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Study and Analysis of the Orbital and Physical Properties of 2010 TK_7

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Abstract

In this paper we calculate the volume, mass, gravitational attraction to the Earth, angular momentum the orbit of the Trojan asteroid (TK_7) [1]. In this paper, we use classical Newtonian mechanics to analyse some of the physical and orbital properties of the Trojan asteroid, which are still experimentally unknown. The asteroid should remain in Earth's orbit for the next 100 years. We conclude by providing informed estimates of 2010 TK_7's yet unknown physical properties: *i.e*. mass, volume, gravitational attraction to Earth and angular momentum.

Keywords: 2010 TK_7, Trojan Asteroid, Universal Gravitation, Horseshoe Orbit, Angular Momentum Newton's Laws of Motion

1. Introduction

TK_7 is located in the L_4 Lagrangian point, 60˚ ahead of the Earth in orbit of the Sun, and was first observed in October 2010 by NASA's Wide-field Infrared Survey Explorer (WISE) [2].

From the data received about TK_7 from NASA's WISE satellite we approximate the volume of TK_7 as circa 1.1310×10^8 m^3. From NASA's JPL Small Body Database Browser [3], 2010 TK_7's orbital properties have the: aphehelion of 1.19095 AU, perihelion of 0.80955 AU, semi-major axis of 1.00025 AU, eccentricity of 1.9066, an orbital period of 365.394 days, a mean anomaly of 217.384˚, inclination of 20.8656˚, a longitude of ascending node of 96.5426˚ and an argument of perihelion of 45.7551˚. It is currently 80,000,000 km from the Earth. TK_7's dimensions are believed to be 300m in diameter [1].

In Section 2, we will review the current literature on Trojan asteroids, and on Lagrangian points. We will not use the three-body problem [4][1]. Instead, in Section 3, we use classical Newtonian mechanics to determine the gravitational attraction of the Trojan to Earth, and we calculate its angular momentum based on its known physical and orbital properties. In Section 4 we provide a synoptic table of the numerical results of this paper. Section 5 concludes the paper discussing its results and possible future uses.

[1]We do not analyse use the three-body problem since only approximations can be made.

2. Review of Literature on Trojan Asteroids and Lagrangian Points

2010 TK_7 was discovered in October 2010 by NASA's satellite WISE. Once discovered, the asteroid has been followed by the Canada-France-Hawaii Telescope [5]. **Figure 1** evaluates the data of orbital properties. These data were published by Connors M. *et al.* 2011 [1].

The suspicion that Earth had a Trojan asteroid had however been present for some time [1]. 2010 TK_7 is the first Trojan asteroid to have been confirmed to be orbiting the Earth. In the past, Trojan asteroids have been found to orbit around many of the gas giants, such as Jupiter or Saturn, however 2010 TK_7 is the first known to have begun orbiting the Earth.

Most Trojan Asteroids originate from the Kuiper Belt [6], and follow the orbit of a planet, 60˚ behind, or 60˚ ahead, with the Sun as the pivot. Joseph-Louis Lagrange determined these points as coordinates where Trojan asteroids can be located. Lagrange also worked on the three-body problem [4], where a gravitationally based calculation is made for a system with three celestial bodies instead of two. Since it is impossible to find an exact answer to the three-body problem, we are not exploring this further (Our limitations have been explained by chaos theory [7]). An example of post-Newtonian work on the three-body problem can be found in Fawzy A. Abd El-Salam, *et al.* (2011) [8]

The Lagrangian points are commonly used to calculate many of the orbital properties of celestial bodies [9].

This paper focuses on celestial planetary mechanics, most of which was developed by Sir Isaac Newton. Other notable examples of modern work on planetary mechanics (although based on a different, Quantum mechanical model) were by Nie Q. X. (2011) [10].

3. Analysis based on Newtonian Physics

3.1. Universal Gravitation

Although the mass of the asteroid TK_7 is unknown, we use the average density of Trojan asteroids (2800 kg/m^3) [11] to calculate its estimated mass (m) (based on spherical approximations) to be 1.4844 × 10^8 kg (based on the fact that the radius is 150 m) by

$$m = \rho v = 53014.3760\ \mathrm{m}^3 \times 2800 \frac{\mathrm{kg}}{\mathrm{m}^3} = 1.4844 \times 10^8\ \mathrm{kg} \quad (1)$$

By applying Newton's Law of Universal Gravitation [12], we obtain the gravitational attraction (F)

$$F = G \frac{\left(5.9736 \times 10^{24}\right)\mathrm{kg}\left(1.4844 \times 10^8\right)\mathrm{kg}}{\left(8 \times 10^7\right)^2} = 9.2466 \times 10^6. \quad (2)$$

3.2. Angular Momentum

The basic law of angular momentum (L) states that $L = r \times p$, r being the distance between the body, and the pivot around which it is orbiting, and p being the linear momentum of the body. We calculate the momentum of TK_7 as:

$$p = \left(1.4844 \times 10^8\,\mathrm{kg}\right)\left(9.4 \frac{\mathrm{km}}{\mathrm{s}}\right) = 1.3953 \times 10^9\ \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}. \quad (3)$$

From the data published in "Earth's Trojan Asteroid" [1], we use the data of the distance between 2010 TK_7 and the Earth; 80,000,000 km. Therefore:

$$r = 8 \times 10^7\ \mathrm{km} = 0.5347\ \mathrm{AU} \quad (4)$$

Now, by substituting Equations (3) and (4), (giving us p and r) into the equation for angular momentum $L = r \times p$, we obtain:

$$L = \left(1.3953 \times 10^9\right) \times \left(8 \times 10^7\right) = 1.1162 \times 10^{17}\ \mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-1} \quad (5)$$

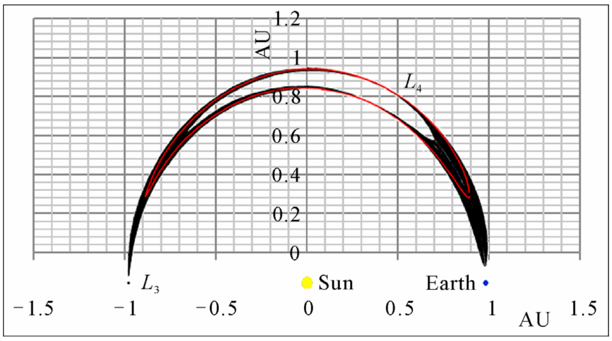

Figure 1. Orbital Dynamics of 2010 TK_7 [1].

3.3. Explanation of Figure 1

Figure 1 [1] shows the orbital path of 2010 TK_7 through the inner Solar system. The graph was compiled with data derived from the images taken by the Canada-France-Hawaii Telescope [5], and from NASA's WISE satellite [2]. Graphical approximation methods suggest that the orbit can be represented by the function: $y = -x^{2.4} + 1$. The horseshoe orbit loosely follows the function of $y = -x^{2.4} + 1$, yet instead of orbiting in a standard fashion, such as other moons or planets, it follows the typical low-eccentricity path of a horseshoe orbiter [13]. The discovery of 2010 SO_{16} was the first horseshoe orbiter of the Earth. The defining aspect of horseshoe orbiters of planets is their low eccentricity, which in this case is 1.9066 [3].

4. Data Table

The table below summarizes the data estimated in this paper from Newton's laws for the asteroid TK_7.

5. Conclusions

In this paper, we have calculated: the estimated volume and mass of the asteroid from an informed estimate of the mean density (2800 kg/m^3) [11]. This data is vital before any other calculations may be made for the physical properties of the asteroid. We have also calculated the gravitational attraction between 2010 TK_7 and the Earth by using classical Newtonian mechanics. We have calculated the angular momentum which is affecting the movement of the asteroid.

Table 1. Data about TK_7 calculated in this paper.

Mass	1.4844 × 10^8 kg
Volume	53014.3760 m^3
Gravitational attraction with Earth	9.2466 × 10^6
Angular Momentum	1.1162 × 10^{17} kg·m^2·s^{-1}

From our analysis above, having used the the data from the WISE satellite [2] and from Earth's Trojan Asteroid [1], our calculations of the physical and orbital properties have served to build on existing knowledge of the Trojan asteroid 2010 TK_7.

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