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A Comparative Study of Resonances in the Dynamics of the Mimas-Tethys System

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Abstract

The probability of capture of Mimas-Tethys in 2:4 resonance was found to be 0.76 by Champenois when they considered the orbit of Tethys to be elliptical (that is eccentricity of Tethys to be 0.0008) and chaos was taken into account. It means probability of capture in i_1^2 or i_3^2 resonance is 0.24 (*i.e.* probability of non capture in $i_1 i_3$ resonance). Here we have done the comparative study of the dynamics of Mimas-Tethys system at $i_1 i_3$, $i_1 i_3 e_3$, $i_1^2 e_3$, $i_3{}^2 e_3$ and i_1^2, $i_1 i_3 e_3$, $i_1^2 e_3$, $i_3^2 e_3$ resonances along with secular resonance of Saturn's six inner satellites and Saturn's oblateness. We have drawn Poincare surface of sections and Time series graphs to compare their effect.

Keywords

Secular Resonance, Three Body Problem, Disturbing Function, Oblateness

1. Introduction

Allan [1] and Sinclair [2] investigated the dynamics of the Mimas-Tethys system under the hypothesis of circular orbits. Allan found (backward in time) the values for the satellites inclinations before capture in the present resonance to be $i_1 = 0.42^o$ and $i_3 = 1.05^o$ and Sinclair found the probability of capture to be 0.04.

Vienne and Duriez [3] discovered, using the frequency analysis method developed by Laskar [4], a particularly interesting 200 yr period in the mean longitude of Mimas which came because oblateness of Saturn and the interaction between Mimas, Enceladus, Tethys, Dione, Rhea and Titan (See [4] [5] [6] [7]) set the eccentricity of Tethys 0.000235, but with some uncertainties, it could amount to 0.001. This is a very small value, which certainly explains why the previous studies would assume Tethys moving on a circular orbit. However, Champenois and Vienne showed that in spite of such a small value, the third order terms with arguments $\psi+\varphi=3\lambda_1-6\lambda_3+2\Omega_1+\varpi_3$, $\psi=\lambda_1-2\lambda_3+\Omega_1-\Omega_3+\varpi_3$, $\psi-\varphi=-\lambda_1+2\lambda_3-2\Omega_3+\varpi_3$ and $\varphi=2\lambda_1-4\lambda_3+\Omega_1+\Omega_3$ brought chaos in the dynamics of Mimas-Tethys system because of near vanishing frequency $n_1-2n_3+\dot{\Omega}_1-\dot{\Omega}_3+\dot{\varpi}_3$.

Greenberg [8] analyzed the effect of i_1^2 and i_3^2 resonances on Mimas-Tethys system along with i_1i_3 resonance.

Champenois and Vienne [5] [6] numerically investigated the role of 200 year period and found that the inclination of Mimas before capture might have been higher (up to 0.7 degree) or lower (down to 0.03 degree) than that of previously considered 0.42 degree. Also Tethys's eccentricity on capture may have been quite higher (0.0008 versus 0). This value of eccentricity was found by a method which took chaos into account. They also found that probability of capture in i_1i_3 resonance was 0.76 when eccentricity of Tethys was 0.0008.

Jha and Agrawal [9] [10] have done the comparative study of the dynamics of Mimas-Tethys at i_1^2, , i_1i_3 and i_3^2 resonances along with three third order resonances with and without considering the secular term of all inner satellites and oblateness of Saturn. Jha and Jha [11] [12] [13] [14] have studied the secular resonance effect of Dione, Rhea and Titan on this system.

Thomas, P. C. *et al.* [15] have measured the shapes and sizes of six icy satellites by Cassini imaging subsystem (ISS) data, employing limb coordinates and stereogrammetic control points. Mimas, Enceladus, Tethys, Dione and Rhea are all well described by triaxial ellipsoids; Iapetus is best represented by an oblate spheroid.

Czechowski *et al.* [16] found that the temperature of Mimas interior was significantly lower than that of Enceladus. Czechowski and Losiak [17] have investigated the early thermal history of Rhea and have found that liquid state convection could delay the differentiation for hundreds of millions years.

The physical model was taken to be Mimas and Tethys on eccentric orbits inclined on the equatorial plane of the Saturn. Saturn's gravitational momenta are essential as they provide the main contribution to the orbital precession rates and we had to take into account the lowest degree oblateness terms J_2, J_4, J_2^2, J_6, J_2J_4 and J_2^3 (see **Table 1** for its values), also the actions of the Japet in the equations whereas the Sun and the small satellites of Saturn are not taken into account because of their weak effects in the generations of the perturbative frequency $\dot{\sigma}$.

Here the notations a_1, n_1, e_1, i_1, γ_1, ϖ_1, Ω_1 and λ_1 are orbital semi-major axis, mean motion, eccentricity, inclination, sine of semi inclination, longitudes of periapse, longitude of ascending node and mean longitude of Mimas respectively. Corresponding notions with subscript 2, 3, 4, 5 and 6 refers to the Enceladus, Tethys, Dione, Rhea and Titan's orbital elements. Small m_1, m_2, m_3, m_4, m_5 and m_6 stands for Mimas, Enceladus, Tethys, Dione, Rhea and Titan's masses with unit of Saturn.

Here we are extending the work of Jha and Agrawal [9] in i_1^2 and i_1i_3 resonance by considering the effect of secular term of all inner satellites along with

Table 1. Parameters of the three-body system Mimas, Enceladus, Tethys, Dione, Rhea, Titan and Saturn $M_{i,i=1-6}$, M and M_S are the masses of the considered satellite, Saturn and the Sun, respectively.

	$m=\frac{M_{i,i=1-6}}{M}$	n (rad/yr)	i (deg)	M_S/M	ae (km)	E	J_2	J_4	J_6
Mimas	6.34×10^{-8}	2422.44	1.62	-	-	0.0194			
Enceladus	0.15×10^{-6}								
Tethys	1.06×10^{-6}	1213.17	1.093	-	-	0.009			
Dione	1.963×10^{-6}								
Rhea	4.32×10^{-6}	-	-	-	-	-			
Titan	236.638×10^{-6}								
Saturn	-	-	-	3498.79	60330	-	0.01298	0.000915	0.000095

Saturn Oblateness.

2. Equation of Motion When the System Is Locked in i_1^2, $i_1^2e_3$, $i_1i_3e_3$ and $i_3^2e_3$ Resonances

(Equations of motion is taken from [9])

$$\ddot{\nu}=A_\nu\mathrm{Sin}[\nu]+A_{\psi+\varphi}\mathrm{Sin}[\psi+\varphi]+A_\psi\mathrm{Sin}[\psi]+A_{\psi-\varphi}\mathrm{Sin}[\psi-\varphi]$$

$$=A_\nu\Bigg[\mathrm{Sin}(\nu)+A_{\nu1}\mathrm{Sin}\left(\frac{3}{2}\nu-\frac{3}{2}(\Omega_1-\Omega_3)+\sigma\right) \quad \text{where, (1)}$$

$$+A_{\nu2}\mathrm{Sin}\left(\frac{1}{2}\nu-\frac{1}{2}(\Omega_1-\Omega_3)+\sigma\right)+A_{\nu3}\mathrm{Sin}\left(-\frac{1}{2}\nu+\frac{1}{2}(\Omega_1-\Omega_3)+\sigma\right)\Bigg]$$

$$\nu=2\lambda_1-4\lambda_3+2\Omega_1\ ,$$

$$\psi+\varphi=3\lambda_1-6\lambda_3+2\Omega_1+\varpi_3,$$

$$\psi+\varphi=\frac{3}{2}\nu-\frac{3}{2}(\Omega_1-\Omega_3)+\sigma,$$

where,

$$\sigma=\frac{1}{2}\Omega_1-\frac{3}{2}\Omega_3+\varpi_3=ft+\sigma_0=\frac{2\pi}{200}t+\sigma_0$$

$$\psi=\lambda_1-2\lambda_3+\Omega_1-\Omega_3+\varpi_3$$

$$=\frac{1}{2}(\nu-\Omega_1+\Omega_3)+\sigma$$

$$=\frac{1}{2}\nu-\frac{1}{2}(\Omega_1-\Omega_3)+ft+\sigma_0$$

and

$$\psi-\varphi=-\lambda_1+2\lambda_3-2\Omega_3+\varpi_3$$

$$=-\frac{1}{2}(\nu-\Omega_1+\Omega_3)+\sigma$$

$$=-\frac{1}{2}\nu+\frac{1}{2}(\Omega_1-\Omega_3)+ft+\sigma_0$$

where f is the frequency of σ at time t. σ_0 is the value of σ at time $t=0$

(J2000) (see [6] [7]). Here $\nu = \varphi - (\Omega_1 - \Omega_3)$ (for values see Table 2).

$$A_\nu = \left(6n_1^2 m_3 \alpha_{13} + 24 n_3^2 m_1\right)\left(-f_0(\alpha_{13})\right)\gamma_1^2$$

$$A_{\nu1} = -3e_3 \frac{f_1(\alpha_{13})}{f_0(\alpha_{13})}, \quad A_{\nu2} = -e_3 \frac{f_2(\alpha_{13})}{f_0(\alpha_{13})}\frac{\gamma_3}{\gamma_1} \quad \text{and} \quad A_{\nu3} = e_3 \frac{f_3(\alpha_{13})}{f_0(\alpha_{13})}\frac{\gamma_3^2}{\gamma_1^2}.$$

$$A_{\psi+\varphi} = \left(18n_1^2 m_3 \alpha_{13} + 72 n_3^2 m_1\right) e_3 f_1(\alpha_{13})\gamma_1^2 = -3A_\nu e_3 \frac{f_1(\alpha_{13})}{f_0(\alpha_{13})}.$$

$$A_\psi = \left(6n_1^2 m_3 \alpha_{13} + 24 n_3^2 m_1\right) e_3 \gamma_1 \gamma_3 f_2(\alpha_{13}) = -A_\nu e_3 \frac{f_2(\alpha_{13})}{f_0(\alpha_{13})}\frac{\gamma_3}{\gamma_1}.$$

$$A_{\psi-\varphi} = -\left(6n_1^2 m_3 \alpha_{13} + 24 n_3^2 m_1\right) e_3 \gamma_3^2 f_3(\alpha_{13}) = A_\nu e_3 \frac{f_3(\alpha_{13})}{f_0(\alpha_{13})}\frac{\gamma_3^2}{\gamma_1^2}.$$

The value of $f_0(\alpha_{13})$, $f_1(\alpha_{13})$, $f_2(\alpha_{13})$ and $f_3(\alpha_{13})$ depend on the Laplace's Coefficients $b_s^{(k)}(\alpha_{ij})$ and its values are given in Table 2.

3. Equation of Motion When the System Is Locked in $i_1 i_3$, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ Resonances

$$\begin{aligned}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = A_0\Bigg(&\mathrm{Sin}\varphi + A_1\mathrm{Sin}\left(\frac{3}{2}\varphi + ft + \sigma_0\right) + A_2\mathrm{Sin}\left(\frac{1}{2}\varphi + ft + \sigma_0\right) \\ &+ A_3\mathrm{Sin}\left(-\frac{1}{2}\varphi + ft + \sigma_0\right)\Bigg) + \left(\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}\right)_o + \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}t^2}\right)_o + \left(\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}\right)_s + \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}t^2}\right)_s\end{aligned} \tag{2}$$

where

$$\varphi = 2\lambda_1 - 4\lambda_3 + \Omega_1 + \Omega_3 ,$$

$$\begin{aligned}\psi + \varphi &= 3\lambda_1 - 6\lambda_3 + 2\Omega_1 + \varpi_3 \\ &= \frac{3}{2}\varphi + \sigma\end{aligned}$$

$$\begin{aligned}\psi &= \lambda_1 - 2\lambda_3 + \Omega_1 - \Omega_3 + \varpi_3 \\ &= \frac{1}{2}\varphi + \sigma\end{aligned}$$

and

Table 2. Analytical expressions of the functions $f_i(\alpha_{13})$ for the arguments φ, $\psi+\varphi$, ψ, $\psi-\varphi$ and ν with their value for $\alpha_{13} = 0.6306386$ (TASS1.6 [7]).

Argument	I	$f_i(\alpha)$	$f_i(\alpha_{13})$
φ	0	$-\alpha_{13} b_{3/2}^3(\alpha_{13})$	−1.65088068
$\psi+\varphi$	1	$3\alpha_{13} b_{3/2}^4(\alpha_{13}) + \frac{1}{4}(\alpha_{13})^2 \frac{\mathrm{d}}{\mathrm{d}\alpha} b_{3/2}^4(\alpha_{13})$	5.23786953
ψ	2	$2\alpha_{13} b_{3/2}^2(\alpha_{13}) + \frac{1}{2}(\alpha_{13})^2 \frac{\mathrm{d}}{\mathrm{d}\alpha} b_{3/2}^2(\alpha_{13})$	9.70821605
$\psi-\varphi$	3	$-\alpha_{13} b_{3/2}^2(\alpha_{13}) + \frac{1}{4}(\alpha_{13})^2 \frac{\mathrm{d}}{\mathrm{d}\alpha} b_{3/2}^2(\alpha_{13})$	0.22188903
ν	4	$\frac{1}{2}\alpha_{13} b_{3/2}^3(\alpha_{13}) = -f_0(\alpha_{13})$	0.82544034

$$\psi-\varphi=-\lambda_1+2\lambda_3-2\Omega_3+\varpi_3$$
$$=-\frac{1}{2}\varphi+\sigma.$$

With $\sigma=ft+\sigma_0$ where $\sigma_0=-0.031391995$ and

$$A_0=\left(12n_1^2m_3\alpha_{13}+48n_3^2m_1\right)f_0\left(\alpha_{13}\right)\gamma_1\gamma_3$$

$$A_1=\frac{3}{2}e_3\frac{f_1\left(\alpha_{13}\right)\gamma_1}{f_0\left(\alpha_{13}\right)\gamma_3},\ A_2=\frac{1}{2}e_3\frac{f_2\left(\alpha_{13}\right)}{f_0\left(\alpha_{13}\right)}\ \text{ and }\ A_3=-\frac{1}{2}\frac{f_3\left(\alpha_{13}\right)\gamma_3}{f_0\left(\alpha_{13}\right)\gamma_1}.$$

The values of $f_0\left(\alpha_{13}\right)$, $f_1\left(\alpha_{13}\right)$, $f_2\left(\alpha_{13}\right)$ and $f_3\left(\alpha_{13}\right)$ depend on the Laplace's Coefficients $b_s^{(k)}\left(\alpha_{ij}\right)$ and given in Table 2.

With

$$\begin{aligned}
\left(\frac{d^2\lambda_1}{dt^2}\right)_S &= -2\left[\sum_{i=2}^{5}m_i\alpha_{1i}^2\frac{d^2A(\alpha_{1i})}{d\alpha_{1i}^2}\right]\frac{dn_1}{dt}\\
\left(\frac{d^2\Omega_1}{dt^2}\right)_S &= \frac{1}{2}\left[\sum_{i=2}^{5}m_i\alpha_{1i}C\left(\alpha_{1i}\right)\right]\frac{dn_1}{dt}\\
\left(\frac{d^2\Omega_3}{dt^2}\right)_S &= \frac{1}{2}\left[\sum_{i=1}^{2}m_iC\left(\alpha_{i3}\right)+\sum_{i=4}^{6}m_i\alpha_{3i}C\left(\alpha_{3i}\right)\right]\frac{dn_3}{dt}\\
\left(\frac{d^2\varpi_3}{dt^2}\right)_S &= 2\left[\sum_{i=1}^{2}m_iB\left(\alpha_{i3}\right)+\sum_{i=4}^{6}m_i\alpha_{3i}B\left(\alpha_{3i}\right)\right]\frac{dn_3}{dt}\\
\left(\frac{d^2\lambda_3}{dt^2}\right)_S &= 2\left[\sum_{i=1}^{2}-m_i\left\{\alpha_{i3}\frac{dA\left(\alpha_{i3}\right)}{d\alpha_{i3}}+A\left(\alpha_{i3}\right)\right\}+\sum_{i=4}^{6}m_i{\alpha_{3i}}^2\frac{dA\left(\alpha_{3i}\right)}{d\alpha_{3i}}\right]\frac{dn_3}{dt}
\end{aligned}\tag{3}$$

(We are not considering any changes in semi major axis of any satellites)

and

$$\begin{aligned}
\left(\frac{d^2\varphi}{dt^2}\right)_S &= \left(\frac{d^2\lambda_1}{dt^2}\right)_S-2\left(\frac{d^2\lambda_3}{dt^2}\right)_S+\frac{1}{2}\left[\left(\frac{d^2\Omega_1}{dt^2}\right)_S+\left(\frac{d^2\Omega_3}{dt^2}\right)_S\right]\\
\left(\frac{d^2\sigma}{dt^2}\right)_S &= \frac{1}{2}\left(\frac{d^2\Omega_1}{dt^2}\right)_S-\frac{3}{2}\left(\frac{d^2\Omega_3}{dt^2}\right)_S+\left(\frac{d^2\varpi_3}{dt^2}\right)_S.
\end{aligned}\tag{4}$$

Now we will find the terms due to Oblateness of Saturn. Saturn's gravitational momenta are quite important so that we have, in order to get the full variations of the mean longitudes, nodes and pericentres due to the oblateness, taken into account the lowest-degree terms with J_2, J_4, J_2^2, J_6, J_2J_4 and J_2^3 (See Table 1 for its values) as a factor (Vienne and Duriez [3] [5]). The other terms are taken constant. Values of α_{ij}, $A\left(\alpha_{ij}\right)$, $B\left(\alpha_{ij}\right)$, $C\left(\alpha_{ij}\right)$ and $\frac{dA\left(\alpha_{ij}\right)}{d\alpha_{ij}}$ for every pair $\left(i,j\right)$, $\left(i<j\right)$ involving Mimas, Enceladus, Tethys, Dione, Rhea and Titan are given in Table 3.

We then get (a_e is the equatorial radius of Saturn).

$$\begin{aligned}
\left(\frac{d^2\Omega_3}{dt^2}\right)_0 &= \left[\frac{-3}{2}J_2\left(\frac{a_e}{a_3}\right)^2\left(1+2e_3^2-2\gamma_3^2\right)+a_3^{-4}\left(\frac{15}{4}J_4-\frac{45}{8}J_2^2\right)\right]\frac{dn_3}{dt}\\
&\quad -6J_2n_3\left(\frac{a_e}{a_3}\right)^2\left(e_3\frac{de_3}{dt}-\gamma_3\frac{d\gamma_3}{dt}\right).
\end{aligned}\tag{5}$$

Table 3. Value of $\alpha_{ij}, A(\alpha_{ij}), B(\alpha_{ij}), C(\alpha_{ij})$ and $\frac{\mathrm{d}A(\alpha_{ij})}{\mathrm{d}\alpha_{ij}}$ for every pair $(i,j),(i<j)$ involving Mimas, Enceladus, Tethys, Dione, Rhea and Titan (α_{ij}) (CV [6]).

$i-j$	α_{ij},	$A(\alpha_{ij})$	$B(\alpha_{ij})$	$C(\alpha_{ij})$	$\frac{\mathrm{d}A(\alpha_{ij})}{\mathrm{d}\alpha_{ij}}$
1 - 2	0.78026	1.2473	1.3674	−5.4695	1.0996
1 - 3	0.63064	1.1306	0.38952	−1.5581	0.55451
1 - 4	0.49258	1.0706	0.15366	−0.61463	0.33609
1 - 5	0.35283	1.0335	0.059940	−0.23976	0.20479
1 - 6	0.15223	1.0059	0.0090810	−0.036324	0.078148
2 - 3	0.80824	1.2807	1.8535	−7.4138	1.2978
3 - 4	0.78108	1.2482	1.3790	−5.5159	1.1047
3 - 5	0.55948	1.0960	0.23856	−0.95425	0.42538
3 - 6	0.24140	1.0151	0.024460	−0.097840	0.12912

$$\left(\frac{\mathrm{d}^2\lambda_1}{\mathrm{d}t^2}\right)_0 = \left[3J_2\left(\frac{a_e}{a_1}\right)^2\left(1+\frac{7}{4}e_1^2-7\gamma_1^2\right)+a_1^{-4}\left(-\frac{15}{4}J_4+\frac{160}{11}J_2^2\right)\right]\frac{\mathrm{d}n_1}{\mathrm{d}t} - 42J_2 n_1\gamma_1\left(\frac{a_e}{a_1}\right)^2\frac{\mathrm{d}\gamma_1}{\mathrm{d}t}. \tag{6}$$

$$\left(\frac{\mathrm{d}^2\lambda_3}{\mathrm{d}t^2}\right)_o = \left[3J_2\left(\frac{a_e}{a_3}\right)^2\left(1+\frac{7}{4}e_3^2-7\gamma_3^2\right)+a_3^{-4}\left(-\frac{15}{4}J_4+\frac{160}{11}J_2^2\right)\right]\frac{\mathrm{d}n_3}{\mathrm{d}t} - 42J_2 n_3\gamma_3\left(\frac{a_e}{a_3}\right)^2\frac{\mathrm{d}\gamma_3}{\mathrm{d}t}. \tag{7}$$

$$\left(\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}\right)_o = \left(\frac{\mathrm{d}^2\lambda_1}{\mathrm{d}t^2}\right)_o - 2\left(\frac{\mathrm{d}^2\lambda_3}{\mathrm{d}t^2}\right)_o + \frac{1}{2}\left\{\left(\frac{\mathrm{d}^2\Omega_1}{\mathrm{d}t^2}\right)_o + \left(\frac{\mathrm{d}^2\Omega_3}{\mathrm{d}t^2}\right)_o\right\}. \tag{8}$$

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}t^2}\right)_o = \frac{1}{2}\left(\frac{\mathrm{d}^2\Omega_1}{\mathrm{d}t^2}\right)_o - \frac{3}{2}\left(\frac{\mathrm{d}^2\Omega_3}{\mathrm{d}t^2}\right)_o + \left(\frac{\mathrm{d}^2\varpi_3}{\mathrm{d}t^2}\right)_o. \tag{9}$$

4. Numerical Integration and Surface of Sections

Our equations were integrated backwards in time. The initial conditions are taken from TASS1.6 [7] for J2000. $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$ are taken to be fixed. Here we have integrated it for -62.8×14000 yrs.

Figure 1 and Figure 2 are Poincare surface of sections with and without oblateness of Saturn respectively at i_1^2, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances. Figure 3 and Figure 4 are the time-series graph for the same. Figure 5 and Figure 6 are the Poincare surface of sections with and without oblateness of Saturn at $i_1 i_3$, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances and Figure 7 and Figure 8 are the time series graph for the same.

5. Discussion

Vienne and Duriez [3] discovered the 200 year period in the mean longitude of

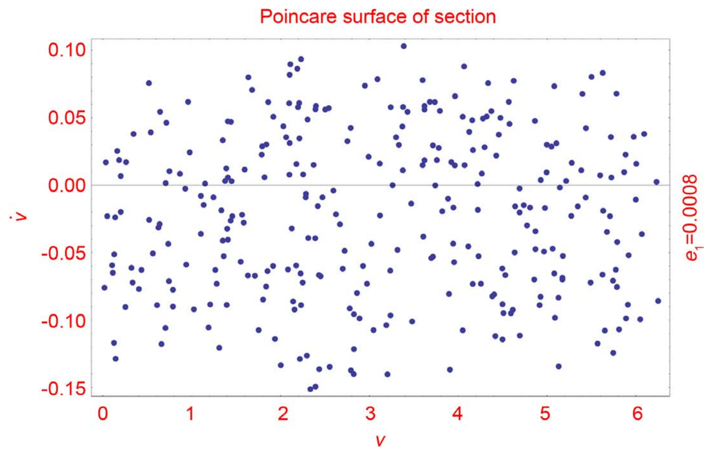

Figure 1. Poincare surface of section at i_1^2, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances with oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

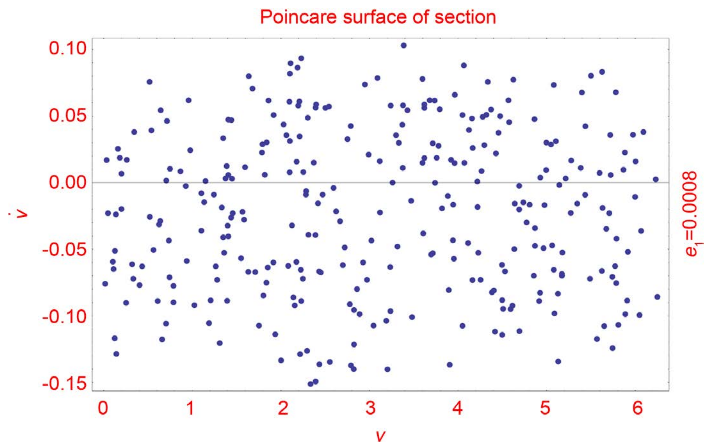

Figure 2. Poincare surface of section at i_1^2, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances without oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

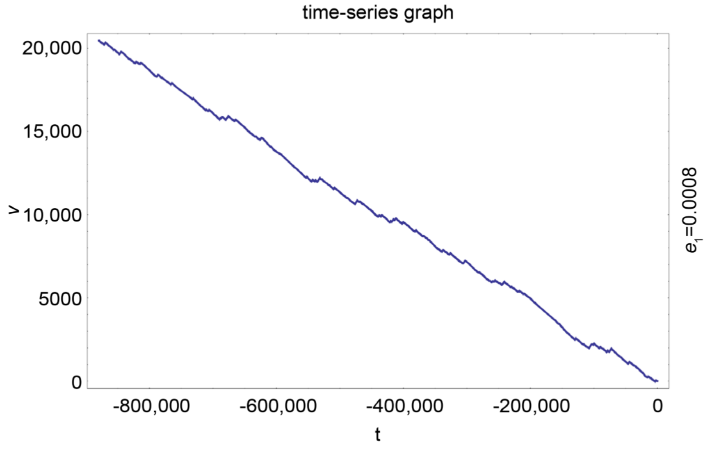

Figure 3. Time-series graph at i_1^2, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances with oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

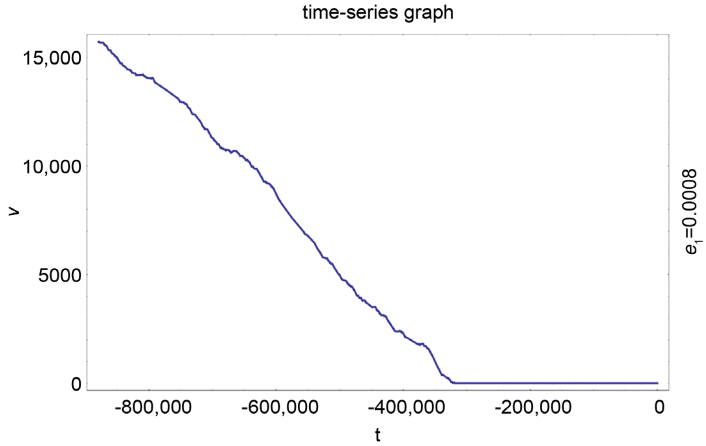

Figure 4. Time-series graph at i_1^2, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances without oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

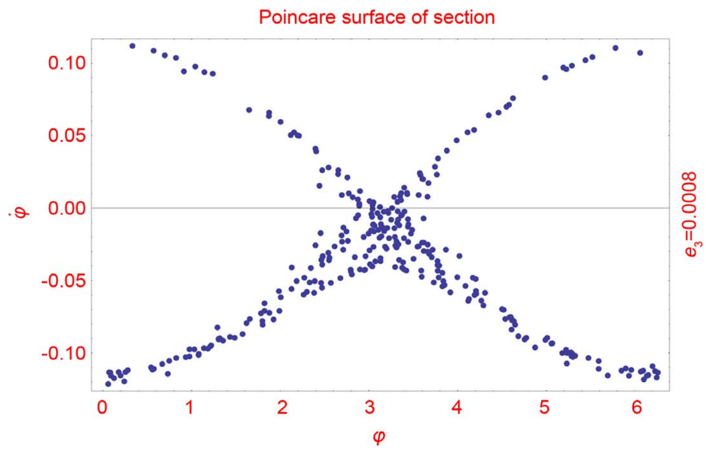

Figure 5. Poincare surface of section at $i_1 i_3$, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances with oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

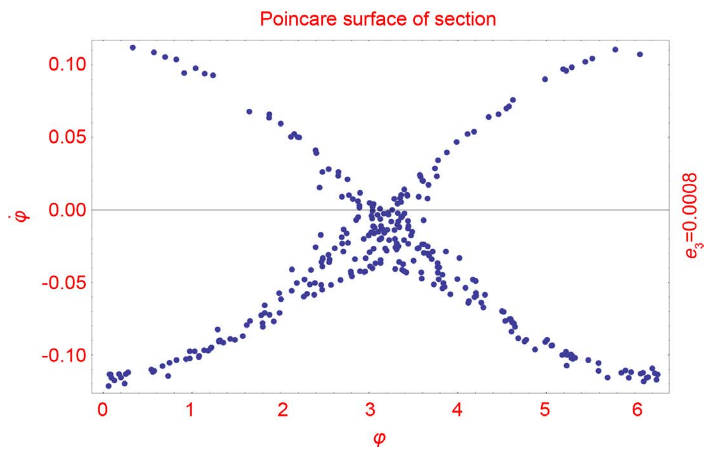

Figure 6. Poincare surface of section at $i_1 i_3$, $i_1^2 e_3$, $i_1 i_3 e_3$ and $i_3^2 e_3$ resonances without oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

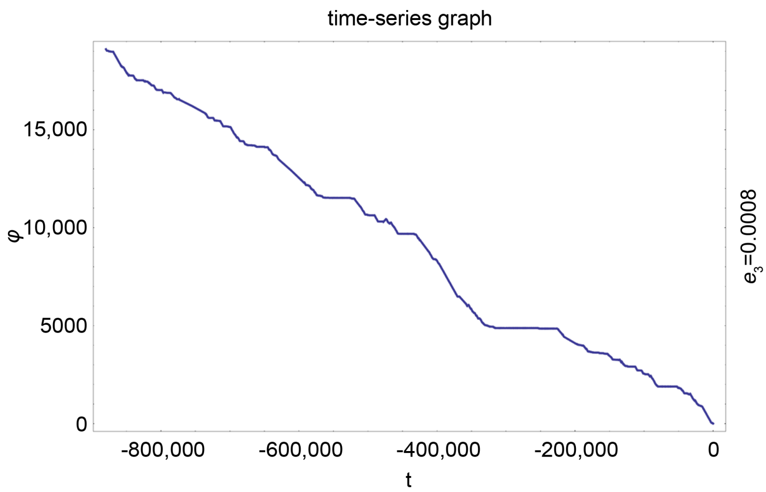

Figure 7. Time-series graph at i_1i_3, $i_1^2e_3$, $i_1i_3e_3$ and $i_3^2e_3$ resonances withoblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

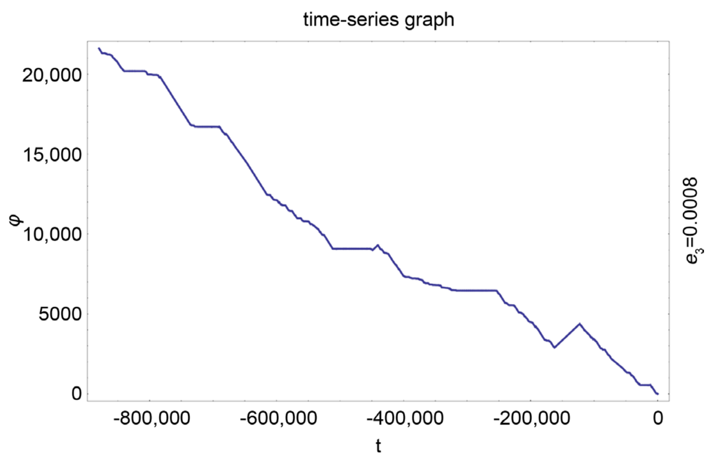

Figure 8. Time-series graph at i_1i_3, $i_1^2e_3$, $i_1i_3e_3$ and $i_3^2e_3$ resonances without oblateness of Saturn. Secular resonances of all inner satellites have been considered at $i_1 = 1.62^o$, $i_3 = 1.093^o$ and $e_3 = 0.0008$.

Mimas. Champenois and Vienne [5] [6] have investigated the role of 200 year long period on the dynamics of Mimas-Tethys system when considered to be presently trapped in i_1i_3 resonance with probability of capture 0.76 at 2:4 commensurability. They found that the sources of this period were both the oblateness of Saturn and the interaction between its six inner satellites. They also realized that considering an eccentric orbit of Tethys upseted the earlier vision of dynamics of the Mimas-Tethys system.

Here, we have analyzed that the system is more chaotic if considered to be (at presently it is) locked in i_1i_3 resonance compared with i_1^2 resonance; we consider the effect of Saturns oblateness and the interaction of its six inner satellites, by the help of Poincare surface of section and time-series graphs which confirm the earlier results too. Figure 2 and Figure 6 are Poincare surface of sections and Figure 4 and Figure 8 are time series graphs without Oblateness of Saturn. Although we could not quantify the chaos, on the basis of above figures, we can say that oblateness of Saturn plays a significant role in the dynamics of Mimas-Tethys system and also it partially controls the chaos.

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