

# Magnetic Properties of a Mixed-Spin-3/2 and Spin-2 Ising Ferrimagnetic System in an Applied Longitudinal Magnetic Field

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## ABSTRACT

The magnetic properties of a mixed Ising ferrimagnetic system consisting of spin-3/2 and spin-2 with different single ion anisotropies and under the effect of an applied longitudinal magnetic field are investigated within the mean-field theory based on Bogoliubov inequality for the Gibbs free energy. The ground-state phase diagram is constructed. The thermal behaviours of magnetizations and magnetic susceptibilities are examined in detail. Finally, we find some interesting phenomena in these quantities, due to applied longitudinal magnetic field.

**Keywords:** Mixed-Spin; Ising; Susceptibility

## 1. Introduction

Recently, there have been many theoretical studies of mixed-spin Ising ferrimagnetic systems. These systems have been of interest because they have less translational symmetry than their single-spin counterparts, since they consist of two interpenetrating nonequivalent sublattices. For this reason, they are studied not only for purely theoretical interest but also because they have been proposed as possible models to describe a certain type of ferrimagnetic systems such as molecular-based magnetic materials [1-3] which are of current interest. Moreover, the increasing interest in these systems is mainly related to the potential technological applications of these systems in the area of thermomagnetic recording [4,5]. Therefore, the synthesis of new ferrimagnetic material is an active field in material science.

One of these models to be studied was the mixed-spin Ising system consisting of spin-1/2 and spin-S ( $S > 1/2$ ) in uniaxial crystal field. The model for different values of S ( $S > 1/2$ ) has been investigated by exact on honeycomb lattice [6-8], as well as on Bethe lattice [9,10], mean-field approximation [11], effective-field theory with correlations [12-16], cluster variational theory [10], renormalization-group technique [17] and Monte-Carlo simulation [18-20]. On the other hand, ferromagnetic and ferrimagnetic systems subjected to longitudinal magnetic

fields have been investigated for many years [21-23]. The results assured that the longitudinal magnetic field has strong effects on the magnetic properties of these systems. Subsequently, the attention has been devoted to mixed-spin systems in a longitudinal field theoretically. The magnetic properties of the mixed spin-1/2 and spin-1 Ising ferromagnetic system with a crystal-field interaction in the presence of a longitudinal magnetic field by using the cluster variational method was investigated by Ekiz and Keskin [24]. Wei *et al.* [25] studied the magnetic properties of a mixed spin-1/2 and spin-3/2 Ising model in a longitudinal magnetic field within the framework of EFT with correlations. They examined the thermal behaviours of the magnetizations, susceptibilities and phase diagrams in detail. The magnetic properties of a mixed spin-1/2 and spin-3/2 Ising system in a longitudinal magnetic field on a Bethe lattice were studied by using the recursion relation scheme [26]. The thermodynamic and magnetic properties of a mixed Ising system on a triangular array in the presence of longitudinal magnetic field were investigated by Aouzi *et al.* by using EFT with correlations [27]. Jiang and Bai also studied the influences of an external longitudinal magnetic field on the magnetic properties of mixed spin-1/2 and spin-3/2 Ising ferromagnetic or ferrimagnetic bilayer system [28]. The Magnetic properties of an anti-ferro-

magnetic and ferrimagnetic mixed spin-1/2 and spin-5/2 Ising model in the longitudinal magnetic field and the Magnetic Properties of a Mixed Spin-3/2 and Spin-2 Ising Ferrimagnetic System within the Effective-field Theory are studied by Bayram Deviren et al by using the effective-field theory with correlations [29,30] and the results show that the longitudinal magnetic field plays an important role in the magnetic properties of the mixed spin Ising systems.

In this paper, our aim is to investigate the magnetic properties of the mixed spin-3/2 and spin-2 Ising system in the presence of longitudinal magnetic field within the framework of the mean-field theory based on Bogoliubov inequality for the Gibbs free energy. The outline of this work is as follows. In Section 2, we define the model and present the mean-field theory based on Bogoliubov inequality for the Gibbs free energy for the mixed-spin system with the applied longitudinal magnetic field. In Section 3, we discuss the temperature dependences of the sublattice and total magnetizations and sublattice and total susceptibilities for selected values of single-ion anisotropies. Finally, In Section 4 we present our conclusions.

## 2. Formulation of the Model and Its Mean-Field Solution

We consider a mixed spin-3/2 and spin-2 Ising model consisting of two sublattices  $A$  and  $B$ , which are arranged alternately. In this system, the sites of sublattice  $A$  are occupied by spin  $S_i^A$ , which take spin values  $\pm 3/2$  and  $\pm 1/2$ , while those of the sublattice  $B$  are occupied by spins  $S_j^B$ , which take spin values  $\pm 2$ ,  $\pm 1$  and  $0$ . The Hamiltonian of the system is given by

$$H = -J \sum_{i,j} S_i^A S_j^B - D_A \sum_{i=1}^{N/2} (S_i^A)^2 - D_B \sum_{j=1}^{N/2} (S_j^B)^2 - h \sum_{i=1}^{N/2} S_i^A - h \sum_{j=1}^{N/2} S_j^B \quad (1)$$

where  $\langle ij \rangle$  indicates a summation over all pairs of nearest-neighboring sites and the first summation is carried out only over nearest neighbour pairs of spins on different sublattices and  $J$  ( $J < 0$ ) is the nearest-neighbour exchange parameter,  $D_A$  is the crystal field interaction constant of spin-2 ions and  $D_B$  is that of spin-3/2 ions.  $h$  is the external magnetic field acting on the lattice.

In order to treat the model approximately we employ a variational method based on the Bogoliubov inequality for the Gibbs free energy which is given by the inequality,  $G(H) \leq \phi$ , where  $G(H)$  is the true free energy of the model described by the Hamiltonian (1) and  $\phi$  is given by the relation

$$\phi \equiv G_0(H_0) + \langle H - H_0 \rangle_0 \quad (2)$$

$G_0(H_0)$  is the average free energy of a trial Hamiltonian  $H_0$  and  $\langle \dots \rangle_0$  denotes a thermal average over the ensemble defined by  $H_0$ .

As the conventional procedure, the trial Hamiltonian is assumed to be in the form

$$H_0 = - \sum_i \left[ \gamma_A S_i^A + D_A (S_i^A)^2 \right] - \sum_j \left[ \gamma_B S_j^B + D_B (S_j^B)^2 \right], \quad (3)$$

where  $\gamma_A$  and  $\gamma_B$  are the two variational parameters related to the molecular fields acting on the two different sublattices, respectively.

By evaluating Equation (2), it is easy to obtain the expression of the free energy per site in *MFA*

$$g \equiv \frac{\phi}{N} = \frac{1}{2\beta} \left\{ \ln \left[ 2 \exp(4\beta D_A) \cosh(2\beta \gamma_A) + \exp(\beta D_A) \cosh(\beta \gamma_A) + 1 \right] + \ln \left[ 2 \exp(9\beta D_B/4) \cosh(3/2 \beta \gamma_B) + 2 \exp(\beta D_B/4) \cosh(\beta \gamma_B/2) \right] \right\} + \frac{1}{2} \left[ -z J m_A m_B + (\gamma_A - h) m_A + (\gamma_B - h) m_B \right], \quad (4)$$

where  $\beta = 1/k_B T$ ,  $N$  is the total number of sites of the lattice and  $z$  is the number of the nearest neighbors of every ion in the lattice.  $m_A$  and  $m_B$  are the sublattice magnetizations per site which are defined by Equations (5) and (6) below:

Now, by minimizing the free energy in Equation (4) with respect to  $\gamma_A$  and  $\gamma_B$ , we obtain

$$\gamma_A = z J m_B + h, \quad \gamma_B = z J m_A + h, \quad (7)$$

The mean field properties of the present system are then given by Equations (5)-(7). As the set of Equations (5)-(7) have in general several solutions for the pair, the

$$m_A = \frac{4 \sinh(2\beta \gamma_A) + 2 \exp(-3\beta D_A) \sinh(\beta \gamma_A)}{2 \cosh(2\beta \gamma_A) + 2 \exp(-3\beta D_A) \cosh(\beta \gamma_A) + \exp(-4\beta D_A)} \quad (5)$$

$$m_B = \frac{3 \sinh(3/2 \beta \gamma_B) + \exp(-2\beta D_B) \sinh(1/2 \beta \gamma_B)}{2 \cosh(3/2 \beta \gamma_B) + 2 \exp(-2\beta D_B) \cosh(1/2 \beta \gamma_B)} \quad (6)$$

pair chosen is that which minimizes the free energy, given in Equation (5). We are here interested in studying the thermal variation of the sublattice magnetizations and the averaged total magnetization per site which defined as

$$M = (m_A + m_B)/2. \quad (8)$$

On the other hand, the sublattice initial susceptibilities ( $\chi_A, \chi_B$ ) are defined by

$$\chi_A = \lim_{h \rightarrow 0} \frac{\partial m_A}{\partial h} \text{ and } \chi_B = \lim_{h \rightarrow 0} \frac{\partial m_B}{\partial h}.$$

From which the total initial susceptibility per site is given by

$$\chi = \lim_{h \rightarrow 0} \frac{\partial M}{\partial h} = \frac{1}{2}(\chi_A + \chi_B). \quad (9)$$

### 3. Results and Discussions

#### 3.1. Phase Diagrams

We begin with the ground-state structure of the system. At zero temperature, we find four phases with different values of  $\{m_A, m_B, q_A, q_B\}$ , namely the ordered ferrimagnetic phases

$$\begin{aligned} O_1 &\equiv \left\{ -2, \frac{3}{2}, 4, \frac{9}{4} \right\} \left( \text{or } \left\{ 2, -\frac{3}{2}, 4, \frac{9}{4} \right\} \text{ as well} \right), \\ O_2 &\equiv \left\{ -1, \frac{3}{2}, 1, \frac{9}{4} \right\} \left( \text{or } \left\{ 1, -\frac{3}{2}, 1, \frac{9}{4} \right\} \text{ as well} \right), \\ O_3 &\equiv \left\{ -2, \frac{1}{2}, 4, \frac{1}{4} \right\} \left( \text{or } \left\{ 2, -\frac{1}{2}, 4, \frac{1}{4} \right\} \text{ as well} \right), \\ O_4 &\equiv \left\{ -1, \frac{1}{2}, 1, \frac{1}{4} \right\} \left( \text{or } \left\{ 1, -\frac{1}{2}, 1, \frac{1}{4} \right\} \text{ as well} \right), \end{aligned}$$

and disordered phases

$$D_1 \equiv \left\{ 0, 0, 0, \frac{9}{4} \right\}, D_2 \equiv \left\{ 0, 0, 0, \frac{1}{4} \right\}$$

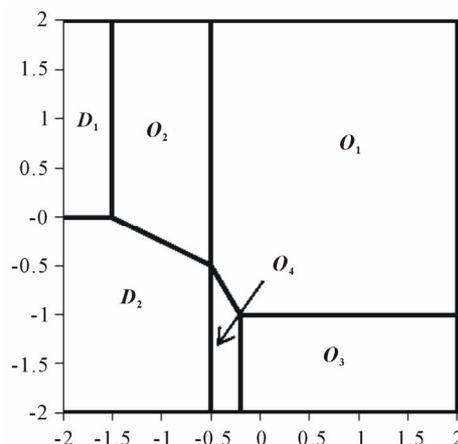
where the parameters  $q_A$  and  $q_B$  are defined by:

$$q_A = \left\langle (S_i^A)^2 \right\rangle, q_B = \left\langle (S_i^B)^2 \right\rangle.$$

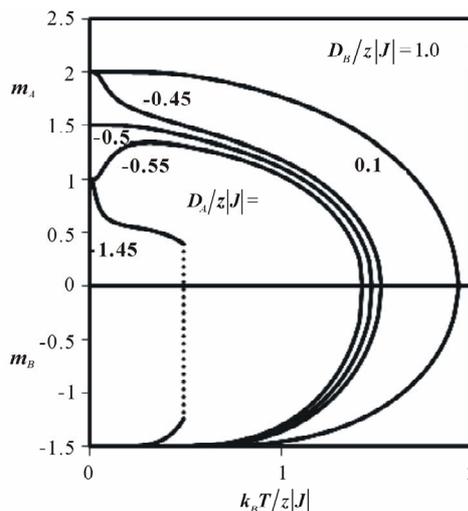
From Hamiltonian (1) and by comparing the ground-state energies of the different phases, the ground-state phase diagram can be determined, and is shown in **Figure 1**.

#### 3.2. Sublattice Magnetizations $m_A$ and $m_B$

In this subsection, let us at first examine the temperature dependence of the sublattice magnetizations  $m_A$  and  $m_B$  for the system. The results are depicted in **Figure 2** with



**Figure 1.** Ground-state phase diagram of the mixed spin-3/2 and spin-2 Ising ferrimagnetic system with the coordination number  $z$  and different single-ion anisotropies  $D_A/z|J|$  and  $D_B/z|J|$ . The four phases: ordered  $O_1, O_2, O_3, O_4$  and disordered  $D_1, D_2$  are separated by first-order transition lines.



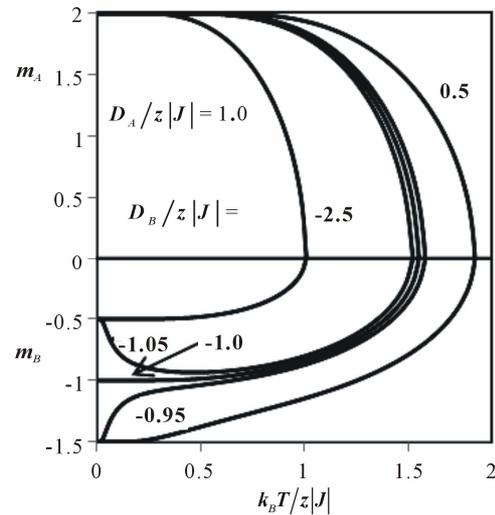
**Figure 2.** Thermal variation of the sublattice magnetizations  $m_A, m_B$  for the mixed-spin Ising ferrimagnet with the coordination number  $z$ , when the value of  $D_A/z|J|$  is changed for fixed  $D_B/z|J|=1.0$ .

$D_B/z|J|=1.0$ , and selected values of  $D_A/z|J|$ . Notice that the selection of  $D_A/z|J|$  corresponds to the crossover from the  $O_1$  to the  $O_2$  phase (see the ground state phase diagram **Figure 1**), therefore the ground state is always ordered. When  $D_A/z|J|=1.0$ , the sublattice magnetizations  $m_A$  and  $m_B$  have standard characteristic convex shape. When  $D_A/z|J|=-0.45$  (slightly above the boundary between the phase  $O_1$  and the phase  $O_3$  in the ground state-phase diagram, where  $D_A/z|J|=-0.5$ ) the sublattice magnetization  $m_A$  may exhibit a rather rapid decrease from its saturation value  $m_A = 2.0$  with the

increase of temperature from  $T = 0$  K to a certain temperature  $T$ . When  $D_A/z|J| = -0.5$  (at the boundary between the ordered phase  $O_1$  and the ordered phase  $O_2$  in the ground state phase-diagram), the saturation value of  $m_A$  is 1.5, which indicates that the half of the spins on the sublattice  $A$  are equal to  $+2$  (or  $-2$  as well) and the other half are equal to  $+1$  (or  $-1$  as well). Note that this mixed state persists as long as  $D_A/z|J| = -0.5$  and  $D_B/z|J| = -0.5$ . When  $D_A/z|J| = -0.55$  the ground state phase is  $O_2$  phase, with  $m_A = 1.0$  at  $T = 0$  K. However, in this case the thermal variation of  $m_A$  exhibits an interesting feature which is the initial rise of  $m_A$  with the increase of temperature before decreasing to zero value at the critical point  $T_c$ .

As shown in **Figure 2**, for  $D_A/z|J| = -1.45$  (slightly below the boundary between the ordered phase  $O_2$  and the disordered phase  $D_I$  in the ground state  $D_A/z|J| = -1.5$ ), the sublattice magnetization  $m_A$  exhibits a rapid decrease before it decreases normally by increasing the value of  $k_B T/z|J|$  to the critical point  $T_c$ . In this case, a large magnetization jump is observed at the critical point, indicating a first-order transition. On the other hand, for all values of  $D_A/z|J|$  the sublattice magnetization  $m_B$  decreases normally by increasing the value of  $k_B T/z|J|$  to the critical point  $T_c$ , even though it is coupled to  $m_A$ . The previous results for sublattice magnetization are similar to those observed in the Mixed Spin-3/2 and Spin-2 Ising Ferrimagnetic System within the Effective-field Theory [30] and in the mixed-spin-1 and spin-3/2 Ising ferrimagnetic system [31-33].

**Figure 3** shows the sublattice magnetization curves as a function of temperature for several values of  $D_B/z|J|$ , when  $D_A/z|J| = 1.0$ . In this case, the selection of  $D_B/z|J|$  corresponds to the crossover from the  $O_1$  to the  $O_3$  phase. When  $D_B/z|J| = 0.5$ , the sublattice magnetization  $m_A$  may show normal behaviour. When  $D_B/z|J| = -0.95$  (slightly above the boundary between the ordered phase  $O_1$  and the ordered phase  $O_2$ , where  $D_B/z|J| = -1.0$ ) the magnetization curve  $m_B$  may exhibit a rather rapid decrease from its saturation value ( $m_B = -3/2$ ) at  $T = 0$  K, while for the value of  $D_B/z|J| = -1.05$  (slightly below that boundary), there is a rapid increase of  $m_B$  from the saturation value ( $m_B = -3/2$ ) with the increase in  $T$ . When the value of  $D_B/z|J| = -1.0$ , the saturation value of the sublattice magnetization  $m_B$  at  $T = 0$  K is ( $m_B = -1.0$ ). It indicates that at this point, the spin configuration of  $S_i^B$  in the ground state consists of the mixed state; half of the spins on the sublattice  $B$  are equal to  $-3/2$  (or  $+3/2$  as well) and the other half are equal to  $-1/2$  (or  $+1/2$  as well). It is also seen from **Figure 3** that when  $D_B/z|J| = -2.5$ , the sublattice magnetization  $m_B$  decreases normally from its saturation value ( $m_B = -1/2$ ) to vanish at the critical tem-



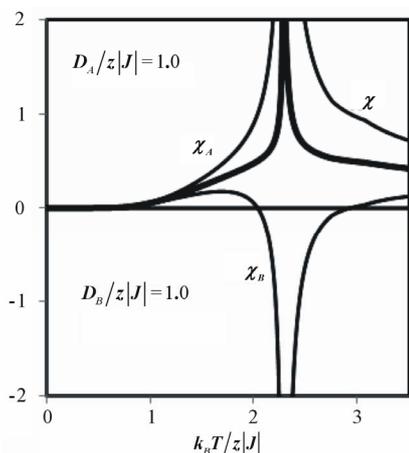
**Figure 3.** Thermal variation of the sublattice magnetizations  $m_A$ ,  $m_B$  for the mixed-spin Ising ferrimagnet with the coordination number  $z$ , when the value of  $D_A/z|J|$  is changed for fixed  $D_A/z|J| = 1.0$ .

perature  $T_c$ .

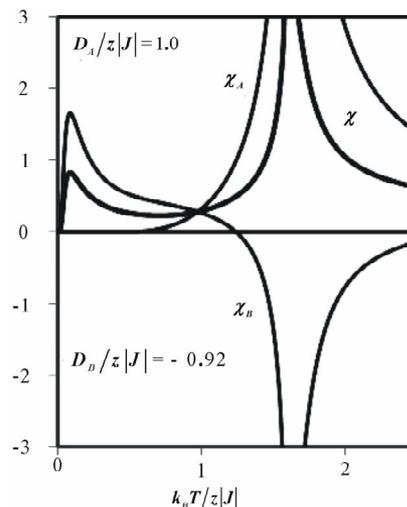
On the other hand, for all the values of  $D_B/z|J|$  the sublattice magnetization  $m_A$  may show normal behaviour, even though it is coupled to  $m_B$ .

### 3.3. Susceptibility

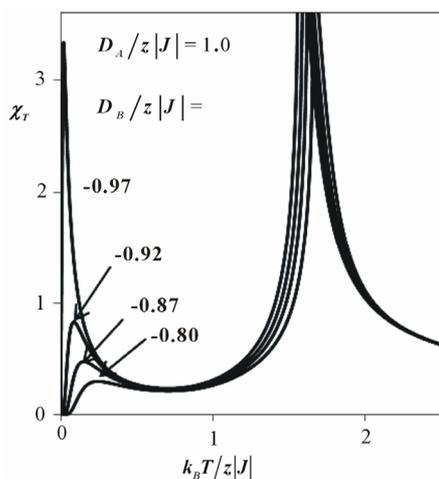
In **Figure 4**, the temperature dependences of the initial susceptibilities (total and sublattices) of the system are shown for  $D_A/z|J| = 1.0$  and  $D_B/z|J| = 1.0$ . We can see clearly that the total susceptibility  $\chi_T$  increases slowly by increasing temperature. When the temperature approaches  $T_c$ , the total susceptibility  $\chi_T$  increases very rapidly and goes to infinity at the critical temperature  $T_c$ . Moreover, one can observe that in the vicinity of  $T_c$ , the sublattice susceptibility  $\chi_A$  rapidly increases for  $T < T_c$  and goes to  $+\infty$  at  $T = T_c$  and rapidly decreases for  $T > T_c$ . On the other hand, the sublattice susceptibility  $\chi_B$  rapidly decreases for  $T < T_c$ , goes to  $-\infty$  at  $T = T_c$  and rapidly increases for  $T > T_c$ . In **Figure 5**, the total initial magnetic susceptibilities  $\chi_T$  are depicted for the system, with  $D_A/z|J| = 1.0$ , when the value of  $D_B/z|J|$  is changed. As is seen from this figure, the total susceptibility diverges at the critical temperature and the point of singularity shifts to lower temperature, upon decreasing the value of  $D_B/z|J|$ . Now, besides the infinite values of  $\chi_T$  at  $T = T_c$  the total susceptibility at low temperatures exhibits a sharp maximum in the low-temperature region (when  $D_B/z|J| = -0.97$ ) and a broad maximum (when  $D_B/z|J| = -0.92, -0.87, -0.80$ ). Particularly, It is seen from **Figure 5** that the largest broad maximum appears



**Figure 4.** Initial susceptibilities (total and sublattices) versus temperature for the mixed spin-3/2 and spin-2 Ising ferrimagnetic system with coordination number  $z$ , when  $D_A/z|J|=1.0$  and  $D_B/z|J|=1.0$ .



**Figure 6.** Initial susceptibilities (total and sublattices) versus temperature for the mixed spin-2 and spin-3/2 Ising ferrimagnet with coordination number  $z$ , when  $D_A/z|J|=1.0$  and  $D_B/z|J|=-0.92$ .



**Figure 5.** Thermal variations of the total initial susceptibility for the mixed spin-3/2 and spin-2 Ising ferrimagnetic system with the coordination number  $z$ , for several values of  $D_B/z|J|$  when the value of  $D_A/z|J|=1.0$ .

for  $D_B/z|J|=-0.92$  and with the increase of  $D_B/z|J|$ , its magnitude gradually decreases, and disappears when  $D_B/z|J| \geq -0.8$ .

In **Figure 6**, temperature dependences of total and sublattice susceptibilities are presented for constant values of  $D_A/z|J|=1.0$  and  $D_B/z|J|=-0.92$ . It is easy to see from this figure that the variation of the total susceptibility at low-temperature region originates from the behaviour of the  $\chi_B$  sublattice susceptibility. It is also seen that the  $\chi_A$  sublattice susceptibility exhibits the usual temperature dependence in the vicinity of  $T_c$ , while the sublattice susceptibility  $\chi_B$  takes negative values. We notice that the existence of the sharp maxima (sharp peak) in the magnetic susceptibility was found by Na

kamura and Tucker for the mixed spin-1 and spin-3/2 Ising ferrimagnet by using Monte Carlo simulation [34]. Moreover, the sharp and broad maxima can be observed in the mixed ferro-ferrimagnetic ternary alloy in reference [35].

In **Figure 7**, we present the temperature dependence of the total susceptibility for the system with  $D_A/z|J|=1.0$  and  $D_B/z|J|=-1.0$  (in the boundary between the phase  $O_1$  and the phase  $O_2$ , in the ground-state phase diagram (**Figure 1**)). In this case, the total susceptibility exhibits two divergences: one at  $T=0$  K indicating a first-order phase transition and another at  $T=T_c$ , indicating a second-order phase transition. We notice that these effects can be observed for ferro-ferrimagnetic ternary alloy [35], site-diluted [36], bond-diluted [37], and site-bond-correlated Ising model [38].

In **Figure 8**, we show the thermal variation of the initial susceptibilities (total and sublattices). For the system with  $D_A/z|J|=1.0$  and  $D_B/z|J|=-1.0$ . It is seen from this figure that the divergence of the total susceptibility at zero temperature originates from the divergence of the sublattice susceptibility  $\chi_B$ .

Now, in order to explain the appearance of the broad maximum in the susceptibility of the sublattice B in low-temperature region (**Figure 6**), we consider the temperature dependence of the sublattice magnetizations  $m_A$  and  $m_B$  (as shown in **Figure 9**) for the system with  $D_A/z|J|=1.0$  and  $D_B/z|J|=-0.92$ , when  $h=0$  (solid lines) and when  $h \neq 0$  (dashed lines). In this figure, it is seen that there is a rapid decrease in  $m_B$  from its saturation value ( $m_B = -3/2$ ) with the increase in  $T$  and it is clear that at any point of temperature in this region,

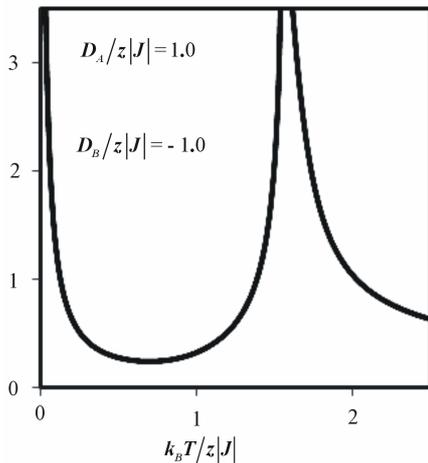


Figure 7. Thermal variations of the total initial susceptibility for the mixed spin-2 and spin-3/2 Ising ferrimagnet with the coordination number  $z$ , when the value of  $D_A/z|J|=1.0$  and the value of  $D_B/z|J|=-1.0$ .

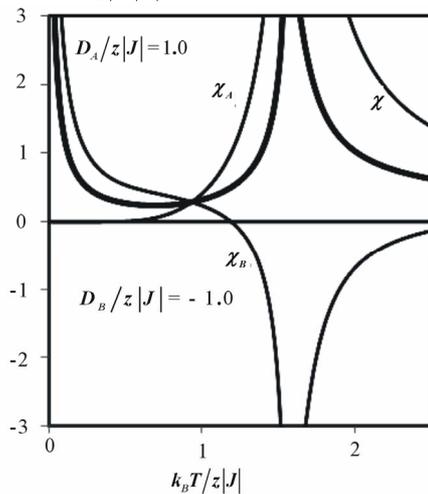


Figure 8. Initial susceptibilities (total and sublattices) versus temperature for the mixed spin-2 and spin-3/2 Ising ferrimagnet with coordination number  $z$ , when  $D_A/z|J|=1.0$  and  $D_B/z|J|=-1.0$ .

there is a jump in  $m_B$  from a certain value, when  $h \neq 0$  to a lower value, when  $h = 0$  resulting in the broad maximum of  $\chi_B$ .

To explain the physical scenario for the appearance of the divergence of the susceptibility of the sublattice  $B$  at zero temperature (Figure 8), we consider the temperature dependence of the sublattice magnetizations  $m_A$  and  $m_B$  (as shown in Figure 10) for the system with  $D_A/z|J|=1.0$  and  $D_B/z|J|=-1.0$ , when  $h = 0$  (solid lines) and when  $h \neq 0$  (dashed lines). In this figure, there is a mixed-spin state on the sublattice  $B$  in the ground state, for  $D_B/z|J|=-1.0$  and  $h = 0$ , consisting from the  $S_j^B = 3/2$  and  $S_j^B = 1/2$  with equal probabilities.

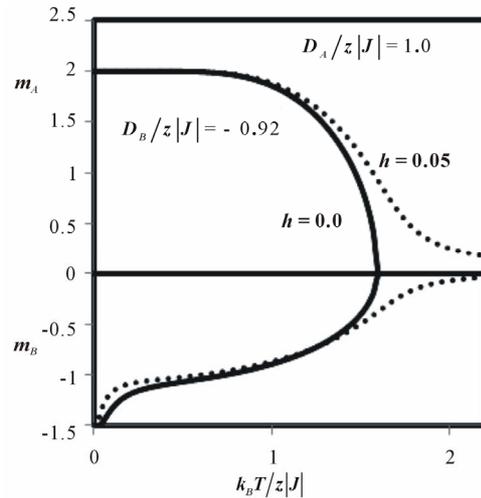


Figure 9. Thermal variation of the sublattice magnetizations  $m_A, m_B$  for the mixed-spin Ising ferrimagnet with the coordination number  $z$ . when the value of  $D_A/z|J|=1.0$  and  $D_B/z|J|=-0.92$ . The solid lines represent the sublattice magnetizations without an external magnetic field effect on the system ( $h = 0.0$ ), while the dashed lines represent the sublattice magnetizations under the effect of an external magnetic field ( $h = 0.05$ ).

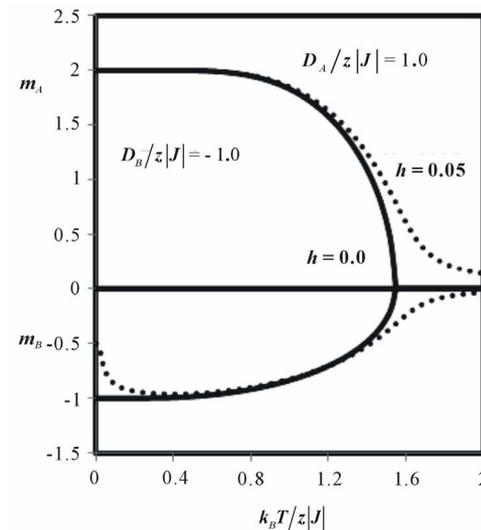


Figure 10. Thermal variation of the sublattice magnetizations  $m_A, m_B$  for the mixed-spin Ising ferrimagnet with the coordination number  $z$ . when the value of  $D_A/z|J|=1.0$  and  $D_B/z|J|=-1.0$ . The solid lines represent the sublattice magnetizations without an external magnetic field effect on the system ( $h = 0.0$ ), while the dashed lines represent the sublattice magnetizations under the effect of an external magnetic field ( $h = 0.05$ ).

However, at  $h \neq 0$  the  $S_j^B = -0.5$  is suddenly favored resulting in a divergence of the  $\chi_B$  sublattice susceptibility at  $T = 0$  K.

#### 4. Conclusions

In this paper, we have studied the magnetic properties of a ferrimagnetic mixed spin-3/2 and spin-2 Ising system with different single ion anisotropies in a longitudinal magnetic field by using the meanfield theory based on Bogoliubov inequality for the Gibbs free energy.

The ground state phase diagram of the model was constructed in  $D_A/z|J|$  and  $D_B/z|J|$  plane. In this phase diagram, we have found four ordered phases and two disordered phases separated by first order lines. We also have investigated the thermal variations of the sublattice magnetizations for selected values of  $D_A/z|J|$  and  $D_B/z|J|$  and we have found some interesting results in the sublattice magnetization curves we have found that the sublattice magnetizations may exhibit a rather rapid decrease from their saturation values with the increase of temperature from  $T = 0$  K to a certain temperature  $T$ . The initial susceptibilities (total and sublattices) of the system are considered for selected values of  $D_A/z|J|$  and  $D_B/z|J|$  and The temperature dependences of the initial susceptibilities (total and sublattices) of the system are considered for selected values of  $D_A/z|J|$  and  $D_B/z|J|$  and we have found that besides the infinite values of  $\chi_T$  at  $T = T_c$  the total susceptibility at low temperatures may exhibits a sharp maximum or broad maximum in the low-temperature region (when  $D_B/z|J|$  approaches the boundary between the phases in the ground state phase diagram). We have also found that the susceptibility (in the boundary between the phases in the ground state phase diagram) may exhibit two divergences: one at  $T = 0$  K indicating a first-order phase transition and another at  $T = T_c$ , indicating a second-order phase transition.

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