks}{Ts+1} \left( \frac{1+T_1s}{1+T_2s} \right)^2 \quad (1)$$

Partial linearization of single machine infinite system linearized at operating point is:

$$\begin{bmatrix} \Delta \dot{P}_e \\ \Delta \dot{w} \\ \Delta \dot{V}_t \end{bmatrix} = \begin{bmatrix} \frac{S_E - S_v}{Td} & S_{E'} & -\frac{R_v S_E}{T_d' R_v} \\ -\frac{w_0}{H} & -\frac{D}{H} & 0 \\ \frac{S_E - S_v}{T_d' R_v S_v} & \frac{S_{E'} - S_v}{R_v} & -\frac{S_E}{T_d' S_v} \end{bmatrix} \begin{bmatrix} \Delta P_e \\ \Delta w \\ \Delta V_t \end{bmatrix} + \begin{bmatrix} \frac{R_{E'} S_E}{T_{d0}'} \\ 0 \\ \frac{R_{E'}}{T_{d0}' R_v} \end{bmatrix} \Delta E_f \quad (2)$$

Related symbols are referenced in literature [3]. When Q、R is chosen, solving the Riccati matrix:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (3)$$

Quadratic target functional is obtained:

$$J = \frac{1}{2} \int_0^\infty (X^T QX + U^T RU) dt \quad (4)$$

Its optimal solution is the optimized feedback gain:

$$K = R^{-1}B^T P \quad (5)$$

LOEC is designed based on the following control law:

$$U = \Delta E_f = -KX \quad (6)$$

Which,  $X = [\Delta P_e \quad \Delta w \quad \Delta V_t]^T$ .

From literature [2], power system in third-order model is satisfied for precise feedback linearized condition, when choose the observe function  $h(x) = \delta - \delta_0 = z_1$ , you can obtain:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = \alpha(x) + \beta(x)u \end{cases} \quad (7)$$

Which  $\alpha(x) = L_f^n h(x)$ ;  $\beta(x) = L_g L_f^{n-1} h(x)$ .

if  $v = \alpha(x) + \beta(x)u$ , the standard type of system is:

$$\dot{z} = Az + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (8)$$

If designed by linear optimal control, and

$$v = -k_1 z_1 - k_2 z_2 - k_3 z_3 \quad (9)$$

The NOEC control law is obtained as:

$$u = \frac{-L_f^3 h(x) + v}{L_g L_f^2 h(x)} \quad (10)$$

## 3. Matrix Pencil Analyze Method

### A. Principle of Matrix Pencil Analyze Method

Matrix pencil method is a parameters estimating method proposed by Hua and Sarkar in 1980s, it can abstract all the eigenvalues of the mode from transient waveform. Compare to traditional prony method or HHT transfer method; it has the low complexity and space complexity, at the same time it can suppress noises effectively. Literature [7-8] addressed that applying matrix pencil method to identify parameters can suppress noises effectively. Literature [9] applied matrix pencil method in identifying the parameters of power system mode. Literature [10] applied matrix pencil method in transient analysis of large-scale ground grid.

For the real observed discrete time series  $y(nTs)$ ,  $n = 1, 2, \dots, N$ , applying the following complex exponent to fit:

$$x(nTs) = \sum_{k=1}^m R_k e^{\lambda_k nTs} = \sum_{k=1}^m R_k z_k^n \quad (11)$$

Which,  $x(nTs)$  is the fitting value at  $nTs$ ;  $T_s$  is the sample time;  $\lambda_k$  is the  $k$ th-order mode character;  $z_k = e^{\lambda_k T_s}$  is the  $k$ th-order mode's pole;  $R_k$  is the  $k$ th-order mode residue,  $m$  is the order of the mode.

As following  $x(nTs)$ 、 $y(nTs)$  are abbreviated to  $x(n)$  and  $y(n)$ . The target of signal's fitting is minimizing the quadratic sum of absolute errors, as follows:

$$\min \sum_{n=1}^N (x(n) - y(n))^2 \quad (12)$$

We should construct the Hankel matrix for applying matrix pencil method to evaluate the eigenvalues, take  $L=N/2$ , so:

$$Y = \begin{bmatrix} y(1) & y(1) & \cdots & y(L+1) \\ y(2) & y(3) & \cdots & y(L+2) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L) & y(N-L+1) & \cdots & y(N) \end{bmatrix} \quad (13)$$

Applying singular decomposing to  $Y$ , so  $Y=UDVT$ ,

the singular value matrix is  $D = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_m)$ , the left eigenvalue vector  $U$  and right eigenvalue vector  $V_L \times m$ , and  $D$  matrix is arranged in descending order in according with  $\sigma$ . For transient wave shape with noises, literature [3] gave an experience threshold value, when the proportion of any singular value to maximum singular value is lower than the threshold value, as  $\sigma_i/\sigma_m < \gamma$ , then this singular value is considered to be caused by noises, so cut the part after this singular value from  $D$  matrix, and the influences of noises are removed.

#### B. Improved Matrix Pencil Analysis Method

The selection of matrix pencil's threshold value  $\gamma$  is decided by the accuracy of signal's mode identification, at the same time, it has a huge randomness, so there is no unified final conclusion. As the singular values after threshold value are cut for considered as noises, the magnitude of the signals and the dynamics of noises will have effect on the selection of threshold value. What's more, the bandwidth of oscillations frequencies is narrow in the process of low frequency oscillations identification. If threshold value is set by experience value, the improper selection of singular value may produce the leak identification of mode.

Borrow the size of variance contributive in principal components analysis to select the order of singular value, can avoid from setting threshold value by experiences, so it is set as follow:

$$\sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_i^2}{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_p^2}} \geq \gamma \quad (14)$$

Which,  $i=1,2,\dots,m$ ,  $p=\min\{N-L, L+1\}$  is the total number of the  $D$  diagonal matrix, and  $m \leq p$ .

In principal component analysis select principle component by variance contribution, when the proportion of first  $i$ -variances' contribution is bigger than  $\gamma$ , considering the first  $i$ -variances are in main role. And  $\sigma_{i+1}$  caused by noises are minor, cut them off can clear the effect of noises up. As for  $\gamma$ , literature [8] gives out similarly referenced data, setting  $\gamma=0.995$ . This paper discovered that for reducing the leak mode of low frequency oscillations' identification, the value of  $\gamma$  should be a little bigger than that, as the frequencies of low frequency oscillations are really close. In the following analyze, setting  $\gamma=0.99995$ , the new singular value matrix is obtained:

$$D' = \begin{bmatrix} \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_m) \\ \mathbf{0}_{(N-L) \times m} \end{bmatrix} \quad (15)$$

Setting  $V1T=(1:L, 1:m)$ ;  $V2T=(2:L+1, 1:m)$ . It means take the 1 to  $L$  row, 1 to  $m$  line of  $V$  to form  $V1T$ ; take the 2 to  $L+1$  row, 1 to  $m$  line of  $V$  to form  $V2T$ , so there formed two new matrices  $Y=UD'V1T$  and  $Y=UD'V2T$ . According from deduction, the pole of signal mode  $z_k$  can be obtained from constructed matrix pencil  $\{Y2:Y1\}$ .  $z_k$  is the generalized eigenvalue of  $Y1+Y2$ .  $Y1+$  is the

pseudo-inverse  $Y1$ , as:

$$z_k = e^{\lambda_k T_s} = \text{pinv}(Y1)Y2 = v_k \quad (16)$$

$$\lambda_k = -\xi_k \omega_k \pm j\omega_k \sqrt{1-\xi_k^2} \quad (17)$$

Which,  $\xi_k$  is damping ratio;  $\omega_k$  is angular frequency of oscillation frequency without damp. So, damping ratio  $\xi_k$  and oscillation frequency  $f_k$  are:

$$\xi_k = \frac{1}{\sqrt{1 + (\text{Im}((\ln v_k)/T_s)/\text{Re}((\ln v_k)/T_s))^2}} \quad (18)$$

$$f_k = \frac{\text{Im}((\ln v_k)/T_s)}{2\pi}$$

## 4. Simulation And Results

#### A. Simulation of Damping to Power Systems Provided by Excitation Controller

This paper uses single machine infinite system as physical model illustrated [2]. For proportional excitation controller, according to the analyze method of literature [3], when excitation magnification  $K_e > 35.6759$ , Hopf branch will appear, at the same time, low excitation magnification has poor transfer performance, this paper took  $K_e=10$ . For PSS, this paper take  $\Delta P_e$  as feedback signals' form, its transfer function is:

$$G(s) = \frac{60s}{3s+1} \left( \frac{1+0.125s}{1+0.05s} \right)^2 \quad (19)$$

According to the theory in literature [9] design the LOEC, selected  $Q=\text{diag}(1,100,500)$ ,  $R=1$ , the optimal feedback gain is obtained as:  $k_p=33.1895$ ,  $k_w=-9.4015$ ,  $k_v=22.082$ .

For regular NOEC, selected  $h(x)=\delta-\delta_0$  as observed variable, this excitation controller is setting by angle deviation, it can improve transient property extraordinarily, but cannot satisfy voltage tracing request[11-12]. This paper combined NOEC to linear optimal excitation control, brought additional linear optimal control to NOEC. Which, part of the linear optimal control parameters are as above, control ratio of nonlinear control part is:

$$u_f = \frac{-L_f h(X) + v}{L_g^T L_f h(X)} = Eq - \frac{DT_{d0} x'_{d\Sigma}}{w_0 V_s \sin \delta} - \frac{E'_q T_{d0} \cos \delta}{\sin \delta} \Delta w - \frac{HT_{d0} x'_{d\Sigma}}{w_0 V_s \sin \delta} v \quad (20)$$

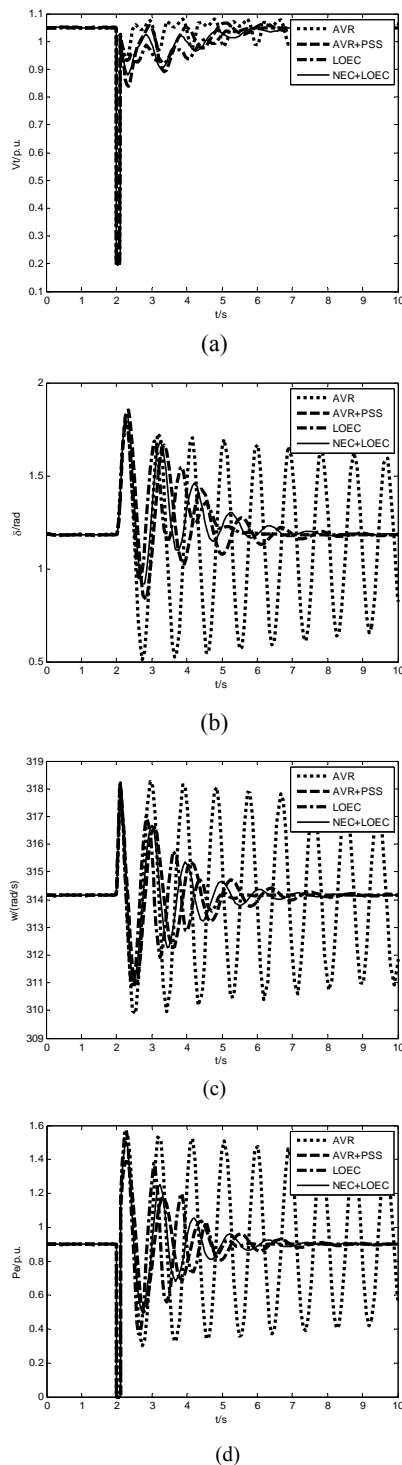
Which:

$$v = -\Delta \delta - 2.3 \Delta w - 2.14 \dot{w} \quad (21)$$

And it is linear optimal control of standard type.

Subject the system to three phase short-circuit, which happened at 2s, lasted for 0.1s then disappeared. The transient curves of terminal voltage, power-angle, angular rate and active power are illustrated as figure 1.

Figure 1 shows that regular proportion excitation controller can be a long time oscillation under large disturbance; PSS excitation control can provide certain damping to suppress low frequency oscillations; LOEC which



**Figure 1** Transient simulation results when three-phase short fault at High-pressure side of the transformer.

(a) terminal voltage (b) power angle (c) speed (d) active power to infinite system

is designed based on accurate feedback linear is better than PSS, LOEC designed based on accurate feedback linearise using angle deviation as observe variables can improve transient process extraordinarily, without bringing terminal voltage deviation's feedback.

The results of active power transient curve analyzed by matrix pencil method are illustrated in table.1. For convenient in comparison, table.1 illustrated 2 main mode results of matrix pencil method and prony method. The identification results are similar.

**Table 1.** Comparison of the results in matrix pencil method and prony method

Excitation control method	Identification results			
	Improved matrix pencil method		Prony method	
	Oscillation frequency	Damping ratio(%)	Oscillation frequency	Damping ratio(%)
AVR	1.0902	1.1604	1.08698 /1.1486	1.4461
AVR+PSS	0.92221 /1.3279	15.431 /12.363	0.943779 /1.84174	12.8554 /17.039
LOEC	1.2939 /1.6263	19.282 /9.9036	1.247572 /1.55915	16.4608 /7.10289 1
NOEC	1.1376 /1.5301	24.803 /13.739	1.212379 /1.61411	23.5717 2 /12.8073

It's easy to see that, after being optimized, excitation controller can provide more than ten times damping to system than unoptimized. So, reasonable excitation controller can provide enough damping to the system to suppress low frequency oscillations in transient process.

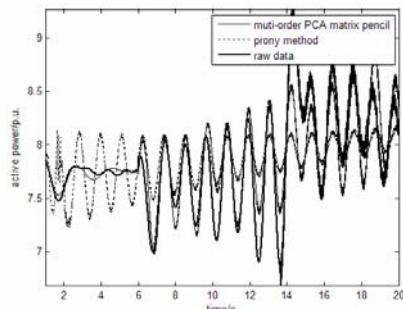
## B. Simulation and Analysis of UHV Transregional Power Grids

In Central China UHV transregional power grids, using power system general program PSASP to simulate disturbed system with low frequency oscillations, simulation time is set to be 20s.

The faults are sanmenxia-luonan 1return to AC line three phases short-circuit fault at 1s, sanxiamen-luonan 2return to AC line three phases break line to AC line at 6s. Then the system has a low frequency oscillation phenomenon. Fig.2 shows the transfer power oscillation curve on fancheng-baihe 1 return line which is the interconnection line between Hubei province and Henan province.

Figure 2 illustrates that there's a much better fitting curve which applied improved matrix pencil method both before and after the second disturbance than prony method. Applying this abstraction of damping ratio to judge the effect of excitation controllers suppressing low frequency oscillations is appropriate and the precision can be guaranteed at the same time.

If the power oscillation curve comes from real UHV transregional power grids' WAMS, this improved matrix pencil method can be very practical in real time monitoring, identification online of low frequency oscillation and excitation controller parameters settlement online.



**Figure 2** The improved matrix pencil method and prony method fit results of active power oscillation curve.

## 5. Conclusions

This paper applies the variance contribution proportion method in principal component analysis to improve matrix pencil method, and then applied the improved matrix pencil method to analyze the damp provided by optimized excitation controller. From simulation experiments' verification, conclusions are obtained:

(1) The improved matrix pencil method proposed in this paper can fit all kinds of curves in UHV transregional power grids, and abstract oscillation mode parameters accurately, judge the effectiveness of excitation controller, it is practical in engineering.

(2) With the combination of linear optimal excitation control and nonlinear excitation control, more damp can be provided to the system with the advantages of both two methods.

(3) With the transient curves simulated, damp provided by excitation controller is related to the regulation time of transient process. If a kind of optimized excitation control strategy is designed to calm transient process in sufficiently short period of time, this strategy can provide more damping without doubt.

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