

# Papert's Microworld and Geogebra: A Proposal to Improve Teaching of Functions

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**How to cite this paper:** De Alencar Carvalho, C. V., De Medeiros, L. G. F., De Medeiros, A. P. M., & Santos, R. M. (2019). Papert's Microworld and Geogebra: A Proposal to Improve Teaching of Functions. *Creative Education*, 10, 1525-1538. <https://doi.org/10.4236/ce.2019.107111>

**Received:** June 6, 2019

**Accepted:** July 14, 2019

**Published:** July 17, 2019

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## Abstract

This paper discusses how to improve teaching of Mathematics in Brazilian schools, based on Seymour Papert's Constructionism associated with Information Technology tools. Specifically, this work introduces the constructionist microworld, a digital environment where students are able to build their knowledge interactively, in this case, using dynamic mathematics software GeoGebra.

## Keywords

Microworld, GeoGebra, Seymour Papert, Information Technologies in Education

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## 1. Introduction

This research's main goal is to present a proposal to help Brazilian teachers improve their educational practices. Many of them are not familiar with Information Technologies (IT) and do not know how to use educational softwares in their classrooms. Abellón (2015) informs that only 2% of Brazilian teachers use TI in their classes, mainly due to lack of access to computers, lack of formal training in pedagogical softwares and low-speed Internet connection. He further details that there is, on average, one computer for every 34 students in Brazil's schools.

Consequentially, several Brazilian teachers still follow traditional pedagogical techniques, where they require students to memorize contents in a rigid, mechanic way. Freire (1987) describes this process as "bank style education", where

teachers “deposit” information in students “empty” minds.

However, this conception is not adequate for modern students, since they use IT massively even before they enter school. They are skilled in using social media and expect to have instant access to information through Internet. Their minds are far from “empty” and traditional educational methods used by many teachers end up frustrating them. D’Ambrosio (1991) defines these methods as “obsolete, uninteresting and useless”.

As a result, this study intends to present a proposal to improve teaching of Mathematics using Seymour Papert’s Constructionism associated with Information Technology (IT) tools. It aims to describe how to use a dynamic mathematics software, like GeoGebra, to build an interactive environment suitable to modern students, to create these components, incorporating the applicable criteria that follow.

## 2. Papert’s Constructionism

School has not changed much for a long time. New technologies and media have little influence on how teaching and learning happen in classrooms. Ripper (1996) affirms that many educational systems still prepare its students to execute repetitive tasks without questioning, a style of work that dates back to XIX century. However, modern jobs demand creative and proactive workers, able to adapt to new challenges and circumstances. In consequence, schools have to depart its traditional posture and change to face those demands.

Papert (1992) describes this situation using a parable, where he imagines teachers traveling through time from a century ago to visit a contemporary classroom. While some objects might look strange, the teachers would easily recognize the place as a classroom, very similar to the ones in their own time. This parable leads to a question: “Why, through a period when so much human activity has been revolutionized, have we not seen a comparable change in the way we help our children learn?” (Papert, 1992: p. 2)

These traditional methods lead to dissatisfaction among modern students.

“To the extent that children reject School as out of touch with contemporary life, they become active agents in creating pressure for a change. Like any other social structure, School needs to be accepted by its participants. It will not survive very long beyond the time if children can no longer be persuaded to accord it a degree of legitimation” (Papert, 1992: p. 6).

Born in South Africa, Papert initially did research in Mathematics at Cambridge University, and later worked with Jean Piaget at University of Genève. This experience in Switzerland motivated him to study how children build their knowledge (Papert, 1992), for Piaget’s theory provided a comprehensive framework to understand how youngsters think of different phases of their lives.

“Piaget’s constructivism offers a window into what children are interested in, and able to achieve, at different stages of their development. The theory describes how children’s ways of doing and thinking evolve over time, and under which circumstance children are more likely to let go of—or hold onto—their

currently held views” (Ackerman, n.d.).

Starting from that perspective, Papert developed his own theory, Constructivism, which focus “on the art of learning, or ‘learning to learn’”, and on the significance of making things in learning” (Ackerman, n.d.). He emphasizes the importance of interacting with educational artifacts to facilitate the construction of new knowledge.

“The word constructionism is a mnemonic for two aspects of the theory of science education underlying this project. From constructivist theories of psychology, we take a view of learning as a reconstruction rather than as a transmission of knowledge. Then we extend the idea of manipulative materials to the idea that learning is most effective when part of an activity the learner experiences as constructing a meaningful product” (Sabelli, 2008).

Later, Papert worked in Massachusetts Institute of Technology (MIT), where he developed LOGO programming language, specially designed as a constructionistic educational environment, quite effective for teaching Mathematics. Papert even created the term mathfobia, to describe the fear and discomfort most students have towards Mathematics (Fainguelernt, 1999).

Constructionism defends computers as a tool for knowledge building, interacting with students in a cycle known as D-E-R-D (description-execution-reflection-debug). Description involves a thorough analysis of the problem, allowing students to understand what has to be accomplished. In the execution phase, students interact with the software and receive instantaneous feedback, which leads to the reflection phase, where results are analyzed. Finally, the debug phase discusses new strategies and views (Valente, 1999), as shown in Figure 1.

Papert considers the tendency to overvalue abstract reasoning an obstacle in Education progress. He believes in an “epistemological reversion to more concrete ways of knowing—a reversal of the traditional idea that intellectual progress consists of moving from the concrete to the abstract” (Papert, 1992: p. 137). His theory preconizes to produce the most learning with the least teaching, allowing students to freely discover new knowledge by themselves. In fact,

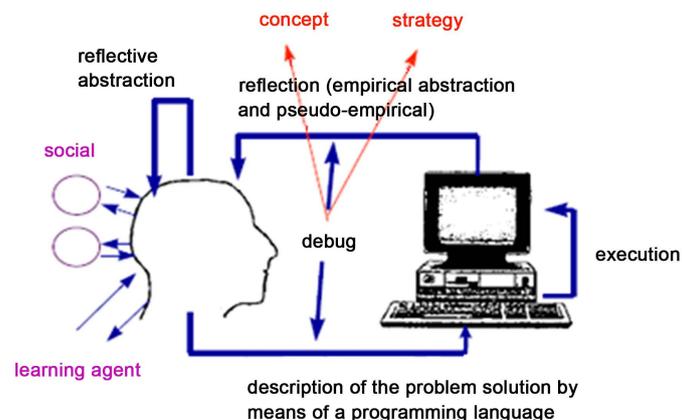


Figure 1. Cycle “description-execution-reflection-debug” (Valente, 1999).

“the kind of knowledge children most need is the knowledge that will help them to get more knowledge” (Papert, 1992: p. 139).

A practical example would be how children learn to play videogames. They assimilate complex rules and strategies without formal training by a teacher, and use their own initiative to look for information in any type of media. In short, they build new knowledge in their personal way and use what they already know as a starting point.

Papert describes these innate skills as learning without teaching, and these abilities help students to lose their fear towards learning Mathematics, previously described as mathfobia. Fainguelernt (1999) states that LOGO programming language is an efficient learning tool for Math students, for its constructionist approach allows them to build their knowledge interacting with the computer at their own pace and time.

However, traditional ways are hard to change, and most schools did not understand how to insert computers and constructionist softwares in their educational routine.

“The computer in the classroom was undermining the division of knowledge into subjects; it was turned into a subject of its own. It undermines the idea of curriculum; it was made the topic of a curriculum of its own. Nevertheless, of course, this mechanism is not confined to computers. In its time, School has normalized other subversive influences too. For example, Piaget was the theorist of learning without curriculum; School spawned the project of developing a Piagetian curriculum” (Papert, 1992: p. 54).

As a result, Courses aiming to achieve computer literacy were developed (Valente, 1995). School started teaching how to use computers as an end in itself, as another discipline in its curriculum, and not as a pedagogic tool to help improve learning of every other discipline.

In sequence, computers were transformed into instructional tools. In other words, they received the exact material used in traditional classes only to transmit it to students, who are passive subjects in the whole process. Computers work as optimized teaching machines, and teachers have little influence on the learning experience (Valente, 1995).

In the constructionist approach, in contrast, students control their learning activities, using computers as tools in building their knowledge, based on the D-E-R-D cycle. This led to what Papert defined as “microworlds”.

### 3. Microworlds

Microworlds are computer environments that simulate real-life scientific models. With them, students can explore, interact and modify these models, building new knowledge in the process.

“In bringing the computer into the education system, the microworld is the richest concept that we have to work with, and it should be used as the central one. My concept of how to create a curriculum (and by this word I mean a co-

herent set of materials to aid learning through the whole school period and before and after, as well) is to create a network of microworlds, each one focusing on different areas of knowledge” (Papert, 1984: p. 86).

Papert named these computer objects as microworlds because he considered them “little slices of reality”. He exemplifies their dynamics with LOGO’s and its turtle, able to move around the computer screen.

“Inside this microworld, a child explores by manipulating the turtle: making it draw squares and circles, repeating and rotating designs, whatever the child can imagine. The microworld is created and designed as a safe place for exploring. You can try all sorts of things. You will never get into trouble. You will never feel ‘stupid’. It will never say a rude thing to you; it will never embarrass you; it will never fall to pieces or bite you or give you a low grade. You are totally safe in this little world. And yet while being safe, it is also designed to be discovery-rich in the sense that little nuggets of knowledge have been scattered around in it for you to find” (Papert, 1984: pp. 79-80).

He also states that microworlds can optimize teaching of science and mathematics.

“If it’s true that knowledge is normally appropriated in a process like microworld construction—that is, something like the creation of little pockets of reality, where you can dominate it and feel at home with it—some kinds of knowledge split up into a form that can be easily appropriated in that way. Others don’t, and that’s where we get into trouble: areas where our culture doesn’t allow that kind of appropriation. Writing, mathematics, and science have been such areas, but the computer now makes it possible to create microworlds which can transform the rather clumsy educational process, as practiced in schools today, into a more natural and spontaneous one, similar to the way children learn language” (Papert, 1984: p. 93).

Today, the main software used to develop mathematic microworlds is GeoGebra. It is a comprehensive environment where students can manipulate all types of mathematic constructions, from the simplest to the most complex ones. It also views each problem both geometrically and algebraically, which enhances the learning experience. Furthermore, millions of people use GeoGebra globally. This community produces a vast amount of pedagogic materials, which are free and available to anyone interested through the site [geogebra.org](http://geogebra.org).

One of those materials is a microworld built by Luiz Geraldo da Silva (2017). Based on Veronica Gitirana Gomes Ferreira’s work Student’s perception of functions articulated in dynamic microworlds, this microworld helps students to conceptualize functions in mathematics by interacting with a computer environment. The construction provides ten different functions, each one identified by a color. Students can select a function using a sliding control. There are also two variables (A and B), and students must analyze how variable B changes in function of variable A, which can be controlled by another sliding control.

This microworld is a helpful tool for teaching Mathematics, since the study of functions is highly relevant. Caraça (1951) considers the search of regularities in

natural phenomena one of the most important tasks in scientific work, and the concept of functions is the best tool to comprehend those regularities.

However, students face several difficulties when working with functions. Markovits, Eylon and Bruckheimer (1995) described in their research that students had trouble to represent graphically values in the domain and image sets, often inverting the axis of each set. They also had trouble in dealing with functions in algebraic form, and associating this form with the geometric one. For Booth (1995), the concept of variable and the need to generalize are the hardest to understand.

So, Silva's microworld was used with a group of students without previous knowledge about functions. The experiment was conducted with 25 (twenty-five) students in their freshman year in a technical course in chemistry integrated with high-school, offered by Centro Federal de Educação Tecnológica Celso Suckow da Fonseca—CEFET/RJ, in Valença, Rio de Janeiro.

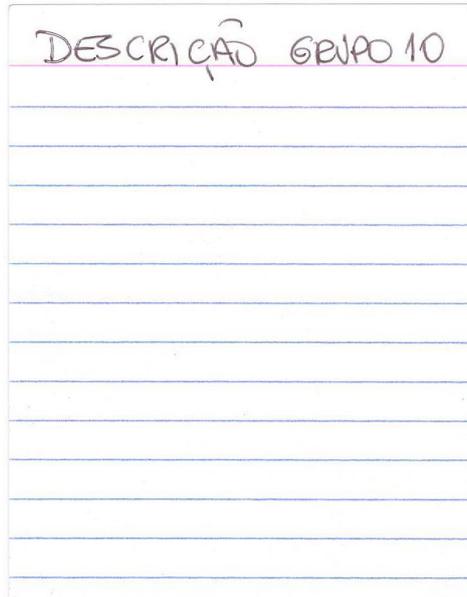
The class was divided into nine groups, seven groups composed of three students and two groups formed by two students. Each group then selected a "color card" (15.2 cm × 10.1 cm), which defined the function they would work with, as shown in Figure 2. In this card, the participants should write down their names and all observations they made with respect to the function analyzed.

Afterwards, they should copy these observations to a smaller card (7,6cm x 10,1cm), as seen in Figure 3. This card has no color identification, and the objective was to describe the function with enough detail to allow other students to identify precisely which graphic corresponded to that specific description.

After all, observations ended, the smaller cards were shuffled and redistributed to the nine groups. Now, each team had to identify the function by its color, using the description in the card as guidance. Next, the results of each group are detailed.



Figure 2. Brown color card.



**Figure 3.** Small card group 10 (unused).

### 3.1. Group 1 (Purple)

The group correctly described the values of variable B in relation to variable A, but did not describe the function's graphic. Even so, Group 5 correctly identified the graphic (Figure 4).

### 3.2. Group 2 (Pink)

The team concluded that variable B did not change in relation to variable A and perceived the graphic as a straight line. Group 6 correctly identified the graphic (Figure 5).

### 3.3. Group 3 (Orange)

Students realized that variable A could be positive and negative, but variable B remained positive for every value of variable A. They described the curve as "forming quadrilaterals with equal sides and external angles". Group 4 correctly identified the graphic (Figure 6).

### 3.4. Group 4 (Light Blue)

Description provided in the small card was very brief. The students only informed that the graphic was a straight line with variables A and B directly proportional to each other. Based on this information, Group 2 wrongly selected the dark green graphic, which also represents a straight line (Figure 7).

### 3.5. Group 5 (Dark Green)

Team did not identify the graphic as a straight line (Figure 8), but stated that variables A and B are always equal. Besides, they concluded, "that every point forms a perfect square". Group 8 correctly identified the graphic.

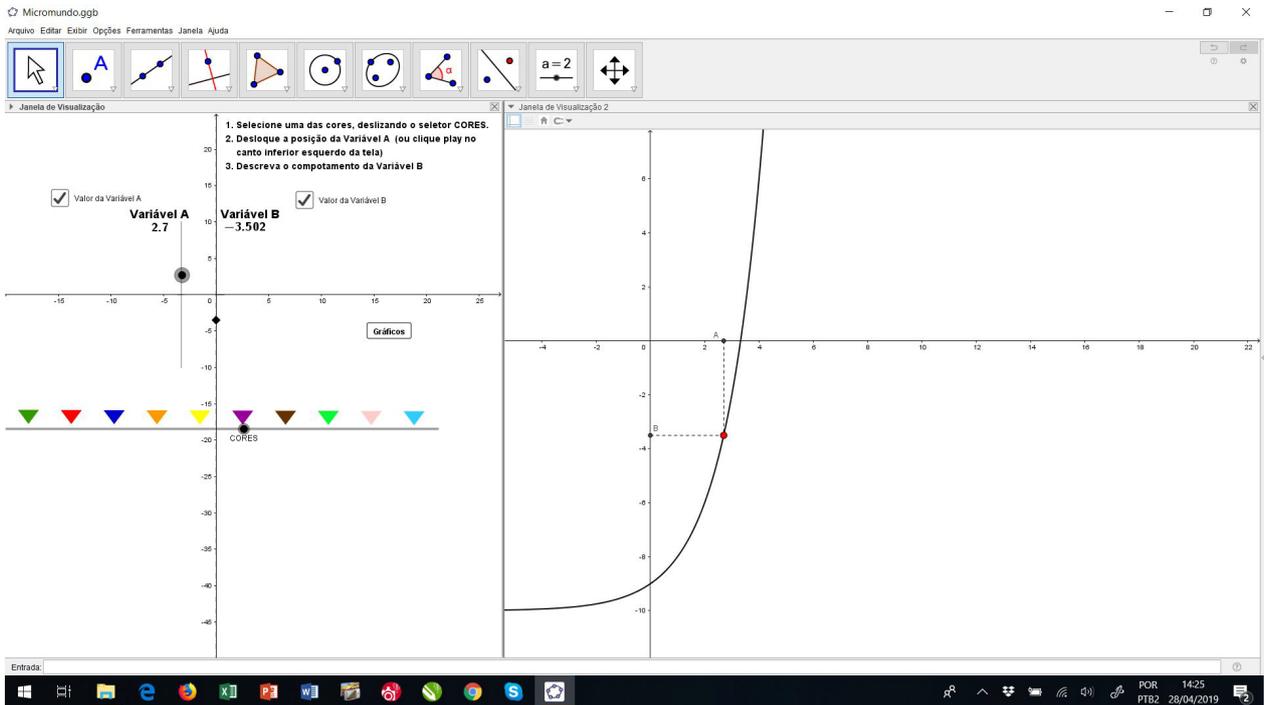


Figure 4. Knowing mathematical functions-purple graphic. <https://www.geogebra.org/m/p55wtfsh>

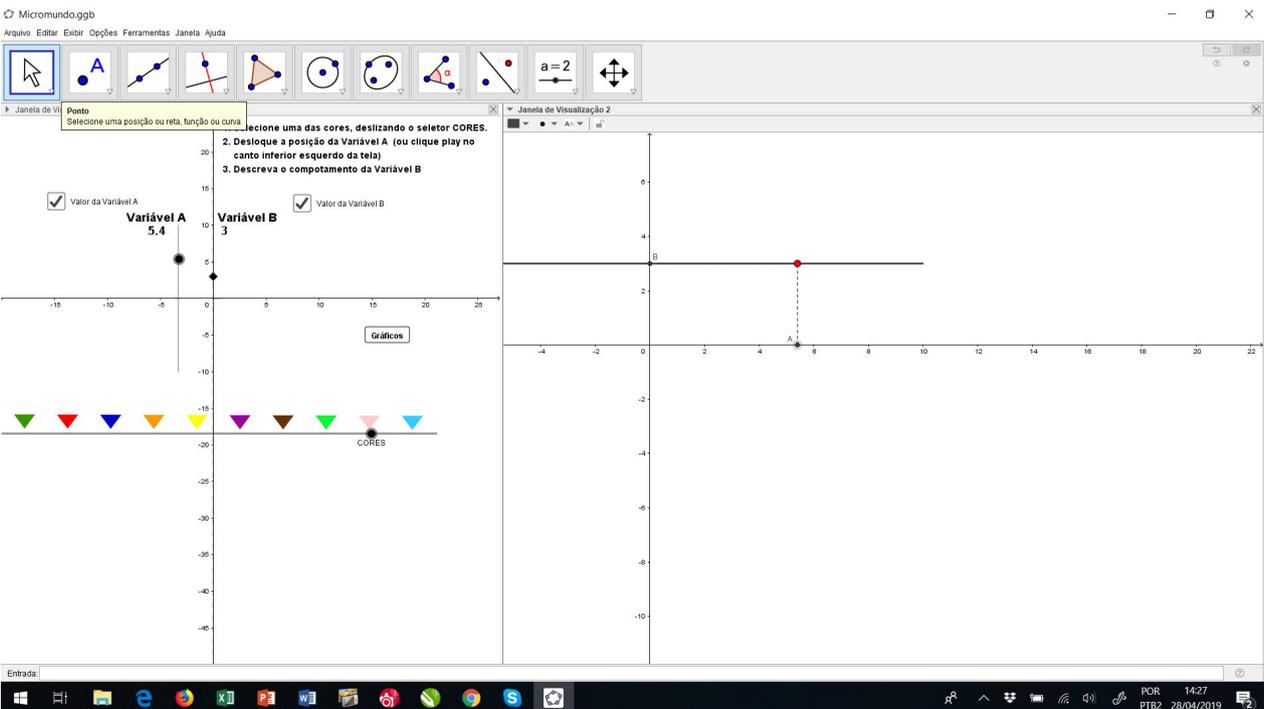


Figure 5. Knowing mathematical functions-pink graphic. <https://www.geogebra.org/m/p55wtfsh>

### 3.6. Group 6 (Light Green)

This group did not use the graphic, nor the relation between variable A and B to describe the problem. Instead, they localized the color as being “on the side of variable B”. They also informed incompatible values for variable B in relation to

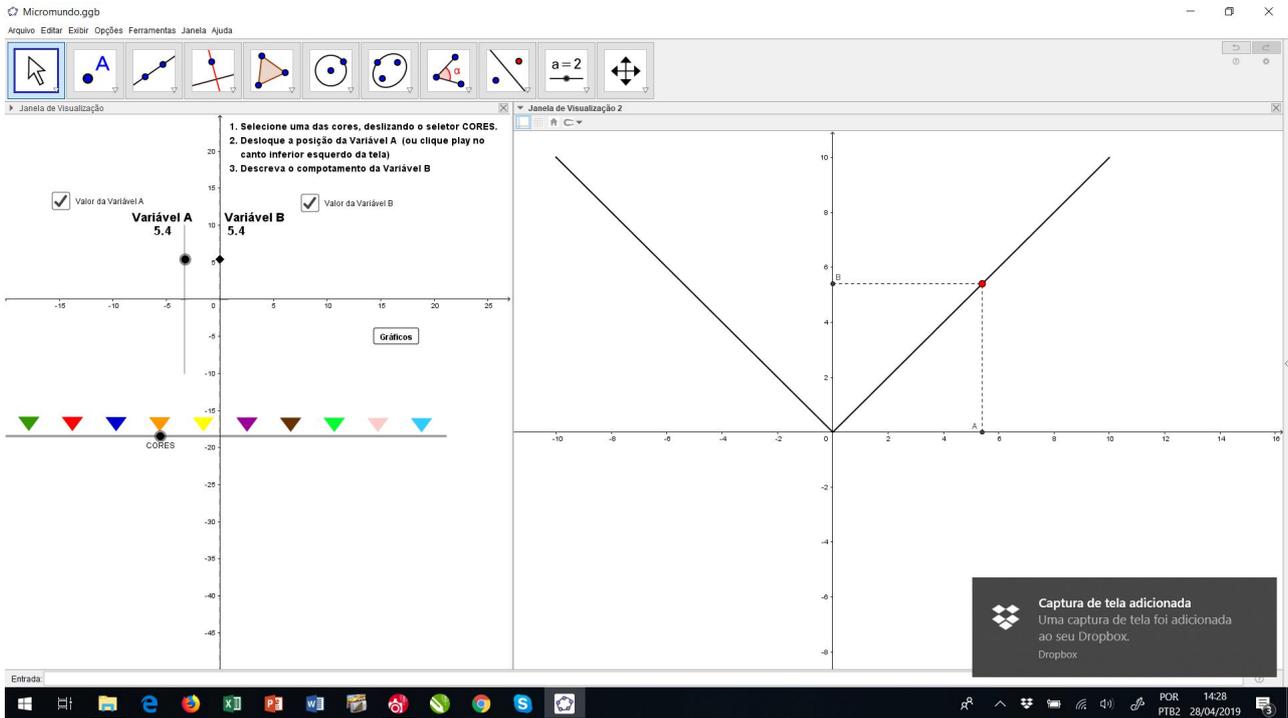


Figure 6. Knowing mathematical functions-orange graphic. <https://www.geogebra.org/m/p55wtfsh>

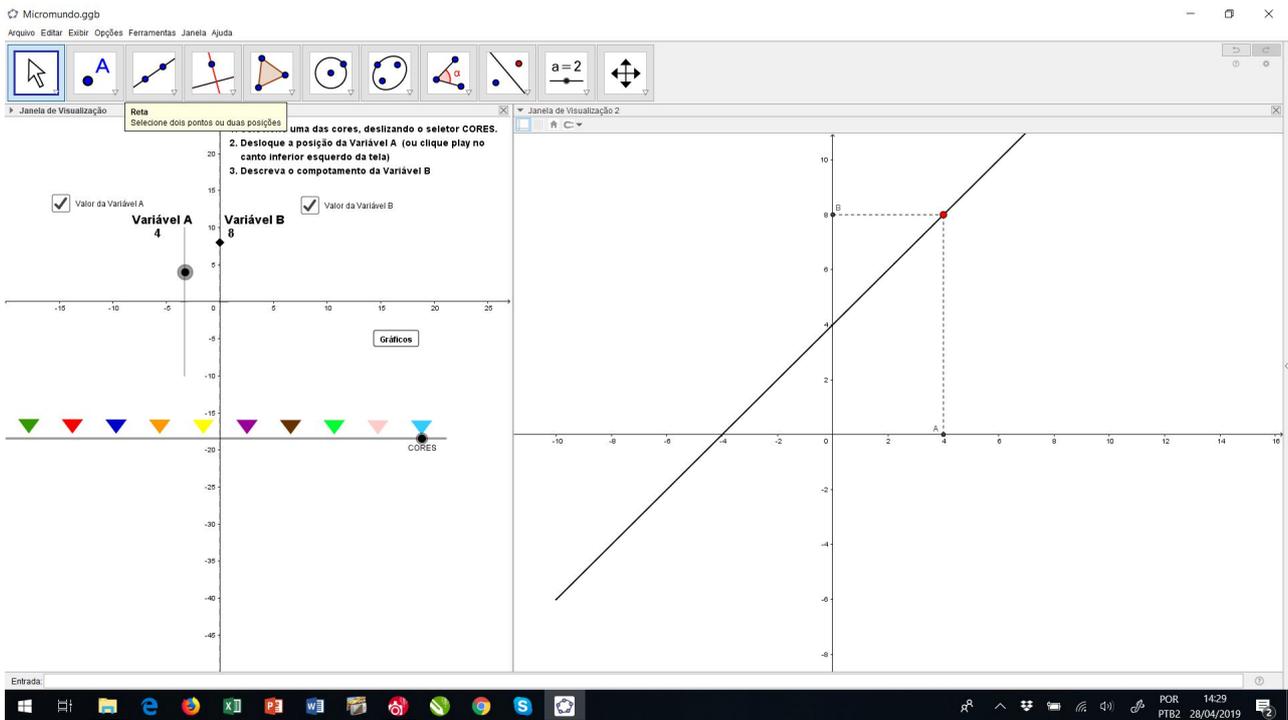


Figure 7. Knowing mathematical functions-light blue graphic. <https://www.geogebra.org/m/p55wtfsh>

variable A, and it took Group 1 a long time to figure out which graphic was the correct one, which they eventually did, using a trial and error approach (Figure 9).

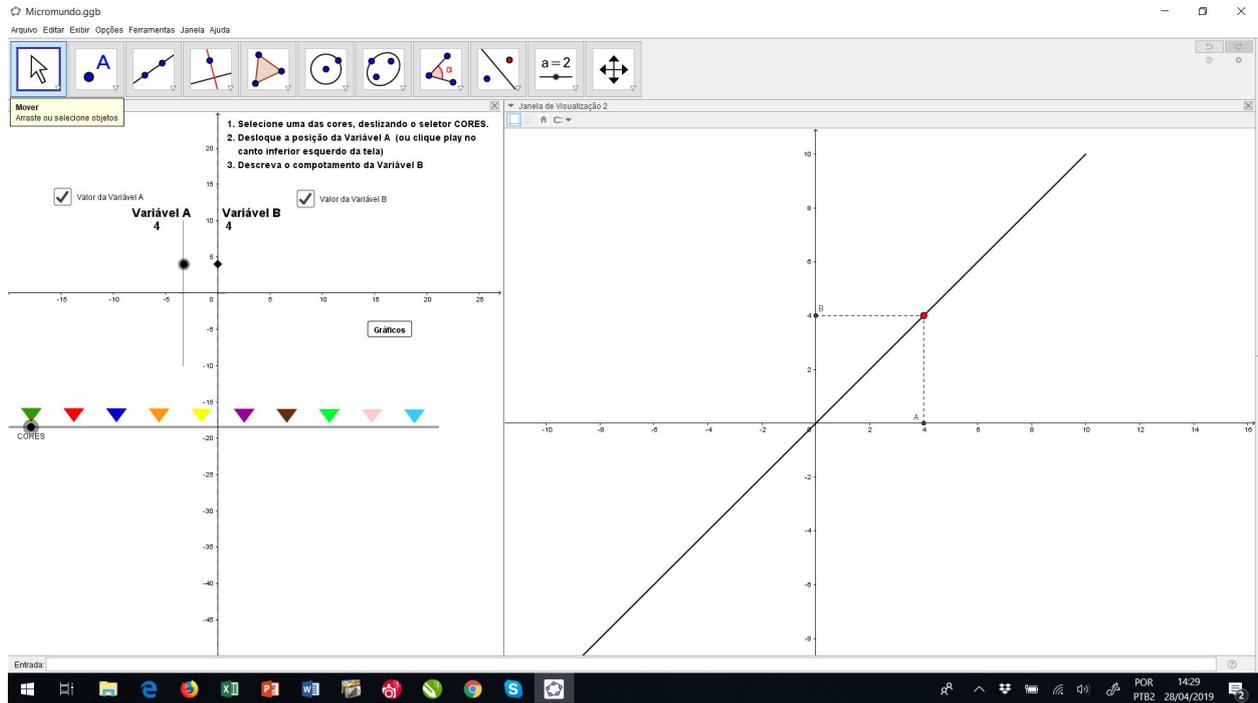


Figure 8. Knowing mathematical functions-dark green graphic. <https://www.geogebra.org/m/p55wtfsh>

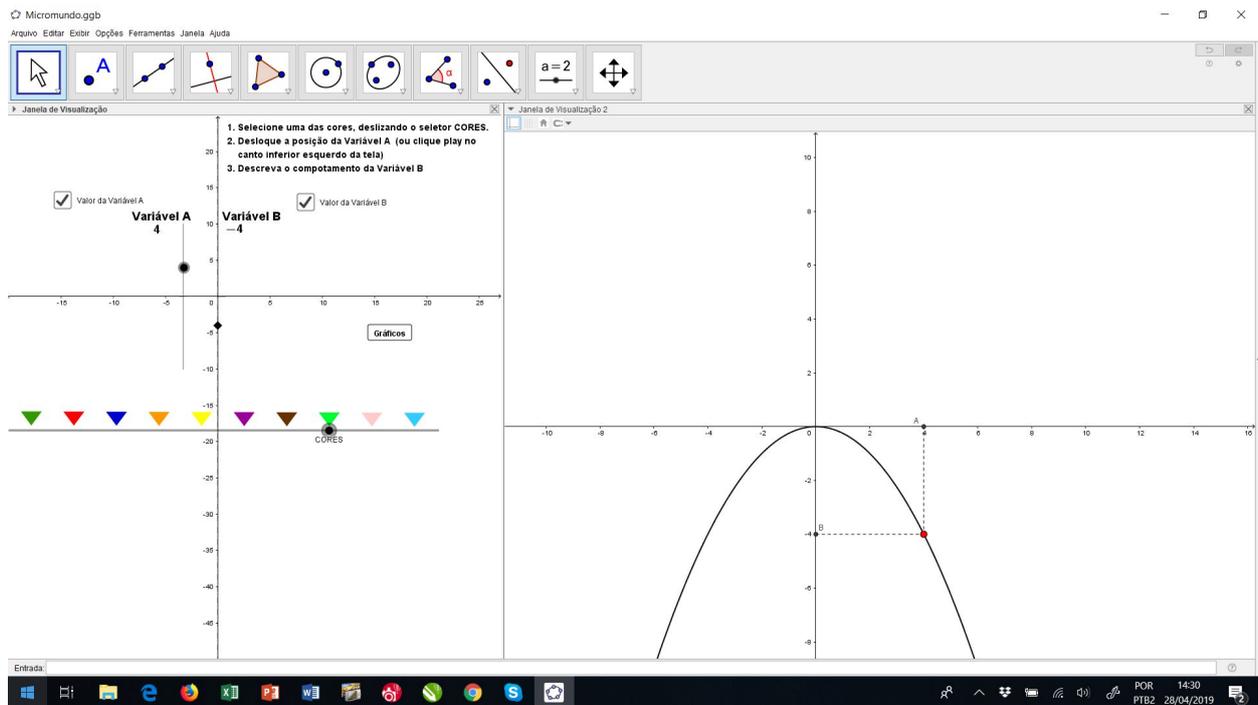


Figure 9. Knowing mathematical functions-light green graphic. <https://www.geogebra.org/m/p55wtfsh>

### 3.7. Group 7 (Dark Blue)

Students did a comprehensive analysis, completely describing the function to the point of stating the relation between variables A and B as “ $2A = B$ ”. Group 3 easily identified the correct graphic (Figure 10).

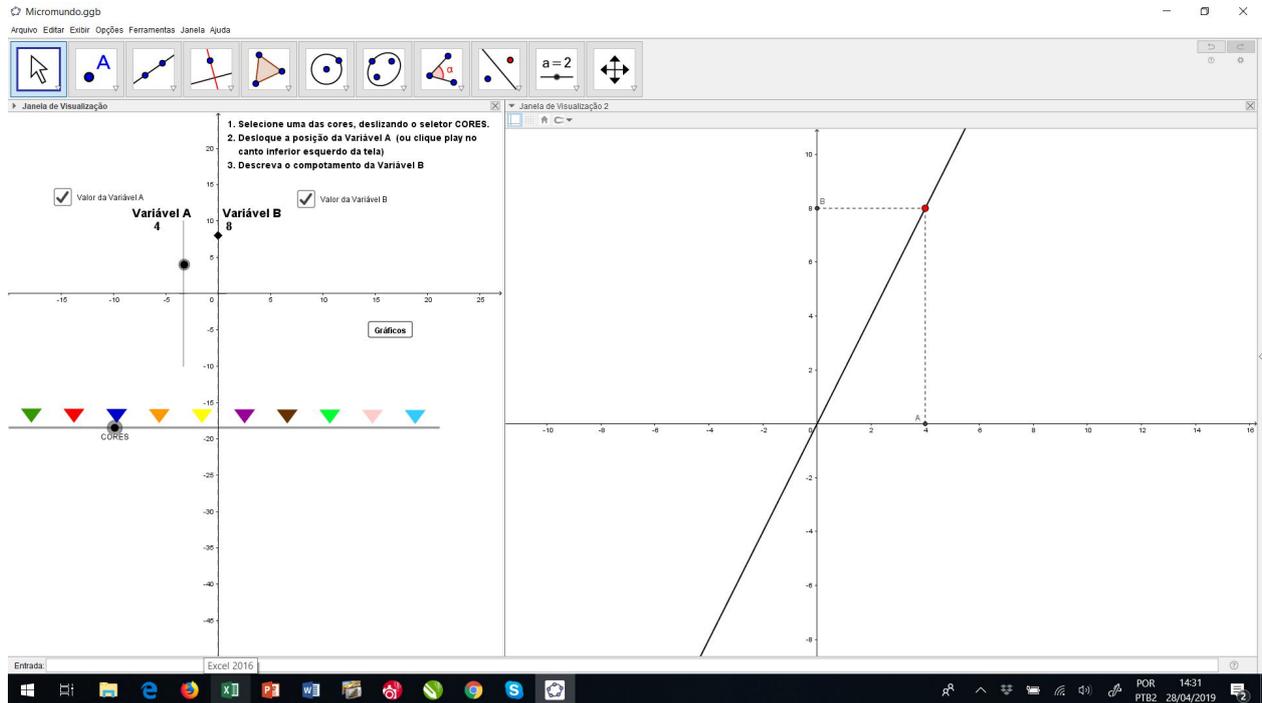


Figure 10. Knowing mathematical functions-dark blue graphic. <https://www.geogebra.org/m/p55wtfsh>

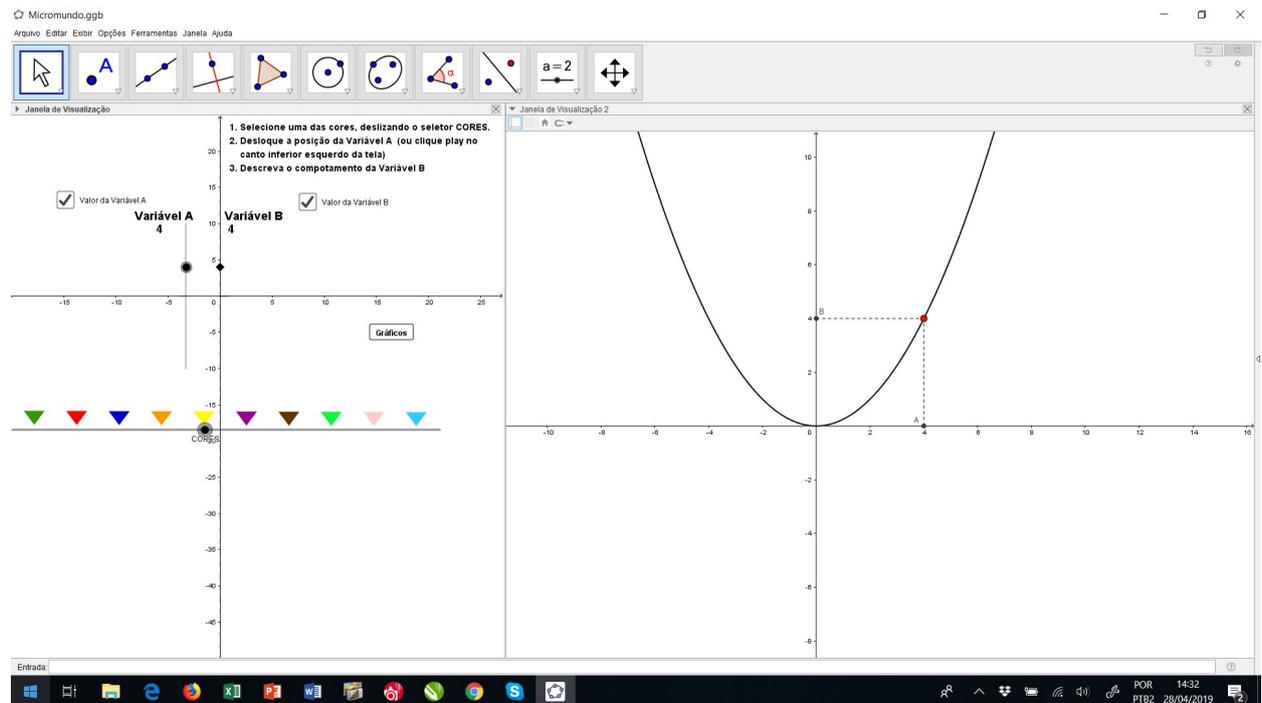


Figure 11. Knowing mathematical functions-yellow graphic. <https://www.geogebra.org/m/p55wtfsh>

### 3.8. Group 8 (Yellow)

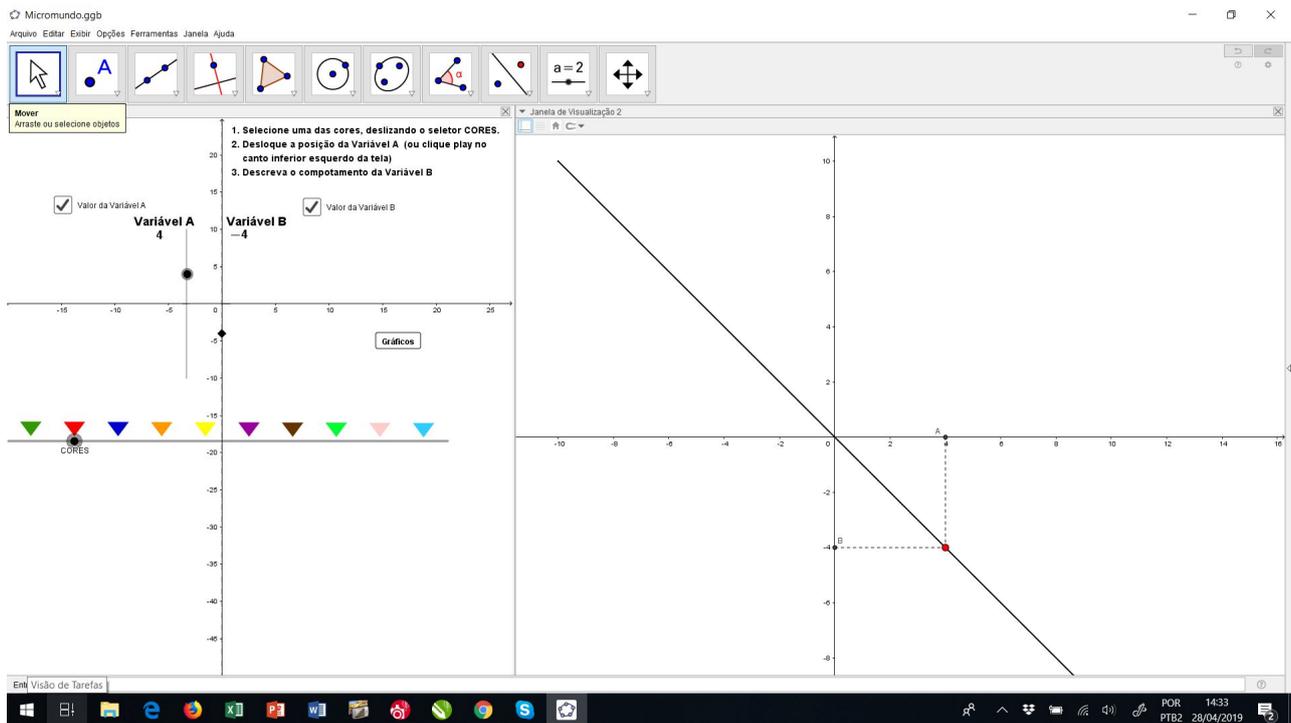
This team described the graphic in **Figure 11** as a parable, placing its vertex in point (0, 0) and maximum value for variable B as 25. Group 9 identified the correct graphic.

### 3.9. Group 9 (Red)

Students described the graphic in **Figure 12** as a decreasing linear function, and Group 7 identified it correctly.

Since only nine groups were formed, the tenth color (brown) was not used in the experiment.

Overall, every student involved participated actively in the project, as seen in **Figure 13**, sharing experiences, questions and discoveries with their peers. The teacher acted as a mediator, helping students to interact with the microworld but



**Figure 12.** Knowing mathematical functions-red graphic. <https://www.geogebra.org/m/p55wtfsh>



**Figure 13.** Students using microworld.

not interfering too much in the process, allowing each student to reach its own conclusions.

It is clearly visible, the description-execution-reflection-debug cycle, proposed by Papert, in the experiment. It is also noticeable the satisfaction students feel when they complete the D-E-R-D cycle and reach the proposed goal. Later on, the teacher used the experience gained with this microworld to construct the formal mathematical concept of functions with this class. Furthermore, GeoGebra proved to be an efficient tool to create constructionists microworlds.

#### 4. Conclusion

Papert's constructionism offers a versatile framework to implement pedagogical models using computers. It provides interactive environments where students investigate problems and try to achieve solutions, reflecting and correcting eventual mistakes found along the way.

Dynamic Mathematics softwares, like GeoGebra, are a major boost for those models. Geogebra is particularly relevant, since millions of people worldwide use it. It is also an open source software, available for free download in several languages, simple to use and a versatile tool to create a constructionist microworld.

This blend between technology and constructionism is a positive factor to improve teaching Mathematics, as shown by the experiment presented in this paper. Instead of memorizing abstract rules, the students built their knowledge about functions in a practical, collective and dynamic way, leading to a better understanding and paving the way for further studies.

The union between microworlds and Geogebra will lead to a more efficient and dynamic classroom, leaving behind outdated pedagogical conceptions and placing the students as the main actor in the learning process.

#### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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