

# The Lerner Index and Economic Efficiency\*

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## Abstract

**This short paper brings together two literatures: the first is the Lerner Index as a measure of imperfect competition familiar from industrial organization and the second is a measure of performance from the efficiency literature, namely the Nerlovian indicator. We show how these may be related and the resulting decomposition of the Lerner index which results.**

## Keywords

**Lerner Index, Nerlovian Indicator, Profit Efficiency**

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In this short paper, we establish a relationship between the Lerner index—familiar from the industrial organization literature—and the less well-known Nerlovian indicator of profit efficiency. We prove that the Lerner index is the first order derivative of the Nerlovian indicator.

We also show that the Lerner index can be decomposed into three components: i) cost elasticity, ii) Farrell output technical efficiency (Farrell, 1957) [1] and iii) return to the dollar (Georgescu-Roegen, 1951) [2].

We begin with the Lerner index, which as Lerner put it, is the

“...formula that I wish to put forward as a measure of monopoly power. If  $P$  = price and  $C$  = marginal cost, then the index of the degree of monopoly power is  $\frac{P-C}{P}$ ” Lerner (1934, p. 169) [3].

With respect to this index of monopoly power, Samuelson (1964, p. 173) [4] wrote

“Today this may seem simple, but I can testify that no one at Chicago or Harvard could tell me in 1935 why  $P = MC$  was a good thing, and I was a persistent Diogenes”.

Turning to the Nerlovian indicator (Chambers, Chung and Färe, 1998) [5]; it was introduced as a measure of profit efficiency, specifically it is the normalized difference between maximal profit and observed profit. It is

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dual to the shortage function (Luenberger, 1992) [6], or equivalently to the directional technology distance function (Chambers, Chung and Färe, 1998) [5].

To continue, let  $\Pi(p, w)$  be a profit function, with input prices  $w$  and output prices  $p$ . Denote observed profit by  $\Pi^o$ , the input and output directional vectors by  $g_x$  and  $g_y$ , respectively. The Nerlovian indicator is then defined as

$$N = \frac{\Pi(p, w) - \Pi^o}{pg_y + wg_x} \tag{1}$$

In our case we assume that there is a single output and output price and that the cost minimization problem is solved so that  $C(y, w)$  is a cost function. In addition we take  $g_x = 0$  and  $g_y = 1$ , then the Nerlovian indicator reads as

$$N = \max_y \frac{py - C(y, w) - \Pi^o}{p \cdot 1} \tag{2}$$

The first order condition is

$$\frac{p - \partial C(y, w) / \partial y}{p} = L \tag{3}$$

which is the Lerner Index.

In addition to showing that the Lerner index is the first order condition of the Nerlovian profit indicator we see that the index is normalized by the output price rather than the marginal cost.

Let us recall some basic notions from duality theory, specifically the duality between Shephard's 1970 output distance function and the revenue function. We begin with technology; thus if

$$P(x) = \{y : x \text{ can produce } y\} \tag{4}$$

with  $x \in \mathfrak{R}_+^N$  being the input vector and  $y \geq 0$  the single output, then the output distance function is

$$D_o(x, y) = \inf \{\theta : y/\theta \in P(x)\} \tag{5}$$

and the corresponding revenue function is given by

$$R(x, p) = \max_y \{py : D_o(x, y) \leq 1\} \tag{6}$$

The distance function may be retrieved from the revenue function by optimizing over  $p$ <sup>1</sup>,

$$D_o(x, y) = \sup_p \{py : R(x, p) \leq 1\} \tag{7}$$

From this duality it follows that

$$py = yD_o(x, 1) \cdot R(x, p) \tag{8}$$

and thus

$$p = D_o(x, 1) \cdot R(x, p) \tag{9}$$

Note that  $D_o(x, y) = yD_o(x, 1)$  in the single output case due to homogeneity of the output distance function in  $y$ .

Using (9) we may write the Lerner Index from (3) as

$$\begin{aligned} L &= \frac{p - \partial C(y, w) / \partial y}{p} = 1 - \frac{\partial C(y, w)}{\partial y} \frac{1}{D_o(x, 1) \cdot R(x, p)} \\ &= 1 - \frac{\partial C(y, w)}{\partial y} \frac{yC(y, w)}{yC(y, w)D_o(x, 1) \cdot R(x, p)} \\ &= 1 - \epsilon \cdot \frac{1}{D_o(x, y)} \cdot \frac{1}{R(x, p) / C(y, w)} \end{aligned} \tag{10}$$

where:

<sup>1</sup>See Färe and Primont (1994) for an exposition of Shephard's duality theories.

- $\epsilon = \frac{\partial C(y, w)}{\partial y} \frac{y}{C(y, p)}$  is the cost elasticity
- $\frac{1}{D_o(x, y)}$  is the Farrell output measure of technical efficiency, and
- $R(x, p)/C(y, w)$  is the Georgescu-Roegen “return to the dollar”.

Thus the Lerner Index may be deduced from a measure of profit efficiency—the Nerlovian Indicator—and may be decomposed into three further economic performance measures. We leave generalization to the multiple output case to future research.

## References

- [1] Farrell, M.J. (1957) The Measurement of Productive Efficiency. *Journal of the Royal Statistical Society Series A, General*, **120**, 253-281. <http://dx.doi.org/10.2307/2343100>
- [2] Georgescu-Roegen, N. (1951) The Aggregate Linear Production Function and Its Application to the Von Neumann Economic Model. In: Koopmans, T., Ed., *Activity Analysis of Production and Allocation*, Wiley, New York.
- [3] Lerner, A.P. (1934) The Concept of Monopoly and the Measurement of Monopoly Power. *The Review of Economic Studies*, **1**, 157-175. <http://dx.doi.org/10.2307/2967480>
- [4] Samuelson, P.A. (1964) A.P. Lerner at Sixty. *The Review of Economic Studies*, **31**, 169-178. <http://dx.doi.org/10.2307/2295906>
- [5] Chambers, R.G., Chung, Y. and Färe, R. (1998) Profit, Directional Distance Functions, and Nerlovian Efficiency. *Journal of Optimization Theory and Applications*, **98**, 351-364. <http://dx.doi.org/10.1023/A:1022637501082>
- [6] Luenberger, D.G. (1992) New Optimality Principles for Economic Efficiency and Equilibrium. *Journal of Optimization Theory and Applications*, **75**, 221-264. <http://dx.doi.org/10.1007/BF00941466>

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