DOA Estimation Based on Improved MUSIC Algorithm

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Abstract: The basic MUSIC algorithm can not effectively estimate the related-signal DOA, and when the antenna array has amplitude and phase errors, the algorithm will decline seriously in its performance. This paper improves the basic MUSIC algorithm by dividing the array steering vector properly and combining the new array steering vector with the low-dimension noise subspace method. The improved algorithm can correctly estimate the non-related signal DOA and the related. Meanwhile it can calibrate the array errors partially. This algorithm can be applied to electronic countermeasure. Simulation results verify the effectiveness of the algorithm.

Keywords: electronic countermeasure; array signal processing; DOA estimation; MUSIC algorithm; relevant signal; array calibration

1 Introduction

MUSIC algorithm [1] will be ineffective when the input signals are relevant or array amplitude-phase errors exist. Scholars have done many researches for the problems. For example, combining matrix reconstruction technique [2-3] or spatial smoothing technique [4-5] with basic MUSIC algorithm is used for relevant-signal DOA estimation; active-calibrations and self-calibrations [6-8] can calibrate array errors. However, most algorithms are either complex, or failure for related-signal DOA estimation and error calibration meantime.

This paper improves basic MUSIC algorithm based on in-depth study of it. Improved MUSIC algorithm divides the array steering vector properly first, and then use the subspace algorithm to estimate DOA. In order to reduce computation complexity, this paper proposes a new method that using low-dimension noise subspace for DOA estimation. Improved MUSIC algorithm can effectively estimate relevant signal DOA and the irrelevant, and it also can calibrate array errors partially. Besides, the algorithm was simple, and easy for practical applications. Finally, this paper does a lot of simulations to verify the effectiveness of the algorithm.

2 Data Model

Assume that there is a N-element uniform linear array. There are K incident signals in space. The array steering vector is $a(\theta)$. Then the direction matrix of the input signals is $A=[a(\theta_1),a(\theta_2),\ldots,a(\theta_K)]$. The K th sample of the received signal is $y(k)=As(k)+n(k)$, where $s(k)=\left[s_1(k) \quad \ldots \quad s_K(k)\right]^T$ is the signal column vector. $n(k)$ is realization of stationary, zero mean Gaussian random process, and there is no correlation between the noise and the signals. Then the received signal covariance matrix is,

$$R_{yy}=E\left\{y(k)y^H(k)\right\} \tag{1}$$

3 Algorithm

3.1 Basic MUSIC Algorithm

Basic MUSIC algorithm makes eigendecomposition of signal covariance matrix and uses subspace algorithm to estimate DOA. The following is a brief discussion.

First of all, make eigenvalue decomposition of R_{yy} in (1), the following can be obtained.

$$R_{yy}=U_S\Sigma_S U_S^H+U_N\Sigma_N U_N^H \tag{2}$$

Then, according to subspace algorithm, $U_S \perp U_N$. Actually the extension space of signal steering vector is the same as signal subspace, so the corresponding MUSIC spectrum is $P_{MUSIC}(\theta)=\dfrac{1}{a^H(\theta)U_N U_N^H a(\theta)}$. The number of input signals and their corresponding DOA can be obtained by searching peaks of $P_{MUSIC}(\theta)$. However, the basic MUSIC algorithm is not available for relevant-signal DOA estimation, and it is invalid when the array amplitude-phase errors exist.

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3.2 Improved Algorithm

Study shows that signal steering vector $a(\theta)$ plays a very important role in MUSIC algorithm. Through an appropriate pretreatment of $a(\theta)$, MUSIC algorithm can estimate the related-signal DOA and calibrate the array errors. Based on this idea, this paper makes improvement of basic MUSIC algorithm. The following is the details.

Divide the steering vector $a(\theta)$ into $a_1(\theta)$ and $a_2(\theta)$, where $M\times 1$ vector $a_1(\theta)$ is consisted of the first M elements of $a(\theta)$, and $L\times L$ ($L=N-M$) diagonal matrix $a_2(\theta)$ is consisted of the last L elements of $a(\theta)$. Assume $\Gamma=[1, \ \dots \ 1]^T_{(L+1)\times 1}$, then $a(\theta)$ can be written in the following form.

$$a(\theta)=\begin{bmatrix} a_1(\theta) & 0_{M\times L} \\ 0_{L\times 1} & a_2(\theta)\end{bmatrix}\begin{bmatrix}1\\ \vdots \\ 1\end{bmatrix}_{(L+1)\times 1}=\tilde{a}(\theta)\Gamma \qquad (3)$$

According to the basic principles of MUSIC algorithm, $\Gamma^H\tilde{a}^H(\theta)U_NU_N^H\tilde{a}(\theta)\Gamma=0$ where $\Gamma\neq 0$, and $\tilde{a}^H(\theta)U_NU_N^H\tilde{a}(\theta)$ is singular. When $M\geq K+1$, only if θ is the actual direction of input signal, $\tilde{a}^H(\theta)U_NU_N^H\tilde{a}(\theta)$ is singular. Therefore,

$$\hat{\theta}=\arg\max_{\theta}\frac{1}{\det\left[\tilde{a}^H(\theta)U_NU_N^H\tilde{a}(\theta)\right]} \qquad (4)$$

where $\det[\ ]$ stands for the matrix determinant.

Supplement of the algorithm:

Firstly, simplify computation of the noise subspace U_N. In fact, as long as the rank of matrix $U_NU_N^H$ is larger than L, (4) is still available. Therefore, computing U_N just need compute the eigenvectors of the smallest $L+1$ eigenvalues. Especially, when $L=0$, one-dimension noise subspace can be obtained to estimate the signal DOA. So, when L is small, the dimension of $\tilde{a}^H(\theta)U_NU_N^H\tilde{a}(\theta)$ is low and the determinant computation is simple. Simplifying the noise subspace is equal to reduce the dimension of it. This method greatly reduces the computational complexity.

Secondly, when the last L elements in steering vector are infected by array errors, the improved algorithm can still estimate the signal DOA, and the corresponding array errors can also be estimated. Assume these L errors are $G_1, \ \dots \ G_L$. Then equation (3) can be rewritten as,

$$a'(\theta)=\begin{bmatrix} a_1(\theta) & 0_{M\times L} \\ 0_{L\times 1} & a_2(\theta)\end{bmatrix}\begin{bmatrix}1\\ G_1\\ \vdots \\ G_L\end{bmatrix}_{(L+1)\times 1}=\tilde{a}'(\theta)\Gamma' \qquad (5)$$

Since $\Gamma'\neq 0$, (4) is still available for DOA estimation. And the array error estimation is,

$$\hat{\Gamma}'=e_{\min}\left[\tilde{a}^H(\hat{\theta})U_NU_N^H\tilde{a}(\hat{\theta})\right] \qquad (6)$$

where $e_{\min}[\ ]$ stands for computing the eigenvector of the smallest eigenvalue, and the first element of $\hat{\Gamma}'$ is 1.

Third, when the inputs are relevant signals, (4) remain available for DOA estimation. However, low-dimension noise subspace is not available for computing (4) here, and the full-dimension noise subspace must be computed.

Summary of the Algorithm:

Step 1, use (3) to compute the pretreated array steering vector $\tilde{a}(\theta)$.

Step 2, make eigendecomposition of the signal covariance matrix R_{yy}, and compute the eigenvectors of the smallest $L+1$ eigenvalues. If the input signals are relevant, compute the eigenvectors of the smallest $N-K$ eigenvalues.

Step 3, use (4) to estimate the signal DOA. When the array errors exist in the last L array elements, use (4-5) for DOA estimation and (6) for array error estimation.

The above discussion shows improved MUSIC algorithm can estimate the relevant-signal and irrelevant-signal DOA both, and also can calibrate array errors. The following are simulations of the algorithm.

4 Simulation

The basic simulation conditions: assume that there is a 10-element uniform linear array, and there are two incident signals with the direction of -30° and 30°.

4.1 Simulation of Algorithm Effectiveness

First, assume that the two signals are not related, $a_1(\theta)$ is consisted of the first 9 elements of the steering vector $a(\theta)$, and $a_2(\theta)$ is consisted of the last element. Then the two-dimension noise subspace can be used in improved algorithm for DOA estimation. The simulation results are shown in Figure 1.

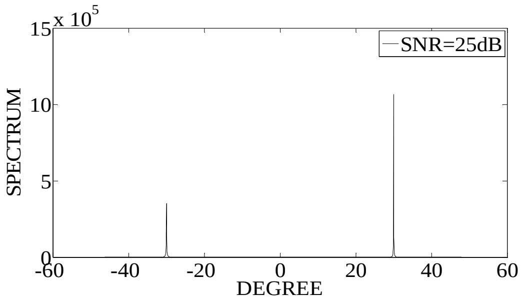

Figure 1. Relevant-signal estimation

Then, assume that the input signals are related, and the other conditions remain the same. Compute full-dimension noise subspace, and then simulate the improved algorithm. The simulation results are shown in Figure 2,

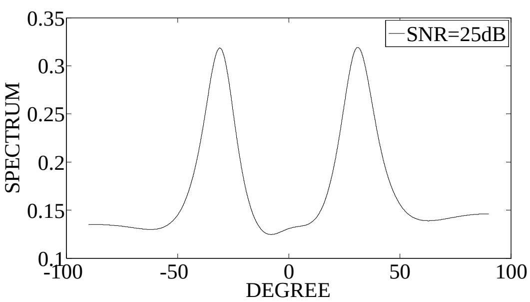

Figure 2. Irrelevant-signal estimation

Finally, assume that there are array errors existing in the last 4 elements of the array, where the amplitude errors are uniform distributing of $[0,\ 2]$, and the phase errors are uniform distributing of $\left[0,\ \frac{\pi}{2}\right]$. The input signals are not relevant. Divide the steering vector into two groups. One group is consisted of the first 6 elements of the steering vector, and the other group is consisted of the last 4 elements of steering vector. Use the two groups to form $\tilde{a}(\theta)$. Compute the corresponding five-dimension noise subspace, and then use the improved algorithm to estimate DOA and array errors. Figure 3 is the DOA estimation results; Table 1 is array error estimation results.

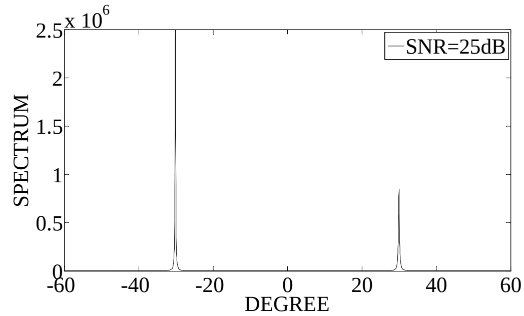

Figure 3. DOA estimation with array errors

Table 1. The array amplitude-phase error estimation

Error Array Elements		1	2	3	4
Amplitude Error	True value	0.4387	0.4983	0.2140	0.6435
	Estimate value	0.4430	0.5015	0.2136	0.6449
Phase Error	True value	0.5027	1.5081	1.1414	0.6471
	Estimate value	0.5042	1.5096	1.1338	0.6481

The simulation results above verified the algorithm effectiveness.

4.2 Simulation of SNR Impact on the Algorithm

Define the estimation relative error as $\varepsilon = \frac{|e-\hat{e}|}{|e|}$, where e is the true value, and $\hat{e}$ is the estimated value. The SNR is $-10 \sim 30$. Firstly, assume that there are no array errors, and the input signals are irrelevant. Use the improved algorithm to estimate the -30° signal based on 100 experiments. The simulation is shown as Figure 4.

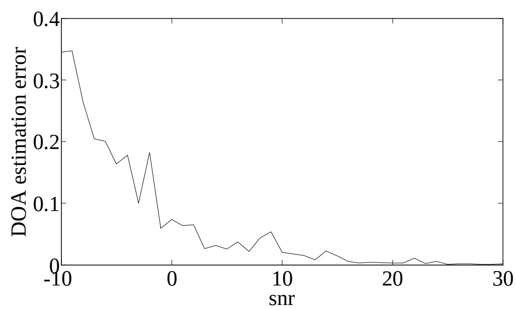

Figure 4. SNR impact on irrelevant signal DOA estimation

Secondly, assume that the input signals are relevant.

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Simulate the algorithm based on 100 experiments. The simulation is shown as Figure 5.

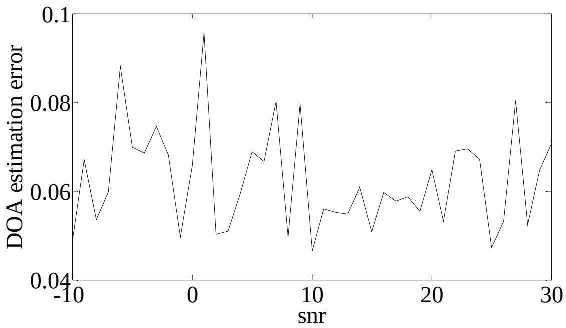

Figure 5. SNR impact on relevant signal DOA estimation

Third, assume that there are array errors existing in the last 4 elements of the array, where the amplitude errors are uniform distributing of $[0, \ 2]$, and the phase errors are uniform distributing of $\left[0, \ \frac{\pi}{2}\right]$. The input signals are not relevant. Use the improved algorithm to estimate irrelevant signal DOA based on 100 experiments. Figure 6 is the simulation result.

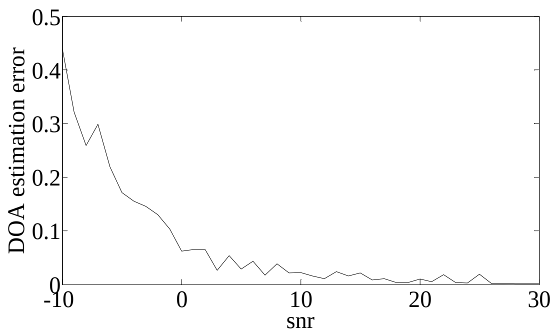

Figure 6. SNR impact on irrelevant signal DOA estimation when array errors exist

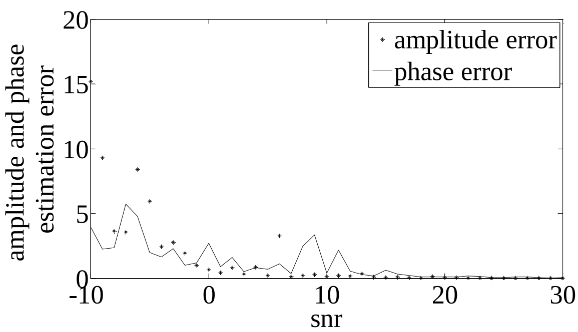

Figure 7. SNR impact on estimating the first array element amplitude-phase errors

Finally, assume that the array errors are the same as the above, and the input signals are irrelevant. Use the improved algorithm to estimate the first array element amplitude-phase errors based on 100 experiments. Figure 7 is the simulation result.

As the above simulations show, increasing SNR can improve the performance of the algorithm.

5 Conclusions

Firstly, the paper briefly discussed the basic MUISC algorithm, and pointed the shortcomings of the basic MUSIC algorithm. Then the paper proposed a method to improve the basic MUSIC algorithm. According to the improved algorithm, make a proper division of the steering vector to form a new one first, and then use the subspace algorithm to yield DOA estimation. In this algorithm, low-dimension noise subspace can be computed to reduce the computation complexity. But when the input signals are relevant, the algorithm must use a full-dimension noise subspace to estimate DOA. Finally, this paper made a lot of simulations. These simulations show that the improved algorithm can efficiently estimate relevant-signal and irrelevant-signal DOA, and calibrate the array errors. This improved algorithm is simple in computation and easy for practice applications.

Acknowledgments

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