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Tailoring Quantum Correlations of a Coupled Central Two Qubits Soaked in a Finite Temperature Antiferromagnetic Environment with Frequency Gap

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Abstract

We revisit the quantum features of an anti-ferromagnetic (AF) spin environment at finite temperature with gap in its frequency spectrum, on the dynamics quantum correlations of a coupled central two qubits system with Dzyaloshinskii-Moriya (DM) interaction, prepared in two entangled Bell states. Using entanglement and quantum discord as quantum meters of decoherence, the prepared entangled states are classified as robust or fragile relative to the degree of information leakage to the AF environment. By tailoring the size of the frequency gap, anisotropy field strength and induced field, due to system AF spin environment coupling, size of the AF environment and DM interaction parameter, a decoherence-free sub-space can be accessed for efficient execution of quantum protocols encoded in the entangled states.

Keywords

Anti-Ferromagnetic Lattice, Anisotropy Field, Magnetic Field, Quantum Correlations, Central Two Qubits

1. Introduction

Entanglement and quantum discord are two different faces of the quantum correlations without classical coun-

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terpart. Due to their notable features in the speed-up of quantum protocols in quantum computers and other quantum information devices, much attention has been devoted over the past recent years to uncover the intricate quantum features [1] [2]. The engineering of these quantum features in order to have control over quantum protocols remains till date a disputable issue due to decoherence and has been a subject of intense investigation [3] [4]. Quantum entanglement being a resource on its own, once quantified becomes a suitable tool of quantum meter of decoherence and various schemes have been theoretically and experimentally proposed to show how the quantum information encoded in the entangled states could be shaded from irreversible leakages into the environment [5] [6]. Amongst others, quantum computing in a decoherence-free sub-space has gained substantial interest involving less computational resources [7]-[9]. In Ref. [9], the formalism for quantum computing in decoherence-free sub-spaces that are partial triangulated to an index related to environment is constructed. The quantum states in the sub-spaces are reported to be the projected states and can be used to perform ideal quantum logical operations with minimal decoherence.

The resurgence in quantum computing in decoherence-free sub-spaces has also revitalized research for constructing different environments as the later produces different decoherence features [10]. The developing interest in the study of spin environment with different geometrical structures especially the role of the latter on quantum correlations, has stimulated us to revisit the dynamics of quantum correlations of two central qubits embedded in an anti-ferromagnetic (AF) environment. The spin structure of the environment is taken to be divided into two interpenetrating sub-lattices a and b with gap in its frequency spectrum. This investigation is an extension of our recent works where the dynamics of a single qubit was considered [11]-[13].

The concept of thermal entanglement is one of the important concepts in quantum correlation because it shows the effect of thermal fluctuations on entanglement [14]. The maximization scheme for thermal entanglement for two interacting qubits has been proposed in Ref. [15] by turning a Hamiltonian under a given interaction, showing that the optimized entanglement does not vanish at any temperature and decays slowly according to $1/(T\log T)$ at high temperatures (where T is the temperature). In Ref [16], the authors observed that the thermal effects can enhance disentanglement, and wash out critical phenomenon of quantum phase transitions as reflected in the disentanglement evolution of bipartite spin-1/2 system coupled to a common surrounding XY chain in transverse fields at non-zero temperature. By engineering the properties of the finite temperature anti-ferromagnetic environment through parameters of the composite system, the dynamics of the central two qubits system can be modulated. To take mechanism for spin-orbit coupled bands, the spin-flip hopping processes, bond distortion for a pair of adjacent magnetic orbitals into consideration, the DM interaction is considered [17] and some authors have investigated its influence on quantum correlations in spin system [18]. Indeed, a setup to couple two central electron spin to an AF environment, considering also the DM interaction has been recently investigated [19]. The authors consider the influence of the DM interaction on the time evolution of the concurrence for different initial pure entangled states at zero temperature showing that in the absence of decoherence, entanglement evolves periodically with frequency and amplitude that increases with the increase in D_Z (z-component of DM interaction). In this work, referring to the setup of Ref. [19] the DM is seen to show a kind of trade-off with the anisotropy field (intrinsic field) as depicted in the entanglement dynamics.

The paper is organized as follows. In Section 2 we call for the model Hamiltonian of the central two qubits spin coupled via XXZ Heisenberg chain soaked in a common AF environment. The spin-wave approximation is applied to map the spin operators of the AF environment to bosonic operators after which, exact tracing of the environmental modes is done to obtain the reduced density matrix of the central system as a function of lattice discrete momentum. This then prepares in Section 3 the ground for the evaluation of the functions characterizing the manipulation of information in the system such as concurrence and discord. Interpretations of results are given in Section 4 and finally the conclusion in Section 5.

2. Model

The model consists of a central two qubits spin system interacting via a Heisenberg XXZ chain with DM interaction soaked in an anti-ferromagnetic environment, driven by an anisotropic and magnetic field. Adiagrammatic picture of the composite system is represented in **Figure 1**.

The total Hamiltonian for a central two qubits spin longitudinally coupled to the AF spin field environment is given by

$$H = H_S + H_{SE} + H_E, \tag{1}$$

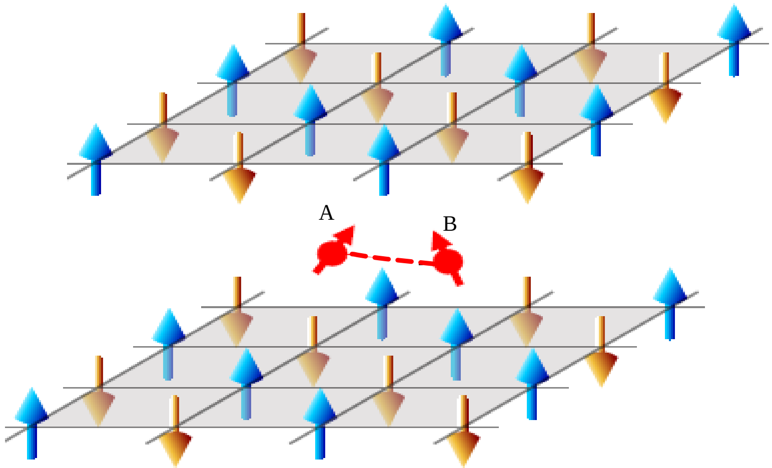


Figure 1. (Color online) Schematic of the two-qubit systems interacting with common anti-ferromagnetic environment.

where H_S and H_E are the Hamiltonians of the central two qubits and of the environment respectively; H_{SE} is the non-demolition coupling of the central two qubit system to the AF environment, they can be written as:

$$H_S = -g\mu_B (B+B_A) S_{1_s}^z - g\mu_B (B-B_A) S_{2_s}^z + \frac{\Omega}{2}\left(S_{1_s}^+ S_{2_s}^- + S_{1_s}^- S_{2_s}^+\right) + iD_Z\left(S_{1_s}^+ S_{2_s}^- - S_{1_s}^- S_{2_s}^+\right) + J_z S_{1_s}^z S_{2_s}^z, \tag{2}$$

$$\begin{aligned} H_E = & -g\mu_B (B+B_A)\sum_i S_{a,i}^z - g\mu_B (B-B_A)\sum_j S_{b,j}^z + J\sum_{i,\delta}\left[S_{a,i}^z S_{b,i+\delta}^z + \frac{1}{2}\left(S_{a,i}^+ S_{b,i+\delta}^- + S_{a,i}^- S_{b,i+\delta}^+\right)\right] \\ & + J\sum_{j+\delta}\left[S_{b,j}^z S_{a,j+\delta}^z + \frac{1}{2}\left(S_{a,j}^+ S_{b,j+\delta}^- + S_{a,j}^- S_{b,j+\delta}^+\right)\right]. \end{aligned} \tag{3}$$

$$H_{SE} = -\frac{J_O}{\sqrt{N}}\left(S_{1_s}^z + S_{2_s}^z\right)\sum_i \left(S_{a,i}^z + S_{b,i}^z\right), \tag{4}$$

For definiteness, the subscript S refers to the system, g is the gyromagnetic factor, μ_B is the Bohr magneton, Ω is the isotropic central two qubits coupling strength along the xy plane and D_Z the DM interaction that models the spin-orbit interaction, J_Z the coupling strength along the *z* axis; considered positive for AF coupling and negative for ferromagnetic coupling. J_O is the coupling constant between system and spin environment, J is the isotropic predominantly nearest neighbor exchange interaction and is positive for AF environment. We consider the model of the AF environment where the spin structure of the environment may be divided into two interpenetrating sub-lattices a and b with the property that all nearest neighbors of an atom of a lie on b, and vice versa, $S_{a,i}$ $\left(S_{b,j}\right)$ represents the spin operator of the i^{th} $\left(j^{\text{th}}\right)$ atom on sub-lattice a (b). Each sub-lattice contains *N* atoms. The vectors $\boldsymbol{\delta}$ is a 3 dimensional vector that connects atom i or j with its nearest neighbor. The parameter B characterizes the intensity of the uniform external magnetic field applied along the *z*-axis. The anisotropy field B_A is assumed to be positive, which approximates the effect of the crystal anisotropy energy, with the property of tending for positive magnetic moment μ_B to align the spins on sub-lattice a in the positive *z*-direction and the spins on sub-lattice b in the negative *z*-direction.

By using the collective angular momentum operators $J_\pm = \sum_{i,j=1}^N S_{a,j}^\pm$ and the Holstein-Primakoff transformation as $J_+ = a^+\sqrt{N - a^+ a}$ and $J_- = \left(\sqrt{N - a^+ a}\right)a$ with $\left[a, a^+\right] = 1$, the Hamiltonians of the bath spin H_E and interaction H_{SE} can be rewritten as

$$H_{SE} = -\frac{J_O}{\sqrt{N}}\left(S_A^z + S_B^z\right)\sum_k \left(b_k^+ b_k - a_k^+ a_k\right), \tag{5}$$

$$H_E = E_0 + \left(2\mathrm{MSJ} + g\mu_B\left(B_z + B_A\right)\right)\sum_k a_k^+ a_k + \left(2\mathrm{MSJ} + g\mu_B\left(B_A - B_z\right)\right)\sum_j b_k^+ b_k + \\ + 2\mathrm{MSJ}\sum_k \gamma_k\left(a_k^+ b_k^+ + a_k b_k\right), \tag{6}$$

With $\gamma_k = M^{-1}\sum_{\delta}\cos\left(\boldsymbol{k}\cdot\boldsymbol{\delta}\right)$. The coefficient M represents the number of nearest neighbors of an atom. The constant $E_0 = -2S^2NJ - 2NSg\mu_B B_A$ may be neglected in subsequent evaluation, k is the lattice wave vector with discreet values ($k = 2\pi m/N$, $m \in a, b$). By performing the Bogoliubov transformation,

$$\left.\begin{aligned} a_k &= \cosh\left(\theta_k\right)\alpha_k - \sinh\left(\theta_k\right)\beta_k^+ \\ b_k &= -\sinh\left(\theta_k\right)\alpha_k^+ + \cosh\left(\theta_k\right)\beta_k \end{aligned}\right\}. \tag{7),}$$

The Hamiltonian H_{SE} and H_E can then be diagonalized as

$$H_{SE} = -\frac{J_O}{\sqrt{N}}\left(S_{1s}^z + S_{2s}^z\right)\sum_k\left(\beta_k^+\beta_k - \alpha_k^+\alpha_k\right), \tag{8}$$

$$H_E = E_0 + \sum_k \omega_k^+\left(\alpha_k^+\alpha_k + 1/2\right) + \sum_k \omega_k^-\left(\beta_k^+\beta_k + 1/2\right), \tag{9}$$

where $\alpha_k^+\left(\alpha_k\right)$ and $\beta_k^+\left(\beta_k\right)$ are the creation (annihilation) operators of the two different magnons with wave vector k and frequency $\omega_k^+\left(\omega_k^-\right)$. While deriving Equations (8) and (9), the function θ_k in the Bogoliubov transformation is chosen such that the coefficient of $\alpha_k\beta_k$ and $\alpha_k^+\beta_k^+$ is zero. We find that, the bath spin is reduced into a bosonic mode thermal field with non-Markovian effect on the dynamics of the two-qubits system. Since this thermal field will not remain in a thermal equilibrium state as usually assumed for an environment with very large degrees of freedom, therefore, the master equation approach cannot be useful. In order to catch the exact non-Markovian dynamics of reduced density matrix for the system at arbitrarily finite temperatures, the time evolving density matrix of the composed system can be written as

$$\rho_{\mathrm{tot}}\left(t\right) = U\left(t\right)\rho_{\mathrm{tot}}\left(0\right)U^+\left(t\right), \tag{10}$$

where

$$U\left(t\right) = \mathrm{e}^{-iH_{\mathrm{tot}}\left(t\right)}. \tag{11}$$

Assuming that the initial density matrix for the total system is separable, *i.e.* the initial state of the central spin system is completely irrelevant to the AF environment then, $\rho\left(0\right) = \rho_{AB}\left(0\right)\otimes\rho_E\left(0\right)$. Where $\rho_E\left(0\right)$ refers to the density matrix describing the initial state of the environment which at thermal equilibrium is represented by the Boltzmann distribution as $\rho_E\left(0\right)$, $\rho_E\left(0\right) = \frac{1}{Z}\exp\left(\beta H_E\right)$, with Z the partition function given as:

$$Z = Tr\left[\exp\left(-H_E/T\right)\right] = \prod_{k\succ 0}\left(\frac{1}{1-\mathrm{e}^{-\omega_k^-/T}}\right)\prod_{k\succ 0}\left(\frac{1}{1-\mathrm{e}^{-\omega_k^+/T}}\right) \tag{12}$$

where k_B is Boltzmanns constant which we henceforth set equal to one, T is the temperature.

$\rho_{AB}\left(0\right) = \left|\Phi\left(0\right)\right\rangle\left\langle\Phi\left(0\right)\right|$ is the density matrix of the central spins in the initial state, $\left|\Phi\left(0\right)\right\rangle$ is assumed to be an initially prepared entangled state. By adopting the approach used in [20] and extending it to multilevel system, the time evolving reduced density matrix of the two-qubits is obtained by tracing out the environmental degrees of freedom

$$\rho_{AB}\left(t\right) = \sum_{\mu,\upsilon=1}^{4} c_\upsilon^* c_\mu F_{\mu\upsilon}\left|\phi_\mu\right\rangle\left\langle\phi_\upsilon\right| \tag{13}$$

where

$$F_{\mu\upsilon} = \left\langle\psi_E\left(t\right)\right|U_E^{+\left(\lambda_\upsilon\right)}\left(t\right)U_E^{\left(\lambda_\mu\right)}\left(t\right)\left|\psi_E\left(t\right)\right\rangle \tag{14}$$

and

$$c_\mu = \exp\left(-i\varepsilon_\mu t\right)\left\langle \varphi_\mu \middle| \Phi \right\rangle, \qquad \mu = 1,2,3,4 \tag{15}$$

With $\left|\phi_\mu\right\rangle$ and ε_μ the μ^{th} eigen function and eigen value respectively of the system Hamiltonian H_S. $U_E^{(\lambda_\upsilon)}(t) = \mathrm{e}^{-iH_E^{\lambda_\mu} t}$ is the projected time evolution operator.

In order to evaluate quantum correlations, the decoherence factors $F_{\mu\upsilon}$ is computed as follows

$$F_{\mu\upsilon} = \prod_{k\succ 0} Tr_k \left[\mathrm{e}^{-i\left(H_E^{\lambda_\mu} + \xi_\mu \sum_k \left(\beta_k^+\beta_k - \alpha_k^+\alpha_k\right)\right)} \rho_E(0)\, \mathrm{e}^{i\left(H_E^{\lambda_\upsilon} + \xi_\upsilon \sum_k \left(\beta_k^+\beta_k - \alpha_k^+\alpha_k\right)\right)} \right] \tag{16}$$

where

$$H_E^{\lambda_\mu} = \sum_k \omega_k^{+,\lambda_\mu}\left(\alpha_k^+\alpha_k\right) + \sum_k \omega_k^{-,\lambda_\mu}\left(\beta_k^+\beta_k\right), \tag{17}$$

represents the effective AF environment-dressed Hamiltonian resulting from the fact that the coupling H_{SE} between the qubits and spin environment exerts an extra magnetic field ξ_μ on the spin environment H_E, resulting to the effective intrinsic magnetic field $\lambda_\mu = B_A + \xi_\mu$, where

$$\xi_{1,4} = \pm \frac{J_O}{\sqrt{N} g \mu_B}, \qquad \xi_{2,3} = 0 \tag{18}$$

In this consideration, the decohorence factors yield

$$F_{\mu\upsilon} = \frac{1}{Z}\prod_{k\succ 0}\left(1 - \exp\left(-\omega_k^+/T + it\Theta_{\mu\nu}^+\right)\right)\prod_{k\succ 0}\left(1 - \exp\left(-\omega_k^-/T + it\Theta_{\mu\nu}^-\right)\right). \tag{19}$$

where $\Theta_{\mu\nu}^+ = \omega_k^{+\lambda_\nu} - \omega_k^{+\lambda_\mu} + \xi_\mu - \xi_\nu$, $\Theta_{\mu\nu}^- = \omega_k^{-\lambda_\nu} - \omega_k^{-\lambda_\mu} + \xi_\nu - \xi_\mu$.

Between the elements of the decoherence factors the following relations exist: $F_{12} = F_{13}$, $F_{23} = 1$, $F_{24} = F_{34}$, and $F_{\mu\upsilon} = 1$ for $\mu = \upsilon$. The absolute value of the decoherence factor F_{14} is evaluated in the thermodynamics limit as in Ref. [21]

$$\left|F_{14}(t)\right| = \exp\left(-t^2\left(\tau^+ + \tau^-\right)\right); \qquad \tau^\pm = \left(\frac{J_0}{\pi g \mu_B}\right)^2 \int_0^\infty \frac{\exp\left(-2\omega^\pm(x)/T\right)}{\left(1 - \exp\left(-\omega^\pm(x)/T\right)\right)^2} x^2 \mathrm{d}x \tag{20}$$

3. Information Transfer in the Couple Spin System

There exist many parameters that characterize the information transfer between system and environment. In this work, we intend to investigate the degree of the information that can be leaked out when the two central qubits spin system in the presence of the DM interaction is coupled to the AF spin chain. We then focus on the evaluation of the concurrence whose numerical result is compared with that of classical correlation and quantum discord.

Using Wootters concurrence [22], the entanglement dynamics is evaluated for the different initially prepared entangled states, $\left|\Phi\right\rangle = \left(\cos\vartheta\left|00\right\rangle + \sin\vartheta\left|11\right\rangle\right)$ and $\left|\Phi\right\rangle = \left(\cos\vartheta\left|01\right\rangle + \sin\vartheta\left|10\right\rangle\right)$ for the reduced density matrix $\rho_{AB}(t)$ in the standard basis spanned by $\left\{\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle\right\}$ as

$$C\left(\rho_{AB}(t)\right) = \max\left\{0, \sqrt{\varpi_1} - \sqrt{\varpi_2} - \sqrt{\varpi_3} - \sqrt{\varpi_4}\right\}, \tag{21}$$

where the quantities ϖ_i are roots of the eigenvalues in decreasing order of the auxiliary matrix $\varsigma = \rho_{AB}(t)\left(\sigma_y \otimes \sigma_y\right)\rho_{AB}^*(t)\left(\sigma_y \otimes \sigma_y\right)$. $\rho_{AB}^*(t)$ denotes the complex conjugate of $\rho_{AB}(t)$ in the standard bases and σ the Pauli matrix. For the initially prepared entangled states taken respectively as $\left|\Phi\right\rangle = \left(\cos\vartheta\left|00\right\rangle + \sin\vartheta\left|11\right\rangle\right)$ and further as $\left|\Phi\right\rangle = \left(\cos\vartheta\left|01\right\rangle + \sin\vartheta\left|10\right\rangle\right)$, the resulting reduced density matrix is obtained as:

$$\rho_{AB}(t)=\begin{pmatrix}\rho_{11} & 0 & 0 & \rho_{14}\\ 0 & \rho_{22} & \rho_{23} & 0\\ 0 & \rho_{32} & \rho_{33} & 0\\ \rho_{14}^{*} & 0 & 0 & \rho_{44}\end{pmatrix} \tag{22}$$

where

$$\rho_{11}=\cos^2\theta,\quad \rho_{44}=\sin^2\theta,\quad \rho_{14}=\mathrm{e}^{\{-i(\varepsilon_1-\varepsilon_4)t\}}F_{14}(t),\quad \rho_{41}=\rho_{14}^{*} \tag{23}$$

$$\rho_{22}=\left[(B_A+u)\left\{\left(\frac{\Omega}{2}+iD_z\right)^2+(B_A+u)^2\right\}^{1/2}\left\{\cos(\theta)(B_A+u)-\left(\frac{\Omega}{2}+iD_z\right)\sin\theta\right\}\right]^2 \tag{24}$$

$$\rho_{33}=\left[(B_A+u)\left\{\left(\frac{\Omega}{2}-iD_z\right)^2+(B_A+u)^2\right\}^{1/2}\left\{\sin\theta(B_A+u)+\left(\frac{\Omega}{2}+iD_z\right)\cos(\theta)\right\}\right]^2 \tag{25}$$

$$\rho_{23}=\left(\cos\theta(B_A+u)-\left(\frac{\Omega}{2}+iD_z\right)\sin\theta\right)\left(\cos\theta\left(\frac{\Omega}{2}-iD_z\right)+(B_A+u)\sin\theta\right)\exp\{-i(\varepsilon_2-\varepsilon_3)\}t \tag{26}$$

$$\rho_{32}=\rho_{23}^{*} \tag{27}$$

To quantify the quantumness of a system more broadly, quantum discord becomes a figure of merit as it quantifies non-classical correlations of more general type than concurrence. Quantum discord is introduced as the difference between the total correlations and the classical correlation and has been evaluated in many works [23]-[27]. In order to discuss the time variation of quantum correlations other than entanglement of the coupled two qubits system bathed in the model of the antiferromagnetic environment, we will only provide results without further derivations since we are concerned with two-qubit X state that has been explicitly derived in [24].

4. Results and Discussions

The time evolution of entanglement as a function of applied global external magnetic field for finite lattice size and the thermodynamic limit are presented in **Figure 2(a)** and **Figure 2(b)** respectively for the initial maximal entangled Bell state $|\Phi\rangle=\left(1/\sqrt{2}\right)\left(\cos\vartheta|00\rangle+\sin\vartheta|11\rangle\right)$. For the case of finite lattice size **Figure 2(a)** we observed entanglement killing for small values of the magnetic field and for increasingly large values of the field, entanglement becomes enhanced with sudden death and revival during the early stage dynamics of the system before hitting zero during the latter stages. The time for the revival is short and with enhancement in the revival peak as the magnetic field increases. We attribute this observation to the fact that, the frequency splitting in the presence of the magnetic field gives birth to two modes which enters the dynamics of entanglement. For a given magnetic field, we distinguish two frequency modes; the high frequency modes ω_k^{+} whose frequency and particle excitation number increases with magnetic field and low frequency modes ω_k^{-} whose frequency and particle excitation number decreases with magnetic field. This shows that the high frequency modes ω_k^{+} enhances memory effect as these modes readily copy and store the information about the state of the system with little leakage to the rest of the environment due to their stability as the field increases. These modes may be save as a decoherence free channel for initializing and carrying out gate operations in quantum computation.

The scenario is different in the thermodynamic limit **Figure 2(b)**, where entanglement dynamics shows an accelerated Gaussian decay as the magnetic field increases. This behavior is related to the overall increase in magnon excitation number difference and fluctuations as magnetic field increases; a situation similarly witnessed in [21] where these fluctuations leads to a decrease in the decoherence time of a central electron spin in an AF environment. The finite time death of entanglement after the peak value is reached is a manifestation of Markovian (memory less) effects that dominates the dynamics of entanglement as the two modes completely lost their information following their interaction with the rest of the thermal environment.

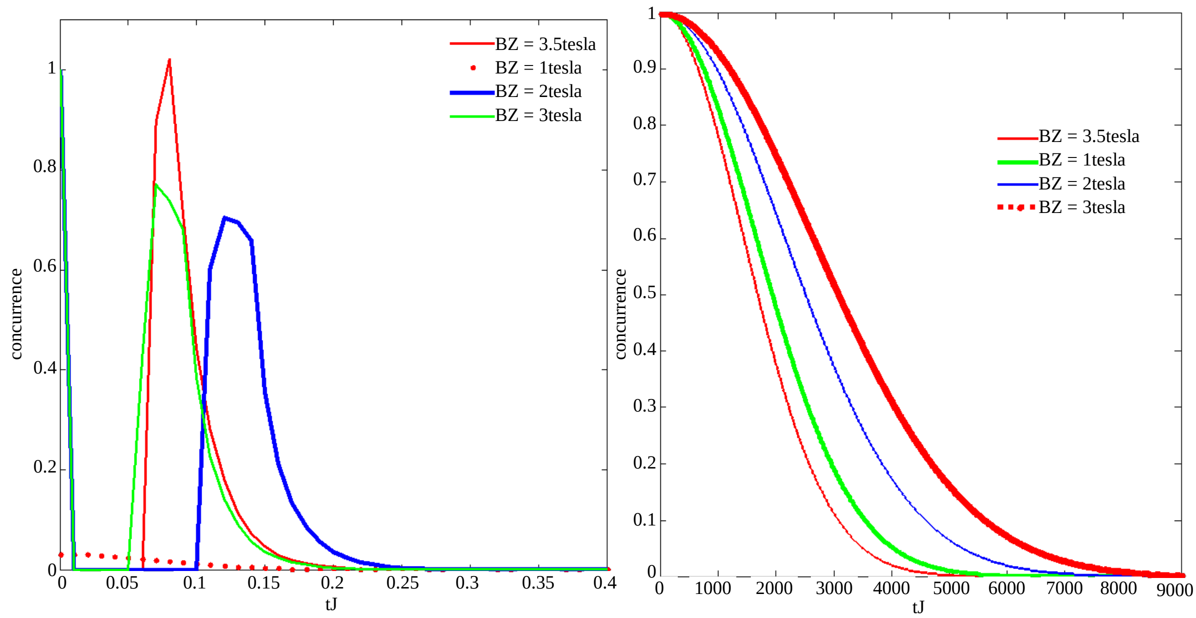
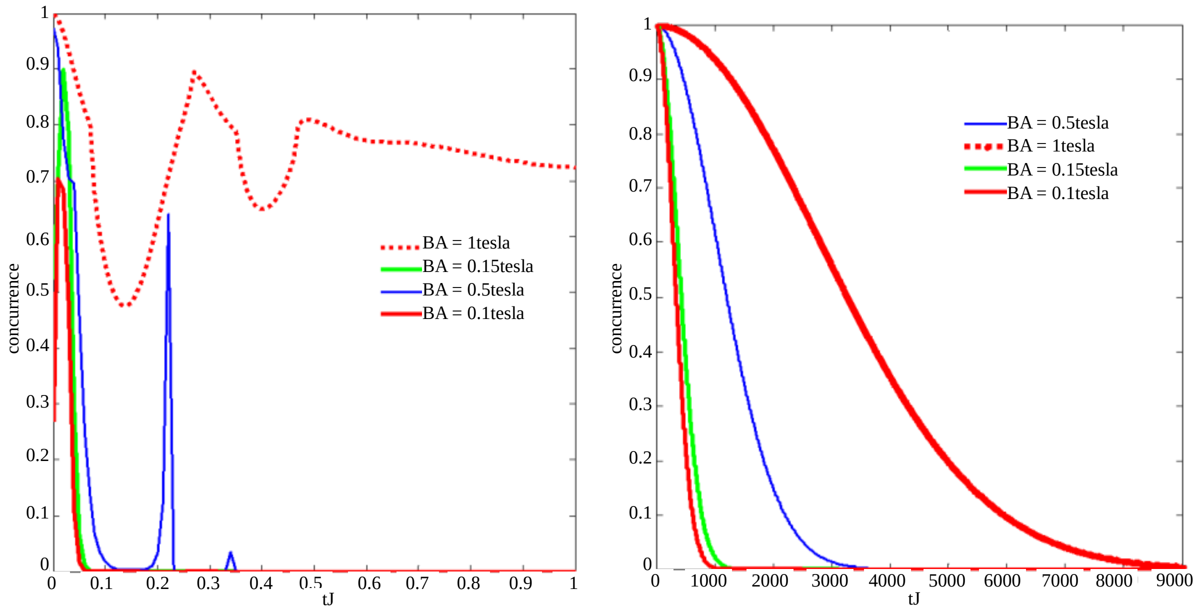


Figure 2. (Color online) The time (tJ) evolution of entanglement for the initial two-qubit state $|\Phi\rangle = (1/\sqrt{2})(\cos\vartheta|00\rangle + \sin\vartheta|11\rangle)$ for different global static external magnetic field (a) finite size of the AF spin environment $N = 100$; $bg = J_O/\sqrt{N} g\mu_B = 1$ tesla is induced magnetic field due to system-environment coupling (b) thermodynamic limit, others parameters are set to $J_Z = 1$, $B_A = 1$ tesla, $\mathrm{MSJ}/g\mu_B = 100$ tesla, $\vartheta = \pi/4$, $T/g\mu_B = 3.5$ tesla.

Entanglement enhancement is seen for a weakly coupled AF spin bath as the anisotropy field increases for the case of the finite size of the system, **Figure 3(a)** and the thermodynamic limit **Figure 3(b)**. In this light, correlations in the anti-ferromagnetic environment are strengthen with increasing value of the anisotropy field; a situation similar to increasing the exchange interaction amongst the environmental spin.

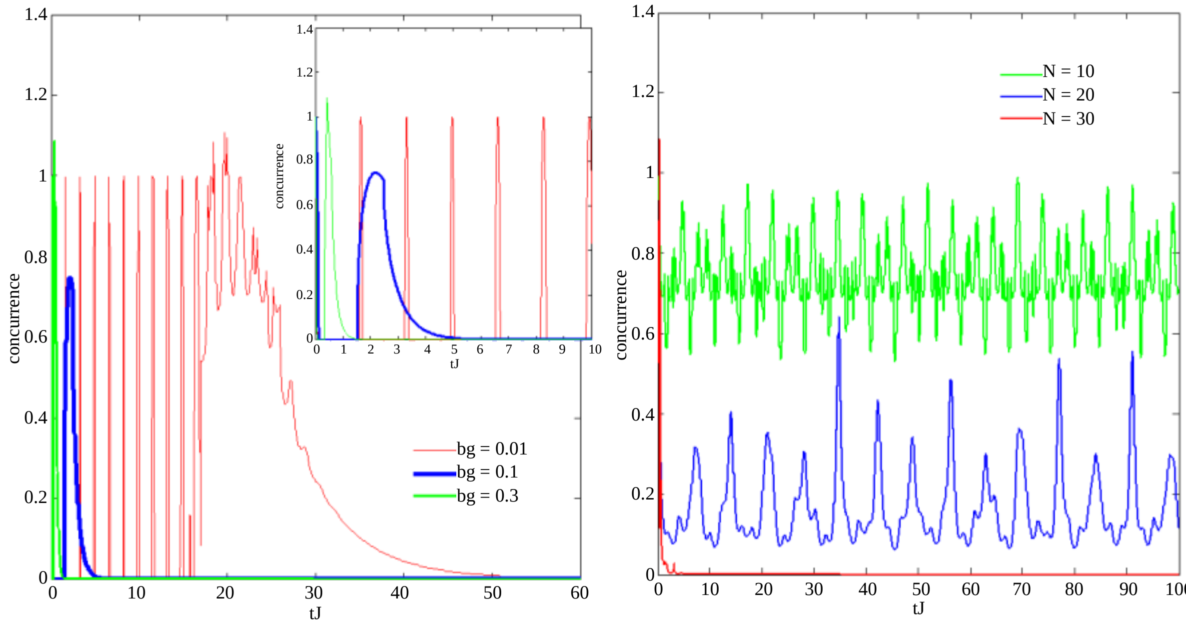
From **Figure 4(a)**, we see that for small values of the induced field strength, memory effects are enhanced as seen from the periodic collapse and revival phenomenon in the entanglement dynamics (see insert for short time dynamics) and only hit zero after a long time interval. Increasing the induced field strength, memory effects are observed only for a short time interval. **Figure 4(b)** show that Entanglement trapping could be achieve for small number of atoms and killing as number of atoms increases. Fluctuations in the entanglement dynamics stems from the strong quantum effects as system size decreases. **Figure 5** shows that increasing the temperature, the sudden death of entanglement is accelerated. As temperature increases, thermally excited fluctuating magnon modes increases and got entangled with the system leading to irreversible flow of information to the AF spin environment.

The zero field oscillatory behavior in the entanglement dynamics **Figure 6(a)** for the initial maximal entangled Bell state $|\Phi\rangle = (1/\sqrt{2})(\cos\vartheta|01\rangle + \sin\vartheta|10\rangle)$ for given central two qubits coupling parameters, is a manifestation of information exchange with feed-back between the computational space and the non-computational spaces driven by the DM interaction. The amplitude of these oscillations increases with reduction in frequency as the value of the DM increases. However further increment in the value of DM interaction drives the entanglement to fall to the stationary state. In the language of quantum computation, this implies tuning the DM parameter to some value, coupling between the computational space and some non computational space could lead to information exchange with little leakages. Turning on, and for increasingly large values of the anisotropy field, the steady state entanglement dynamics becomes enhanced before the birth of periodic oscillations for larger values of the anisotropy field **Figure 6(b)**. The scenario witnessed here is similar to that explained in [6] where suppression of leakage errors due to spin orbit interaction is done by the application of strong magnetic field in a proposed implementation of the quantum gates on the encoded two-spin singlet-triplet qubits. In our scenario, the influence of the AF environment on to this initial state is inherent in the anisotropy field (intrinsic



(a) (b)

Figure 3. (Color online) The time (tJ) evolution of entanglement for the initial two-qubits state $|\Phi\rangle = \left(1/\sqrt{2}\right)\left(\cos\vartheta|00\rangle + \sin\vartheta|11\rangle\right)$ for different anisotropy field; (a) finite size of the AF spin environment $N = 100$; $bg = 1$ tesla, $\text{MSJ}/g\mu_B = 1$ tesla; (b) thermodynamic limit, $\text{MSJ}/g\mu_B = 100$ tesla, others parameters are set to $J_z = 1$, $B_z = 0.5$ tesla, $T/g\mu_B = 3.5$ tesla, $\vartheta = \pi/4$.



(a) (b)

Figure 4. (Color online) The time (tJ) evolution of entanglement for the initial two-qubit state $|\Phi\rangle = \left(1/\sqrt{2}\right)\left(\cos\vartheta|00\rangle + \sin\vartheta|11\rangle\right)$; (a) for different induced field $bg = J_O/\sqrt{N}g\mu_B$ for $N = 100$, (b) different sizes of the AF spin environment N for $bg = 1$ tesla; $N = 10$ (green curve), $N = 20$ (blue curve), $N = 30$ (red curve), others parameters are set to $J_z = 1$, $B_A = 0.15$ tesla, $\text{MSJ}/g\mu_B = 100$ tesla, $T/g\mu_B = 1.7$ tesla, $B_z = 1$ tesla, $\vartheta = \pi/4$.

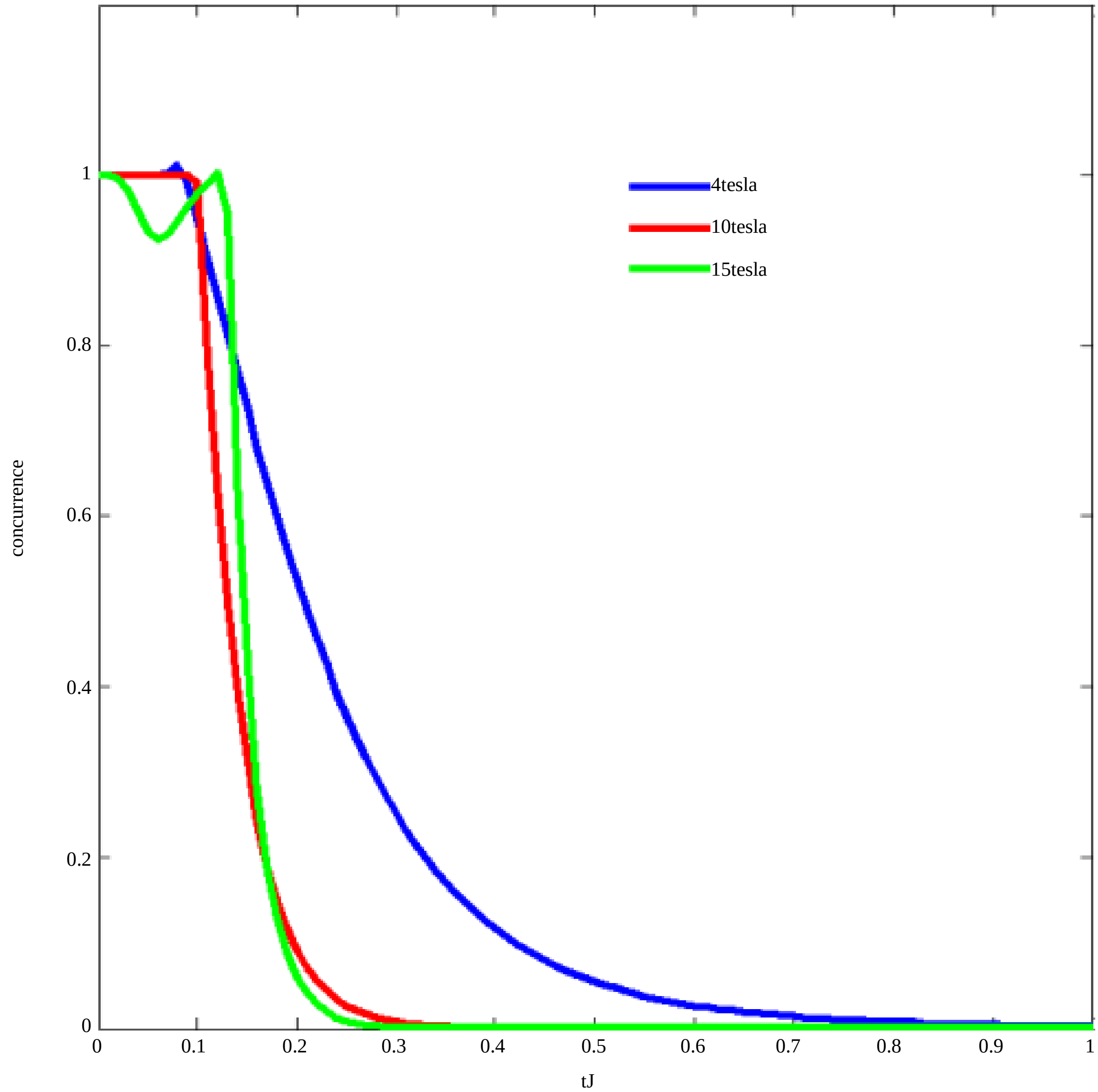
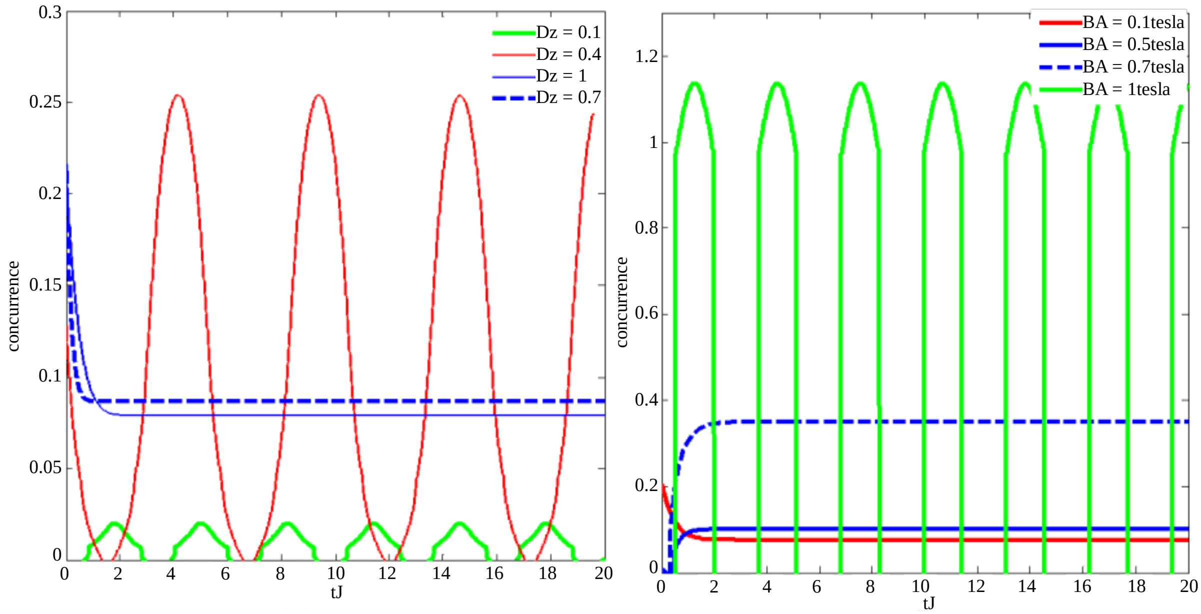


Figure 5. (Color online) The time (tJ) evolution of entanglement for the initial two-qubit state $|\Phi\rangle = (1/\sqrt{2})(\cos\vartheta|00\rangle + \sin\vartheta|11\rangle)$; for different temperatures scaled as $T/g\mu_B$; $T/g\mu_B = 4$ tesla (blue curve), $T/g\mu_B = 10$ tesla (red curve), $T/g\mu_B = 15$ tesla (green curve), others parameters are set to $J_Z = 1$, $B_A = 0.15$ tesla, $\mathrm{MSJ}/g\mu_B = 100$ tesla, $B_Z = 1$ tesla, $bg = 1$ tesla, $\vartheta = \pi/4$.

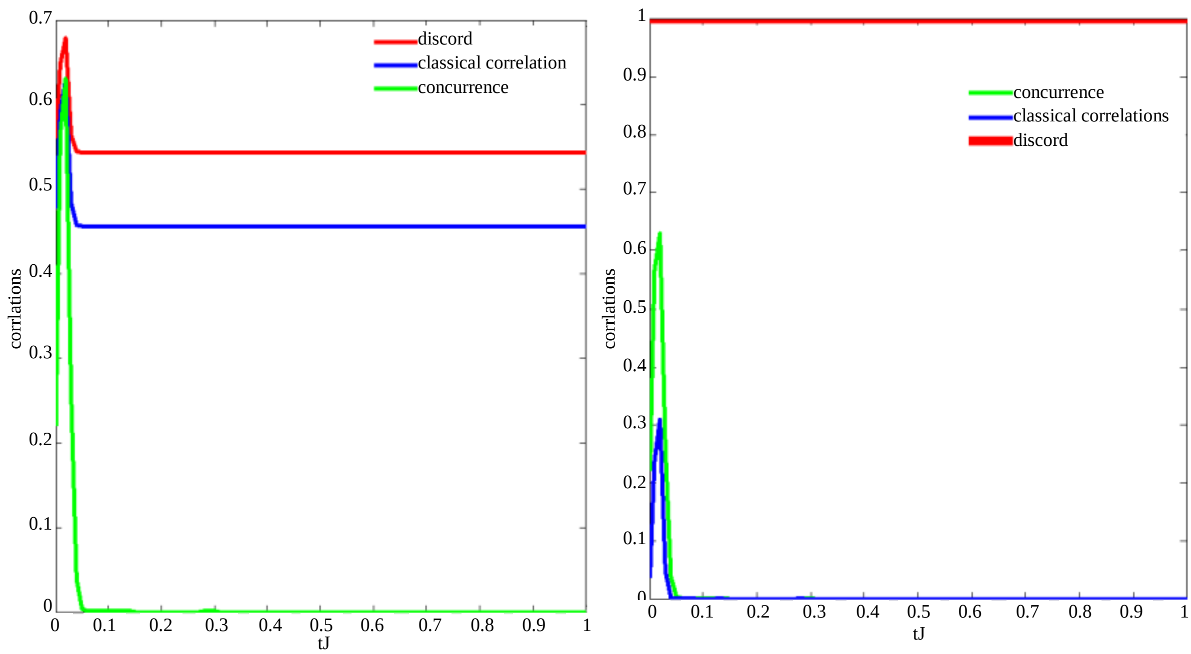
field) as depicted in the Hamiltonian Equation (2), as this initial state does not coupled to the AF environment since the total z-magnetization equal to zero. We may then say here that there is entanglement amplification when the DM interaction drives it to fall to stable entanglement once the initial state is in contact with the AF environment with appropriate anisotropy field. Since this initial state preparation is insensitive to system-environment coupling parameter, the appellation robust is used.

Further investigations are done when studying discord and classical correlation effects. Figure 7 shows the behavior of the time evolution of quantum correlations considering different minimization parameters. It is observed that quantum discord is more resistant to the action of the environment than the quantum entanglement and it can persist even in the asymptotic long time regime as compared to entanglement which hit zero during



(a) (b)

Figure 6. (Color online) The time (tJ) evolution of entanglement for the initial two-qubit entangled state $|\Phi\rangle = \left(1/\sqrt{2}\right)\left(\cos\vartheta|01\rangle + \sin\vartheta|10\rangle\right)$; (a) for different values of D_Z, setting: $B_Z = B_A = 0$ tesla, (b) for different values of the anisotropy field B_A, setting $D_Z = 1$, other parameters are set to $J_Z = 1 = \Omega = 1$, $B_Z = 1$ tesla, $\vartheta = \pi/4$.



(a) (b)

Figure 7. (Color online) The time (tJ) evolution of various correlation for the initial two-qubit entangled state $|\Phi\rangle = \left(\cos\vartheta|00\rangle + \sin\vartheta|11\rangle\right)$; (a) for minimization parameters $l = 1/8$, $k = 7/8$; (b) for minimization parameters $l = k = 1/2$, others parameters are set to $J_Z = 1$, $B_A = 0.1$ tesla, $bg = 1$ tesla, $\mathrm{MSJ}/g\mu_B = 1$ tesla, $T/g\mu_B = 3.5$ tesla, $B_Z = 2$ tesla, $\vartheta = \pi/4$.

this time limit. This is from the fact that, during the dissipative time evolution even without quantum entanglement, the correlations introduced by the environment are transferred to the two central qubits, producing a finite quantum discord. We equally notice from **Figure 7** that discord is larger than classical correlation, a scenario similarly observed in [27]. This results provide the description of the behavior of the classical and quantum correlations and their robustness under the decoherence process. In fact, the quantum discord behavior between the two-qubit depends on the minimization of the classical correlations during the time evolution. We have notice that for all arbitrary system parameters, quantum discord has a steady maximum value "1" once the minimization parameter are set to $l = k = 1/2$ (**Figure 7(a)**) as compared to values outside this setting (**Figure 7(b)**). **Figure 8** finally shows the evolutions of the various correlations in the thermodynamic limit and most importantly is the fact that the correlations approach their maximum values asymptotically.

5. Conclusion

This work investigates the influence of finite temperature AF spin environment, with gap in its energy spectrum, on the dynamics quantum correlations of a central two qubits system interacting via a Heisenberg XXZ chain with Dzyaloshinski-Moriya interaction, prepared in two entangled Bell states; commonly identified as fragile

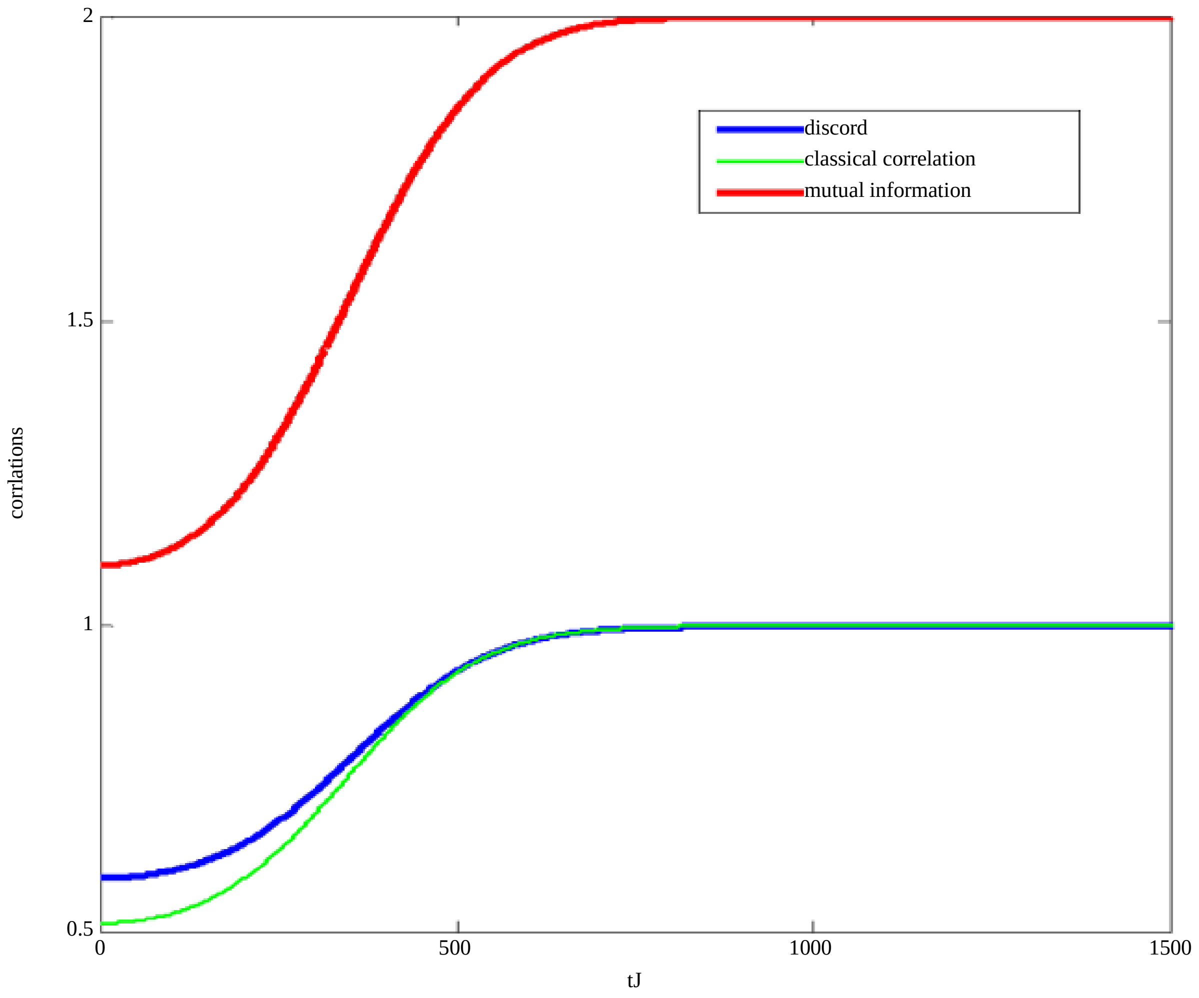


Figure 8. (Color online) The time (tJ) evolution of quantum correlations in the thermodynamic limit : discord (blue curve), classical correlation (green curve), mutual information (red curve) for the initial two-qubit state $|\Phi\rangle = (\cos\vartheta|00\rangle + \sin\vartheta|11\rangle)$; system parameters are set to $J_z = 1$, $B_A = 0.1$ tesla, $\mathrm{MSJ}/g\mu_B = 100$ tesla, $T/g\mu_B = 3.5$ tesla, $B_z = 2$ tesla, $\vartheta = \pi/4$, $l = 1/8$, $k = 7/8$.

and robust. For the fragile Bell state, two opposite scenes are seen to influence the entanglement dynamics as the frequency gap increases via increasing the magnetic field for the finite lattice size and the thermodynamic limit, and are explained in terms of the stability, excitation number difference and fluctuations of the two magnon modes that enter the dynamics of quantum correlations. Scenarios where entanglement trapping could be achieved by tailoring the anisotropy field and size of the AF spin environment are discussed. Endurance of feed-back effects is seen when the induced magnetic field is tune to small values and a general killing for large values. For the robust initial Bell state, while entanglement amplification is witnessed as the DM interaction parameter increases, regime exists where further increment in this parameter drives entanglement to fall to stable state, and can be restored by an appropriate anisotropy field. The robustness of quantum discord is manifested as it never hit zero irrespective of the composite system parameter and attends it maximum value for a rightly chosen minimization parameter. We have also conducted a comparative study of the various correlations in the thermodynamic limit. Our theoretical investigation provides insight on how quantum state initialization and gate operations for quantum computing and information technology using this type of AF spin environment as host, could be tailored by engineering the properties of the environment. We look forward to investigate the influence of the lattice vibration on the dynamics of quantum correlations central two qubits system considering both finite and zero temperature of the environment.

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