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Predictive formulas expressing relationship among dose rate, duration of exposure and mortality probability in total body irradiation in humans

Sung Jang Chung

Morristown-Hamblen Healthcare System, Morristown, USA.
Email: sung.chung@comcast.net

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ABSTRACT

A clear and exact quantitative relationship between dose of radiation and mortality in humans is still not known because of lack of human data that would enable to determine LD50 for humans in total body irradiation. Analysis of human data has been primarily from radiation accidents, radiotherapy and the atomic bomb victims.The death rate equation derived from the 'probacent'-probability model of survival probability is employed in this study to construct the general formula of mortality probability as a function of dose rate and duration of exposure in total body irradiation in humans. There is a remarkable agreement between formula-predicted and published estimated LD50 and also between both mortality probabilities. The formulas of LD50 ans mortality probability in lethal radiation exposure for humans might be helpful in preventing radiation hazard and injury, and further for safety in radiotherapy.

Keywords: Mortality Probability; Total Body Irradiation

1. INTRODUCTION

A clear and exact quantitative relationship between dose of radiation and mortality in humans is still not known because of lack of human data that would enable to determine LD_{50} for humans in total body irradiation. Analysis of human data has been primarily from radiation accidents, radiotherapy and the atomic bomb victims [1-9].

Van Middlesworth published worldwide increased levels of ^{131}I fallout in animal thyroid glands after nuclear weapon tests and also immediately after the nuclear reactor accident at Chernobyl during the period from 1954-1987 [9-11].

Consequently, laboratory animals have been used to investigate the relationship between radiation exposures and biomedical effects in total body irradiation, and further to possibly derive a general predictive mathematical formula expressing a dose-effect curve [1,12-17].

Sacher reported comprehensive experimental data on average survival times in mice irradiated daily during the duration of life [12,13].He stated in his article that the Gompertz model seemed to be approximately applicable to the data on age-specific death rates. A general formula of the Gompertz model that might express death rate as a function of dose rate of radiation is, however, not presented in the Sacher's article.

The Gompertz model (1825) is one of the well-known mathematical expressions among mortality models in the literature that are used to describe mortality and survival data of a population [18-20].

The author reported that a formula expressing death rate, **Eq. 1** [21-23] was found to better fit the US national mortality data of total elderly population than the Gompertz model [23].

$$(\log D)^c = a + b \cdot \log t \quad (1)$$

where D is death rate, t is age; c, a and b are constants.

Eq. (1) also seemed to better fit death rate of the US total elderly population, 2001 than the exponential, the Weibull and the lognormal distributions as well as the Gompertz model [23].

Eq. (1) was derived from the author's 'probacent'-probability model, Eq. (2) that expresses survival probability of a population [21].

$$P^{\gamma} = A - B \cdot \log t \quad (2a)$$

$$S = 10/\sqrt{2\pi} \int_{-\infty}^{P} \exp\left[-(P-50)^2/200\right] dp \quad (2b)$$

where S is percent survival probability; P is 'probacent' (abbreviation of probability percentage) and considered to be a relative biological amount of reserve for survival; 'probacent' of 0, 50 and 100 corresponds to -5 SD, mean and mean + 5 SD, respectively. The unit of 'probacent' is

0.1 SD. t is time (age or time after biomedical insult); γ, A and B are constants; A is an intercept and B a slope; γ represents a curvature (a shape of curve). If the value of γ becomes equal to one, Eq. (2) represents a lognormal distribution. Eq. (2) seems to represent a generalized lognormal distribution.

Eq. (2) is considered to be fundamentally based on the Gaussian normal distribution.

Eq. (1) that expresses death rate of a population is derivable from the 'probacent'-probability equation of survival probability, Eq. (2) [21]. If the value of its constant c is one, the Eq. (1) becomes essentially similar to the Weibull model [23].

The mathematical model of the 'probacent'-probability equation, Eq. (2) of survival probability was constructed from experimental studies: to express relationship among intensity of stimulus or environmental agent (such as drugs [24,25], heat [26]), and duration of exposure and biological response in animals [24].

The model has been applied to data in the biomedical literature; to express carboxyhemoglobin levels of blood as a function of carbon monoxide concentration in air and duration of exposure [27]; to express a relationship among plasma acetaminophen concentration, time after ingestion and occurrence of hepatotoxicity in man [28]; to express survival probability in patients with heart transplantation [29]; to predict survival probability in patients with malignant melanoma [30], and express relationship among age, height and weight, and percentile in Saudi and US children of ages 6-16 years [31].

Mehta and Joshi successfully applied the 'probacent'-probability model to use model-derived data as an input for radiation risk evaluation of Indian adult population in their studies [32].

The author [33] finds that Eq. (1) is applicable to the above described Sacher's data [13] on the relationship between dose rates and survival times in mice irradiated daily during the duration of life.

To my knowledge, however, there seem to be no general mathematical models in the literature that express the quantitative relationship among dose rate of radiation, duration of exposure and mortality probability and /or to determine LD_{50} for humans in ionizing total body irradiation.

The purpose of this study is to derive a general mathematical formula that expresses the relationship among dose rate, duration of exposure and mortality probability in total body irradiation in humans.

The mathematical model of death rate that was derived from the 'probacent'-probability model [24] and employed in the author's previous studies [21, 22, 23] is applied in this study to predict mortality probability as a function of dose rate and duration of exposure as well as to predict LD_{50} for humans in lethal radiation exposures.

2. MATERIALS AND METHODS

Data shown in a table of animal-model predictions of lethal radiation doses to humans published by Cerveny, MacVittie and Young [1] are used to construct predictive formulas expressing relationships among dose rate in rad/min, duration of exposure in minutes and mortality probability in percentage in ionizing total body irradiation in humans. The data are based on an extensive study of mortality resulting from radiation exposure and a compilation of animal experimental data published by Jones, Morris, Wells and Young at the Oak Ridge National Laboratory [2].

The data that are used in this study are LD_5, LD_{10}, LD_{50}, LD_{90} and LD_{95} of animal-model-predicted lethal radiation doses in ionizing total body irradiation to humans without subsequent medical treatment that are shown in **Table 1.** The data are plotted on a log-log graph paper as illustrated in Fig. 1 for a better mathematical analysis.

A close look at the data points suggests that all data points of each group of $LD_{5\text{-}95}$ appear to fall on each of the five straight lines, respectively. The range of dose rate is from o.01 to 0.50 Gy/min (1 to 50 rad/min); the duration of exposure is from 2.3 to 360 minutes.

It seems to the author that a general formula of death rate , **Eq.1** be applicable to the data points illustrated in **Figure 1**. The straight lines indicate that the constant c in **Eq.1** is one.

2.1. Formula of Lethal Doses, $LD_{5\text{-}95}$

The following five equations, (3)-(7) are constructed on the basis of the animal-model-predicted LD_5 to LD_{95} for humans [1].

$$\log D_5 = 2.01805 - 0.88209 \text{ x } \log T \quad (3)$$

$$\log D_{10} = 2.06134 - 0.88766 \text{ x } \log T \quad (4)$$

$$\log D_{50} = 2.21767 - 0.90913 \text{ x } \log T \quad (5)$$

$$\log D_{90} = 2.33089 - 0.9203 \text{ x } \log T \quad (6)$$

$$\log D_{95} = 2.35353 - 0.92068 \text{ x } \log T \quad (7)$$

here $D_5, D_{10}, D_{50}, D_{90}$ and D_{95} are dose rates in rad/min in LD_5, LD_{10}, LD_{50}, LD_{90} and LD_{95}, respectively. Dose rate D (rad/min) is a function of duration of exposure T (minute).

If the duration of exposure, time T is given or at any given time T, the dose rate D_{50} and LD_{50} (a product of D_{50} x T) can be expressed by **Eqs.8** and **9**, respectively.

$$D_{50} = 10^{2.21767 - 0.90913 \text{ x } \log T} \quad (8)$$

$$LD_{50} = 10^{2.21767 - 0.90913 \text{ x } \log T} \text{ x } T \quad (9)$$

If the dose rate D is given or with a known dose rate D, the duration of exposure, time T that would cause 50 % mortality probability, LD_{50} (a product of D_{50} x T) can be

Table 1. Comparison of Formula-derived and animal-model-predicted lethal radiation doses to humans.

Lethal		Dose Rate (Gy/minute)					
Dose		0.01	0.02	0.05	0.10	0.20	0.50
LD_5	Formula-Derived *	194.0	176.9	156.4	142.7	130.0	115.0
	Model-Predicted **	194	177	156	143	130	115
LD_{10}	Formula-Derived	210.0	192.3	171.3	156.9	143.8	128.0
	Model-Predicted	210	192	171	157	144	128
LD_{50}	Formula-Derived	275.0	256.6	234.1	218.4	203.9	186.0
	Model-Predicted	275	257	234	218	204	186
LD_{90}	Formula-Derived	341.0	321.1	296.7	279.3	263.1	243.0
	Model-Predicted	341	321	297	279	263	243
LD_{95}	Formula-Derived	360.0	339.1	313.4	295.2	278.1	257.0
	Model-Predicted	360	339	313	295	278	257

*Formula-derived lethal radiation doses are calculated from Eqs. (3)-(7), obtaining total doses (rad) by dose rate (rad/minute), *D* multiplied by duration of exposure, time *T*(minute). (see text). **Model-predicted lethal radiation doses are obtained from Reference [1]. $P > 0.995$.

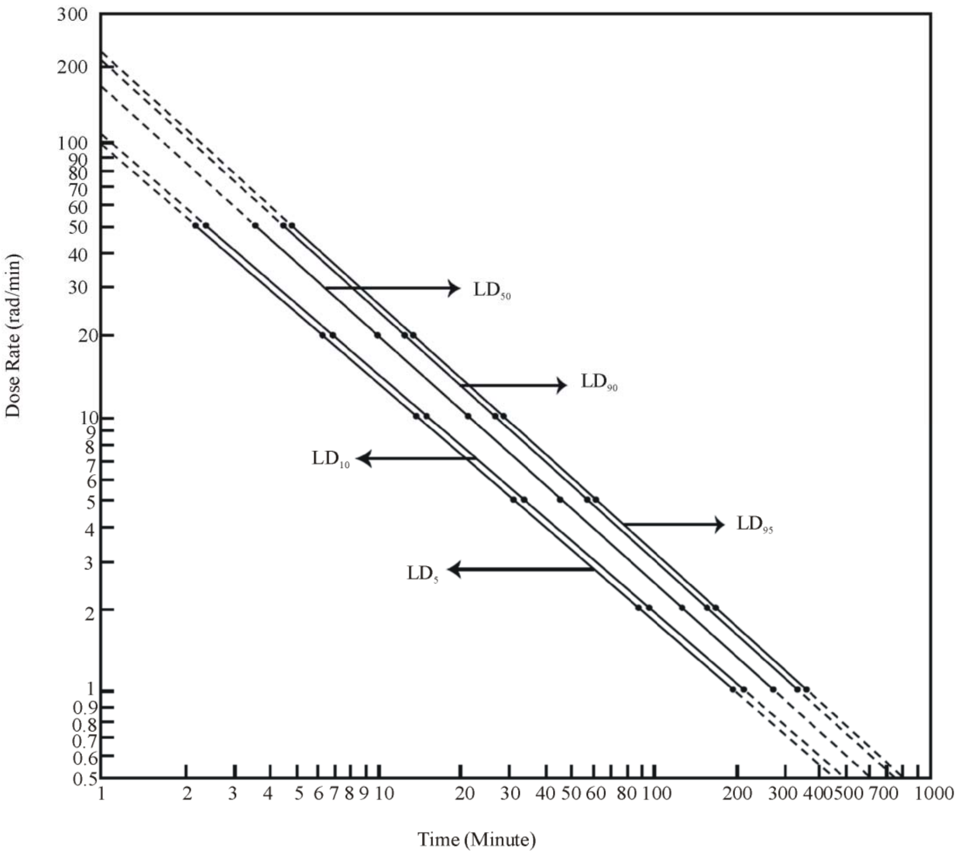

Figure 1. Relationship among dose rate of radiation, duration of exposure and lethal radiation dose (LD_{5-95}) in total radiation body irradiation to humans. The abscissa represents duration of exposure in minutes (log scale). The ordinate represents dose rate in rad/min (log scale). Data points indicate lethal doses of LD_{5-95} and appear to fall on the five formula-predicted straight lines in each group, respectively (see text).

expressed by Eqs. (10) and (11), respectively.

$$T = 10^{(1/0.90913) \times (2.21767 - \log D)} \quad (10)$$

$$LD_{50} = 10^{(1/0.90913) \times (2.21767 - \log D)} \times D \quad (11)$$

Therefore, LD_{50} is dependent on both dose rate and duration of exposure.

If the duration of exposure, time T is given as 10 or 100 minutes, dose rates that would cause 5, 10, 50, 90 and 95 % of mortality probability can be calculated by Eqs. (3)-(7), respectively as aforementioned. **Table 2** shows formula-predicted relationship between dose rates, LD values, mortality probabilities or 'probacent' in cases of the durations of 10 or 100 minutes. Data points in those two cases are illustrated in **Figure 2**. 'Probacent'

Table 2. Formula-predicted relationship between dose rate and mortality probability or 'probacent' in cases of duration of exposure, 10 and 100 minutes in total body irradiation.

Duration of exposure	10 minutes					100 minutes				
Dose rate(rad/min)	13.7	14.9	20.3	25.7	27.1	1.79	1.93	2.51	3.09	3.25
Lethal dose (LD_{5-95})	LD_5	LD_{10}	LD_{50}	LD_{90}	LD_{95}	LD_5	LD_{10}	LD_{50}	LD_{90}	LD_{95}
Mortality probability	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%
'Probacent' (P)**	33.55	37.18	50.0	62.82	66.45	33.55	37.15	50.0	62.82	66.45

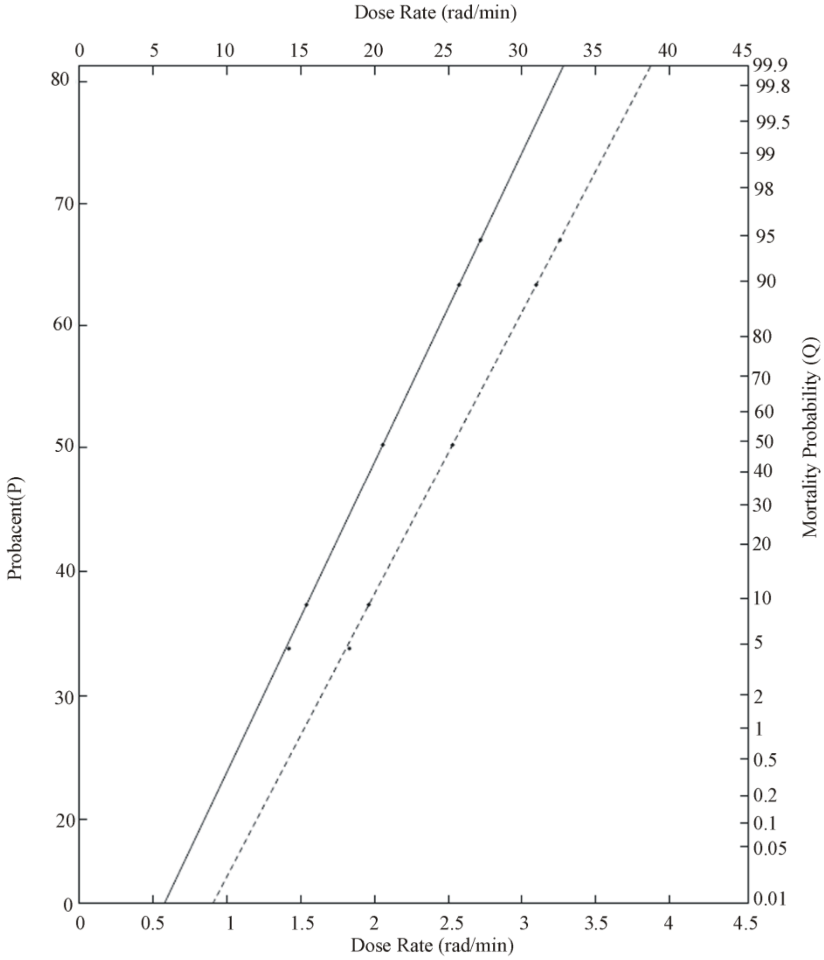

Figure 2. Relationship among dose rate, mortality probability (Q) and 'probacent' (P) in cases of durations of exposure, times 10 and 100 minutes in total body irradiation to humans. The upper abscissa represents dose rate in rad/min in case of duration of 10 minutes; the lower abscissa represents dose rate in rad/min in case of duration exposure of 100 minutes, respectively. The ordinate on the right side represents percent mortality probability (Q). The ordinate on the left side represents 'probacent' corresponding to mortality probability (Q) as shown in Table 2. Data points indicate 5, 10, 50, 90 and 95 % of mortality probabilities corresponding to LD_5, LD_{10}, LD_{50}, LD_{90} and LD_{95}. Data points of durations of exposure of 10 and 100 minutes appear to fall overall on the two straight 'probacent' lines, respectively (see text).

values corresponding to mortality probabilities can be readily obtainable from a table of conversion of percent probability of response into 'probacent' (P) published by the author [25,6].

Data points of each group of 10 or 100 minutes duration appear to overall fall on a straight line.

In addition, the two straight lines show noticeably different 'probacent' slopes. It is assumed that data points of LD_5 to LD_{95} at any time of duration of exposure would fall similarly on a straight 'probacent' line with a different slope.

2.2. General Formula of Mortality Probability, Q

A general formula, Eq. (12) that expresses a quantitative relationship among dose rate, duration of exposure and mortality probability in total body irradiation in humans without medical support is constructed on the basis of above findings revealed in **Figures 1** and **2**, and an aforementioned assumption as follows:

$$K = 2.21767 – 0.90913 \times \log T \quad (12a)$$

$$L = 2.01805 – 0.88209 \times \log T \quad (12b)$$

$$P = 100 \times [(D – 10^K) + (10^K – 10^L) \times 50/16.45]/[(10^K - 10^L) \times 100/16,45] \quad (12c)$$

$$Q = 10/\sqrt{2\pi} \int_{-\infty}^{P} \exp [- (P – 50)^2/200]\, dP \quad (12d)$$

where Q is percent mortality probability based on the Gaussian normal distribution; D is dose rate in rad/min, T is duration of exposure in minutes. P is 'probacent' and is considered to correspond to a relative biologic amount of loss of reserve for survival. 'Probacent' of 0, 50 and 100 corresponds to – 5 SD, mean and mean + 5 SD, respectively. The unit of 'probacent' is 0.1 SD in the Gaussian normal distribution of percent mortality probability, Q (12d).

2.2. Description of the Computer Program

The computer programs were written in UBASIC for IBM PC microcomputer and compatibles for Eq. (12). The computer program for **Eq.12** uses a formula of approximation instead of the integral of **Eq.12d** because the computer cannot execute integral [24,25,34]. Mathematical transformation of the formula of integral, Eq. (12d) to the formula of approximation in computer programming is described in detail in the author's book [34]. A representative program for Eq. (12) is illustrated in **Figure 3** to calculate mortality probabilities in humans as a function of dose rate and duration of exposure.

2.3. Statistical Analysis

A χ^2 goodness-of-fit test (logrank test) [35] is used to test the fit of mathematical models to the data on animal-model predictions of lethal radiation doses for humans [1]. The differences are considered statistically significant when p <0.05.

3. RESULTS

Table 1 shows the results of lethal radiation doses for humans, LD_5, LD_{10}, LD_{50}, LD_{90} and LD_{95} calculated from Eqs. (3)-(11) as a function of dose rate and duration of exposure in total body irradiation. Table 1 also shows comparison of the formula-derived values with the animal-model-predicted lethal doses [1].

Differences between both values of lethal radiation doses are statistically not significant (p >0.995). A close agreement is seen in Table 1. The maximum difference between both values is ± 0.4 rads.

Figure 1 illustrates the relationship between dose rate of radiation and duration of exposure in each group of LD_5, LD_{10}, LD_{50}, LD_{90} and LD_{95}. It seems to the author that the distributions of animal-model-predicted lethal doses ($LD_{5\text{-}95}$) appear to be closely represented by the formulas-derived straight lines.

Table 3 shows comparison of formula (Eq. (12)-derived and animal-model-predicted mortality probabilities (%) in terms of lethal radiation doses ($LD_{5\text{-}95}$) in total body irradiation to humans. Mortality probabilities are determined as a function of dose rate and duration of exposure as shown in Table 3. Values of formula-derived mortality probabilities are obtained by **e**xecution of the computer program illustrated in Fig. 3 as well as manual calculation. Both values from the computer program and manual calculation are found to show a complete agreement with accuracy. Differences between both values of formula-derived and animal-model-predicted lethal radiation doses , $LD_{5\text{-}95}$, are statistically not significant (p >0.995). The maximum difference is ± 1.4 %. A close agreement is present in both values in **Table 3**.

4. DISCUSSION

Table 1 and Fig. 1 reveal that a close agreement between formula-derived and animal-model-predicted data on lethal radiation doses, $LD_{5\text{-}95}$ for humans in the total body irradiation [1] (p >0.995). The lines representing Eqs. (3)-(7) in Fig. 1 are straight, indicating that the constant c in Eq. (1) of death rate is one and becomes essentiall similar to the Weibull distribution [18] and suggesting that Eq. (1) seems to be a generalized Weibull model.

If the dose rate D is reduced and the duration of exposure T is proportionately increased, asame amount of total dose $D \times T$ of radiation would decrease the mortality probability as shown in Table 1. Ellington reported that simple dose fractionation decreases the mortality rate caused by the same doses when given in one exposure [15] as suggested by that the mortality probability is

Table 3. Comparison of formula-derived and animal-model-predicted mortality probabilities (%),(lethal radiation doses, $LD_{5\text{-}95}$) in total body irradiation to humans.

Dose Rate	Dose Rate (Gy/minute)						
Gy/minute		0.01	0.02	0.05	0.10	0.20	0.50
(rad/minute)		(1)	(2)	(5)	(10)	(20)	(50)
Duration of exposure (min)		194	88.5	31.2	14.3	6.5	2.3
	Formula						
Mortality	Derived*	5.0	5.0	4.9	5.1	5.0	5.0
probability (%)	Model						
(LD_{50})	Predicted**	5	5	5	5	5	5
Duration of exposure (min)		210	96	34.2	15.7	7.2	2.56
	Formula-						
Mortality	derived	9.3	9.1	9.0	9.1	9.1	8.9
probability (%)	Model						
(LD_{10})	predicted	10	10	10	10	10	10
Duration of Exposure (min)		275	128.5	46.8	21.8	10.2	3.72
	Formula-						
Mortality	derived	50.0	50.3	49.9	49.6	50.1	50.0
probability (%)	Model						
(LD_{50})	predicted	50	50	50	50	50	50
Duration of Exposure (min)		341	160.5	59.4	27.9	13.15	4.86
	Formula-						
Mortality	derived	91.4	91.1	91.1	90.7	90.8	90.8
probability (%)	Model						
(LD_{90})	predicted	90	90	90	90	90	90
Duration of Exposure (min)		360	169.5	62.6	29.5	13.9	5.14
	Formula-						
Mortality	derived	96.1	95.8	95.5	95.3	95.2	95.1
probability (%)	Model						
(LD_{95})	predicted	95	95	95	95	95	95

*Formula-derived mortality probabilities (%), ($LD_{5\text{-}95}$) are calculated from Eq. (8); **Model-predicted mortality probabilities (%) are obtained from Reference [1].p >0.995.

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10   lprint "RELATIONSHIP AMONG DOSE RATE, DURATION, PROBACENT AND MORTALITY"
20   lprint "PROBABILITY IN TOTAL BODY IRRADIATION IN HUMANS"
30   lprint
40   lprint "Dose rate",tab(11);"Time",tab(21);"Probacent",tab(53);"Mortality
50   lprint tab(1);"rad/min",tab(11);"min",tab(55);"%"
60   read D,T
70   DeffnK=(2.21767-0.90913*log(T)/log(10))
80   DeffnL=(2.01805-0.88209*log(T)/log(10))
90   DeffnKA=10^DeffnK
100  DeffnLA=10^DeffnL
110  DeffnKLH=(DeffnKA-DeffnLA)*50/16.45
120  DeffnKLT=(DeffnKA-DeffnLA)*100/16.45
130  DeffnP=100*(D-DeffnKA+DeffnKLH)/DeffnKLT
140  P=DeffnP
150  A1=0.278393
160  A2=0.230389
170  A3=0.000972
180  A4=0.078108
190  if (P-50)<0 then 200 else 230
200  X=(50-P)/sqrt(200)
210  Q=50/(1+A1*X+A2*X^2+A3*X^3+A4*X^4)^4
220  goto 250
230  X=(P-50)/sqrt(200)
240  Q=100-50/(1+A1*X+A2*X^2+A3*X^3+A4*X^4)^4
250  lprint D,tab(10);T,tab(20);P,tab(52);Q
260  goto 60
270  data 1,194,2,88.5,5,31.2,10,14.3,20,6.5,50,2.3
280  data 1,210,2,96,5,34.2,10,15.7,20,7.2,50,2.56
290  data 1,275,2,128.5,5,46.8,10,21.8,20,10.2,50,3.72
300  data 1,341,2,160.5,5,59.4,10,27.9,20,13.15,50,4.86
310  data 1,360,2,169.5,5,62.6,10,29.5,20,13.9,50,5.14
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Figure 3. The computer program to calculate percent mortality probability (Q) as a function of dose rate of radiation in rad/min (D) and duration of exposure in minutes (T), expressed by Eq. (12), in total body irradiation in humans. Results of execution of the program are shown in the rows of 'Formula-derived mortality probabilities (%)' in Table 3 (see text).

a function of dose rate and duration of exposure as expressed by Eq.(12).

Extrapolation of the five straight lines in Fig. 1 might be possible as shown with dashed lines beyond the upper and lower ends of the lines. However, it would require further animal-model-predicted data on lethal radiation doses and relevant human data to examine the extrapolation.

A quantitative dose-response relationship in lethal ionizing radiation exposure in humans is not known [1]. Several investigators have derived hypothetical dose-response curve based on experiences with reactor accidents and the atomic exposure in Japan. From these observations, the $LD_{50(60)}$ for humans exposed to a single dose of radiation delivered over a period of less than 24 hours is believed to be in the range of 2.50 to 4.0 Gy (250 to 400 rads) [36].

If extrapolation is allowed and the duration of exposure is assumed to be 1,400 minutes (less than 24 hours), LD_{50} calculated from Eq. (9) will be 3.19 Gy that is in the range of 2.50 to 4.0 Gy.

Levin, Young and Stohler [37] published an estimate of the median lethal dose on humans exposed to ionizing total body irradiation and not subsequently treated for radiation sickness. The median lethal dose was estimated from calculated doses to young adults who were inside two reinforced concrete buildings that remained standing in Nagasaki, Japan after the atomic detonation. Median lethal dose estimates were calculated using both logarithmic (2.9 Gy) and linear (3.4 Gy) dose scales. Both calculations supported previous estimates of the median lethal dose based solely on human data, which clustered around 3 Gy. The LD_{50} estimate, based on a log probit calculation, was 2.9 Gy to the bone marrow. The LD_{50} of 2.9 Gy was surprisingly consistent with estimates made by other researchers; 2.45 Gy by Langham (1967), 2.86 Gy by Lushbaugh et al. (1967), 2.65-2.70 Gy by Bond and Robertson (1957) [37].

Fujita, Kato and Schull [38] reported that the $LD_{50(60)}$ is 2.3-2.6 Gy on the basis on a total of 7,593 persons exposed to the atomic bombing of Hiroshima. The range of 2.3-2.6 Gy is noticeably in a good agreement with the values of LD_{50} shown in Table 1.

Therefore, if it is taken into consideration that LD_{50} is a function of dose rate and duration of exposure, there seems to be a remarkable agreement between formula-predicted LD_{50} in Tables 1 and 3 and above described published-estimated LD_{50} for humans [36-38].

According to Tokyo Electric Power Company's report, radiation leaking from the damaged reactors of the Fukushima Daiich Nuclear Power Plant caused by the 9.0 magnitude earthquake and tsunami recently hitting Japan (March 11, 2011, [40]) was 600 mSv per hour [41]. If Eq. (12) is possibly applied in this grave situation to calculate mortality probabilities for one hour, 25 or 10 minutes of exposure to ionizing radiation, results are as follows:

In one hour of exposure to radiation, mortality probability (Q) = 5.295×10^{-3} %, and 'probacent' (P) = 8.148. In 25 minutes of exposure that was for total 250 mSv as a maximum limit dose allowed to each emergency rescue worker, mortality probability (Q) = 1.318×10^{-3} %, and 'probacent' (P) = 3.084. In 10 minutes of exposure that was for total 100 mSv as a low dose limit allowed to each rescue worker, mortality probability (Q) = 1.063×10^{-3} %, and 'probacent' (P) = 2.278 [42].

The above results might suggest a potential risk of 5-6 deaths (one-hour exposure), and 1-2 deaths (25-minutes exposure) and one death (10-minutes exposure) in 100,0000 ordinary persons exposed to radiation without clinical support, respectively.

Hematopoietic cells of bone marrow, intestinal tract, and central nervous system are most vulnerable to radiation effects [5,8,17,39]. Body responses to lethal radiation effects reflect status of living body in which physiologic response, repair and regeneration of recovery process, pathologic changes and aging process are concurrently occurring [4,6,13,14,36,39].

Death is caused by multi-organ failure. In cases of relatively high doses, infection and hemorrhage are earliest contributing factors to death, resulting from the damage to the most sensitive hematopoietic cells in total body irradiation [1,13,36]. Clinical support with transfusion of fresh platelets and granulocytes, antibiotics, and infusion of fluids is capable of treating radiation sickness and raising LD_{50} [1].

5. CONCLUSIONS

A formula of death rate, Eq. (1) derived from the 'probacent'-probability equation for survival probability [21] is applied in this study to construct a general formula that expresses a quantitative relationship among dose rate of radiation, duration of exposure and mortality probability in total body irradiation in humans. The general formula, Eq. (12) is developed on the basis of the animal- model predictions of lethal radiation doses for humans published by Cerveny, MacVitte and Young [1]. The data on animal-model predictions are based on the extensive study of mortality resulting from radiation exposure and a compilation of animal experimental data published by Jones, Morris, Wells and Young [2]. The LD_{50} for humans is mathematically predictable as a function of dose rate and/or duration exposure.

A close agreement is present between both values of formula-derived and animal-model-predicted LD_{50} as well as mortality probabilities. The formula-predicted

LD_{50} seems to remarkably agree with published estimates of LD_{50} [36-38]. The general formula, Eq. (12) might be hopefully helpful in preventing radiation hazard and injury, and further for safety in radiotherapy.

The general formulas, Eqs. (9)-(11) and (12) that estimates human LD_{50} and mortality probability are constructed based on animal-model-predicted lethal radiation doses and so would need further research on animal experiments and relevant human data for their verification.

6. ABSTRACT

Jones, Morris, Wells and Young at the Oak Ridge National Laboratory, Oak Ridge, Tennessee published an extensive study on mortality resulting from radiation exposure and a compilation of animal experimental data [2]. Cerveny, MacVittie and Young [1] published a table of animal-model predictions of lethal radiation doses for humans that is based on the publication of the Oak Ridge National Laboratory by Jones, Morris, Wells and Young [2]. A general formula, Eq. (12) that expresses a relationship among dose rate, duration of exposure and percent mortality probability in total body irradiation in humans without subsequent medical treatment is constructed on the basis of the data published by Cerveny, MacVittie and Young [1]. The LD_{50} for humans is expressed by Eq. (9) and (11). A close agreement is present between formula-derived and animal-model-predicted mortality probabilities as well as LD_{50}.

The death rate equation (1) derived from the 'probacent'-probability model of survival probability, Eq. (2) is employed in this study to construct the general formula, Eq. (12) of mortality probability as a function of dose rate and duration of exposure in total body irradiation in humans. There seems to be a remarkable agreement between formula-predicted and published estimated LD_{50} (36-38) and also between both mortality probabilities (see **Tables 1 and 3**).

The formulas, Eqs. (9)-(11), and (12) for LD_{50} and mortality probability in lethal radiation exposures might be hopefully helpful in preventing radiation hazard and injury, and further for safety in radiotherapy. The general formulas are constructed based on animal-model- predicted lethal radiation doses for humans in this study and so would need further research on animal experiments and relevant human data for their verification.

7. KEY WORDS

Total Body Irradiation, Lethal Radiation Dose, Formula of Mortality Probability, Formula of LD_{50}, Computer Program, Nuclear Reactor Accident, Safety in Radiotherapy, Radiation Hazard and Injury.

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