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Scientific Research

Expansion of U_{PMNS} and Neutrino Mass Matrix M_ν in Terms of $\sin\theta_{13}$ for Inverted Hierarchical Case

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ABSTRACT

The recent observational data supports the deviation from Tri-bimaximal (TBM) mixing. Different theories suggest the interdependency among the observational parameters involving the mixing angles. On phenomenological ground, we try to construct the PMNS matrix, U_{PMNS} with certain analytic structure satisfying spontaneously the unitary condition, in terms of a single observational parameter $\sin\theta_{13}$. We hypothesise the three neutrino masses, m_i as functions of $\sin\theta_{13}$ and then construct the neutrino mass matrix M_ν with certain exact and expandable form. We assume the convergence of the model to TBM mixing when θ_{13} is taken 0. The mass matrix so far obtained can be employed for various applications including the estimation of matter-antimatter asymmetry of the universe.

Keywords: Inverted Hierarchy; Tri-Bimaximal Mixing; Neutrino Masses; μ-τ Symmetry

1. Introduction

Recent results published by Double Chooz [1], Daya Bay [2], RENO [3], T2K [4] and MINOS [5] collaborations assure relatively large reactor angle (θ_{13}). Also recent global neutrino data analysis [6] insists on $\theta_{23} < \pi/4$. Tri-bimaximal mixing [7] is associated with $\theta_{13} = 0$, and $\theta_{23} = \pi/4$. This symmetry has a strong theoretical support because of its relation with so called $\mu - \tau$ symmetry of neutrino mass matrix. $\mu - \tau$ symmetry, in turn is associated with A_4 [8-12], one of the candidates of discrete flavour symmetry groups. But in order to comply with the recent experimental results, some perturbations have to be introduced in this mixing pattern. Whether the corrections [13,14] are needed or a new mixing scheme is to be introduced, is still an open question [15].

Literature [16,17] shows the dependency of the mixing angles on one another. If this is true, then we are allowed to choose a single parameter capable of describing all the three mixing angles.

We move a step ahead and express the three masses under this parameter. This helps us to define a simplified neutrino mass model with a single parameter only.

Out of all the three observational parameters concerning the mixing angles $\sin\theta_{13}$ is the smallest one. So, we choose $\sin\theta_{13}$ as the guiding parameter. We consider tri-bimaximal mixing pattern and μ-τ symmetry as the first approximation. Hence the model is supposed to produce T. B. M mixing when we put $\sin\theta_{13} = 0$. We try to keep the structure of the three rotation matrices $U(\theta_{13})$, $U(\theta_{12})$ and $U(\theta_{23})$ in analytical form so that they can satisfy the unitary condition $\left[U(\theta_{ij})\right]^\dagger U(\theta_{ij}) = I$, without any prior approximation.

We start with the following ansatz,

$$s_{13} = \epsilon\,, \tag{1}$$

$$s_{12} = \frac{1}{\sqrt{3}} - \frac{\epsilon}{5}, \tag{2}$$

$$s_{23} = \frac{1}{\sqrt{2}} - \frac{\epsilon}{2}\,. \tag{3}$$

where, $s_{ij} = \sin\theta_{ij}$, and then construct the PMNS mixing matrix and then the neutrino mass matrix in the usual way.

2. Construction of the PMNS Matrix

We consider the charged lepton mass matrix to be diagonal. Hence we can choose $U_{\text{PMNS}} = U_\nu$. We propose the three rotation matrices as:

$$U(\theta_{13}) = \begin{pmatrix} (1-\epsilon^2)^{\frac{1}{2}} & 0 & \epsilon e^{-i\delta} \\ 0 & 1 & 0 \\ -\epsilon e^{-i\delta} & 0 & (1-\epsilon^2)^{\frac{1}{2}} \end{pmatrix}, \quad (4)$$

$$U(\theta_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \left(\frac{1}{2}+\frac{\epsilon}{\sqrt{2}}-\frac{\epsilon^2}{4}\right)^{\frac{1}{2}} & \frac{\epsilon}{2}-\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}}-\frac{\epsilon}{2} & \left(\frac{1}{2}+\frac{\epsilon}{\sqrt{2}}-\frac{\epsilon^2}{4}\right)^{\frac{1}{2}} \end{pmatrix}, \quad (5)$$

$$U(\theta_{12}) = \begin{pmatrix} \left(\frac{2}{3}+\frac{2\epsilon}{5\sqrt{3}}-\frac{\epsilon^2}{25}\right)^{\frac{1}{2}} & \frac{\epsilon}{5}-\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}}-\frac{\epsilon}{5} & \left(\frac{2}{3}+\frac{2\epsilon}{5\sqrt{3}}-\frac{\epsilon^2}{25}\right)^{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

We have,

$$U_{\text{PMNS}} = U(\theta_{23})U(\theta_{13})U(\theta_{12}) = \begin{pmatrix} u_{e1} & u_{e2} & u_{e3} \\ u_{\mu1} & u_{\mu2} & u_{\mu3} \\ u_{\tau1} & u_{\tau2} & u_{\tau3} \end{pmatrix} \quad (7)$$

where,

$$u_{e1} = (1-\epsilon^2)^{\frac{1}{2}} a(\epsilon),$$

$$u_{e2} = \left(\frac{\epsilon}{5}-\frac{1}{\sqrt{3}}\right)(1-\epsilon^2)^{\frac{1}{2}},$$

$$u_{e3} = \epsilon e^{-i\delta},$$

$$u_{\mu1} = \frac{1}{30}\left(5\sqrt{3}-3\epsilon\right)b(\epsilon) + \epsilon\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)a(\epsilon)e^{i\delta},$$

$$u_{\mu2} = \frac{1}{30}\left\{c(\epsilon)b(\epsilon) + \epsilon\left(\sqrt{2}-\epsilon\right)\left(3\epsilon-5\sqrt{3}\right)e^{i\delta}\right\},$$

$$u_{\mu3} = \frac{1}{2}\left(\epsilon-\sqrt{2}\right)(1-\epsilon^2)^{\frac{1}{2}},$$

$$u_{\tau1} = \left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right) - \frac{1}{10}\epsilon b(\epsilon)d(\epsilon)e^{i\delta},$$

$$u_{\tau2} = \left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)a(\epsilon) - \frac{1}{30}\epsilon\left(3\epsilon-5\sqrt{3}\right)b(\epsilon)e^{i\delta},$$

$$u_{\tau3} = \frac{1}{2}(1-\epsilon^2)b(\epsilon),$$

and,

$$a(\epsilon) = \left(\frac{1}{3}+\frac{2\epsilon}{5\sqrt{3}}-\frac{\epsilon^2}{25}\right)^{\frac{1}{2}},$$

$$b(\epsilon) = \left(2-\epsilon^2+2\sqrt{2}\epsilon\right)^{\frac{1}{2}},$$

$$c(\epsilon) = \left(150+30\sqrt{\epsilon}-9\epsilon^2\right)^{\frac{1}{2}},$$

$$d(\epsilon) = \left(\frac{50}{3}+\frac{10\epsilon}{\sqrt{3}}-\epsilon^2\right)^{\frac{1}{2}}.$$

It can be checked that,

$$\left[U(\theta_{ij})\right]^{\dagger} U(\theta_{ij}) = \left[U_{\text{PMNS}}\right]^{\dagger} U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

And we get,

$$\tan^2\theta_{12} = \left|\frac{u_{e2}}{u_{e1}}\right|^2 = \frac{25-10\sqrt{3}\epsilon+3\epsilon^2}{50+10\sqrt{3}\epsilon-3\epsilon^2} \quad (9)$$

$$\tan^2\theta_{12} = \left|\frac{u_{e2}}{u_{e1}}\right|^2 = \frac{25-10\sqrt{3}\epsilon+3\epsilon^2}{50+10\sqrt{3}\epsilon-3\epsilon^2} \quad (10)$$

After interpreting the above two relations in terms of $\sin\theta_{13}$, we have,

$$\tan^2\theta_{12} = \frac{1}{2} - \frac{1}{2}\sin\theta_{13} + \frac{1}{4}\sin^2\theta_{13} - \frac{2}{25}\sin^3\theta_{13}, \quad (11)$$

$$\tan^2\theta_{23} = 1 - \frac{13}{5}\sin\theta_{13} + 2\sin^2\theta_{13} - 2\sin^3\theta_{13}. \quad (12)$$

U_{PMNS} for $\epsilon = 0$ and $\epsilon = 0.156$ are shown below,

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix},$$

$$U = \begin{pmatrix} 0.8274 & -0.5395 & 0.156e^{-i\delta} \\ 0.4245+0.0822e^{i\delta} & 0.6511-0.0536e^{i\delta} & -0.6214 \\ 0.3435-0.1015e^{i\delta} & 0.5270+0.0662e^{i\delta} & 0.7678 \end{pmatrix}.$$

This is clear from the above analysis that $\tan^2\theta_{12} = 0.5$ and $\tan^2\theta_{23} = 1$, if $\sin\theta_{13} = \epsilon = 0$ (TBM mixing). At $\epsilon = 0.155$ (N. H), 0.156 (I. H) (the best-fit value of $\sin\theta_{13}$) [6], we get $\tan^2\theta_{12} = 0.425$ and $\tan^2\theta_{23} = 0.657, 0.654$, which are very close to the best fit results [6]: $\tan^2\theta_{12}$ = 0.443 (N. H or I. H) and $\tan^2\theta_{23} = 0.628$ (N. H) and 0.644 (I. H). This is shown in **Figure 1**, where the variations of $\tan^2\theta_{12}$ and $\tan^2\theta_{23}$ are plotted against $\sin\theta_{13}$.

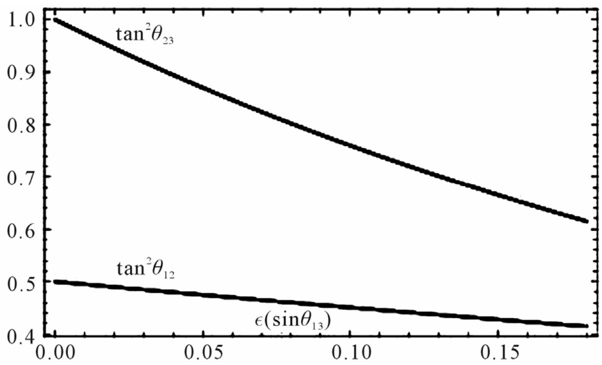

Figure 1. Variation of $\tan^2\theta_{12}$ and $\tan^2\theta_{23}$ with ϵ.

3. Jarkslog Parameter (J_{cp})

We introduce the CP phase δ in U_{13} as shown in Equation (8). The inclusion of δ_{cp} does not affect $\tan^2\theta_{12}$ or $\tan^2\theta_{23}$ (Equations (9) and (10)).

We obtain the J_{cp} as,

$$J_{cp} = \mathrm{Im}\left[u_{e1}^* u_{\mu 1}^* u_{e3} u_{\mu 1}\right] = \epsilon\left(1-\epsilon^2\right)\left(\frac{1}{\sqrt{2}} - \frac{\epsilon}{2}\right)\left(\frac{1}{\sqrt{3}} - \frac{\epsilon}{5}\right) \cdot \left(\frac{1}{2} + \frac{\epsilon}{\sqrt{2}} - \frac{\epsilon^2}{4}\right)^{\frac{1}{2}}\left(\frac{2}{3} + \frac{2\epsilon}{5\sqrt{3}} - \frac{\epsilon^2}{25}\right)^{\frac{1}{2}} \sin\delta \tag{13}$$

Maximum J_{cp}, *i.e.*, $J_{\max}$ is obtained for $\delta = \pi/2$. For, $\epsilon = 0.156$, $J_{\max}$ is obtained as 0.0341. The variation of $J_{\max}$ with respect to $\sin\theta_{13}$ is shown in **Figure 2**. Also the variation of J_{cp} with δ (with ϵ or $\sin\theta_{13}$ fixed at 0.156), is plotted in **Figure 3**.

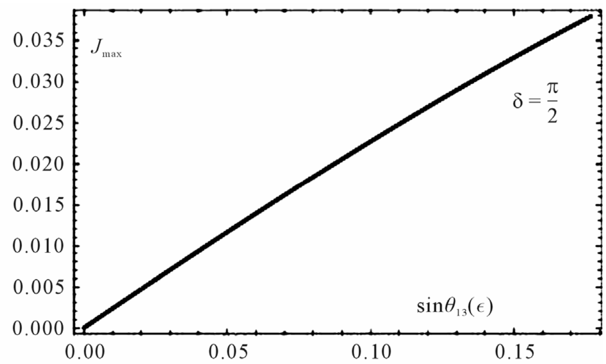

Figure 2. Variation of $J_{\max}$ with ϵ.

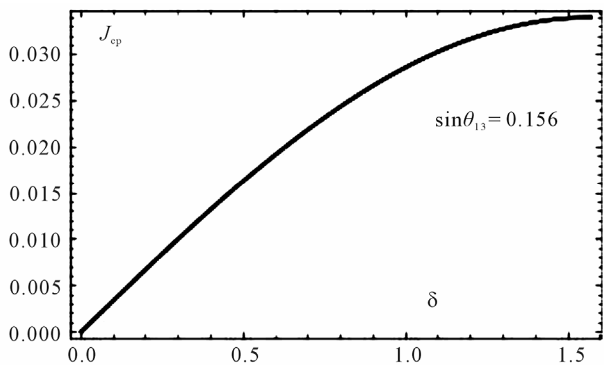

Figure 3. Variation of J_{cp} with δ.

4. Generation of Neutrino Mass Matrix with Inverted Hierarchy

We now apply the PMNS mixing matrix from Equation (7), to construct the neutrino mass matrix with inverted hierarchy (I. H). We try to interpret the masses in terms of the same parameter $\sin\theta_{13} = \epsilon$.

We choose on phenomenological ground the absolute values of three neutrino masses in units of eV as,

$$m_1 = \frac{60}{1250} + \frac{7}{8}\epsilon^4, \tag{14}$$

$$m_2 = \frac{61}{1250} + \frac{7}{8}\epsilon^4 - \frac{\epsilon^5}{3}, \tag{15}$$

$$m_3 = \frac{7}{8}\epsilon^4. \tag{16}$$

leading to,

$$\Delta m_{21}^2 = \frac{121}{1562500} + \frac{7\epsilon^4}{5000} - \frac{61}{1875}\epsilon^5 - \frac{7}{12}\epsilon^9 + \frac{1}{9}\epsilon^{10}, \tag{17}$$

$$\Delta m_{23}^2 = \frac{3721}{1562500} + \frac{427\epsilon^4}{5000} - \frac{61}{1875}\epsilon^5 - \frac{7}{12}\epsilon^9 + \frac{1}{9}\epsilon^{10}. \tag{18}$$

The variations of Δm_{21}^2 and Δm_{23}^2 with ϵ are shown in **Figures 4** and **5**. At TBM mixing condition, *i.e.*, at $\epsilon = 0$, we get, $\Delta m_{21}^2 = 7.74\times10^{-5}\ \mathrm{eV}^2$, $\Delta m_{23}^2 = 2.38\times10^{-3}\ \mathrm{eV}^2$, and $\Delta m_{21}^2 = 7.53\times10^{-5}\ \mathrm{eV}^2$, $\Delta m_{23}^2 = 2.43\times10^{-3}\,\mathrm{eV}^2$ are obtained at $\epsilon = 0.156$.

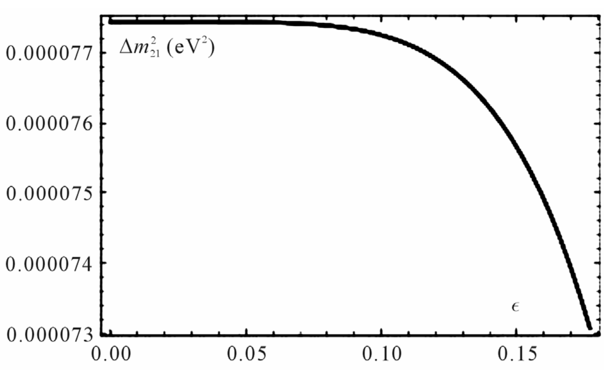

Figure 4. Variation of Δm_{21}^2 with ϵ.

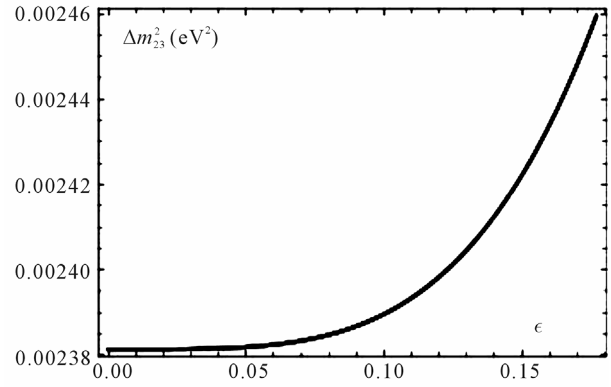

Figure 5. Variation of Δm_{23}^2 with ϵ.

For simplicity in the texture of the neutrino mass matrix, we avoid the inclusion of δ_{cp}. Using Equation (7) for U_{PMNS} (with $\delta_{cp}=0$) and Equations (14)-(16) for m_i, we construct the neutrino mass matrix M_ν as follows,

$$M_\nu = U_{\text{PMNS}}^{\text{T}}\cdot\begin{pmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3\end{pmatrix}\cdot U_{\text{PMNS}} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33}\end{pmatrix}. \tag{19}$$

where,

$$m_{11} = \frac{7}{8}\epsilon^6 + A^2(\epsilon)\left(1-\epsilon^2\right)\left(\frac{6}{125}+\frac{7}{8}\epsilon^4\right) - B(\epsilon)\left(1-\epsilon^2\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)^2,$$

$$m_{12} = -\frac{7}{8}\epsilon^5\left(1-\epsilon^2\right)^{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right) + A(\epsilon)\left(1-\epsilon^2\right)^{\frac{1}{2}}\left(\frac{6}{125}+\frac{7}{8}\epsilon^4\right)\left\{C(\epsilon)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+A(\epsilon)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\right\}$$
$$+B(\epsilon)\left(1-\epsilon^2\right)^{\frac{1}{2}}\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)\left\{\epsilon\left(\frac{\epsilon}{2}-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+C(\epsilon)A(\epsilon)\right\},$$

$$m_{13} = \frac{7}{8}C(\epsilon)\epsilon^5\left(1-\epsilon^2\right)^{\frac{1}{2}} + B(\epsilon)\left(1-\epsilon^2\right)^{\frac{1}{2}}\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)\left\{\epsilon C(\epsilon)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+A(\epsilon)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\right\}$$
$$+A(\epsilon)\left(1-\epsilon^2\right)^{\frac{1}{2}}\left(\frac{6}{125}+\frac{7\epsilon^4}{8}\right)\left\{\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)-\epsilon A(\epsilon)C(\epsilon)\right\},$$

$$m_{22} = \frac{7}{8}\epsilon^4\left(1-\epsilon^2\right)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)^2 + \left(\frac{6}{125}+\frac{7\epsilon^4}{8}\right)\left\{C(\epsilon)\left(\frac{1}{\sqrt{3}}-\frac{2}{5}\right)+\epsilon A(\epsilon)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\right\}^2$$
$$-B(\epsilon)\left\{\epsilon\left(\frac{\epsilon}{2}-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+A(\epsilon)C(\epsilon)\right\}^2,$$

$$m_{23} = -\frac{7\epsilon^4}{8}C(\epsilon)\left(1-\epsilon^2\right)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)$$
$$-B(\epsilon)\left\{\epsilon C(\epsilon)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+A(\epsilon)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\right\}\left\{\epsilon(\frac{\epsilon}{2}-\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+C(\epsilon)A(\epsilon)\right\}$$
$$+\epsilon A(\epsilon)\left(\frac{6}{125}+\frac{7\epsilon^4}{8}\right)\left\{C(\epsilon)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\right\}\left\{\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)-\epsilon C(\epsilon)A(\epsilon)\right\},$$

$$m_{33} = -\frac{7\epsilon^4}{8}C(\epsilon)\left(1-\epsilon^2\right) - B(\epsilon)\left\{\epsilon C(\epsilon)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)+A(\epsilon)\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\right\}^2$$
$$+\left(\frac{6}{125}+7\frac{\epsilon^4}{8}\right)\left\{\left(\frac{1}{\sqrt{2}}-\frac{\epsilon}{2}\right)\left(\frac{1}{\sqrt{3}}-\frac{\epsilon}{5}\right)-\epsilon C(\epsilon)A(\epsilon)\right\}^2.$$

and,

$$A(\epsilon)=\left(\frac{2}{3}+\frac{2\epsilon}{5\sqrt{3}}-\frac{\epsilon^2}{25}\right)^{\frac{1}{2}},$$

$$B(\epsilon)=\left(\frac{61}{1250}+\frac{7\epsilon^4}{8}-\frac{\epsilon^5}{3}\right)^{\frac{1}{2}},$$

$$C(\epsilon)=\left(\frac{1}{2}+\frac{\epsilon}{\sqrt{2}}-\frac{\epsilon^2}{4}\right)^{\frac{1}{2}}.$$

At, $\epsilon=0$ (T. B. M mixing), Equation (22) reduces to μ-τ symmetric mass matrix form,

$$M_{\mu\tau}=\begin{pmatrix}\delta_1 & 1 & 1\\ 1 & \delta_2 & \delta_2\\ 1 & \delta_2 & \delta_2\end{pmatrix}m_0, \tag{20}$$

with $\delta_{1,2}\ll 1$, for inverted hierarchy. Equation (19) leads to

$$M_\nu=M_{\mu\tau}=\frac{121}{3750}\begin{pmatrix}\frac{1}{2} & 1 & 1\\ 1 & -\frac{1}{4} & -\frac{1}{4}\\ 1 & -\frac{1}{4} & -\frac{1}{4}\end{pmatrix}$$
$$=\begin{pmatrix}0.0157 & 0.0323 & 0.0323\\ 0.0323 & -0.0083 & -0.0083\\ 0.0323 & -0.0083 & -0.0083\end{pmatrix},$$

$$m_1=\frac{60}{1250},\ m_2=-\frac{61}{1250},\ m_3=0 \text{ in eV} \tag{21}$$

At $\epsilon=0.156$, Equation (22) leads to

$$M_\nu=M_{\mu\tau}+\Delta M_\nu$$
$$=\begin{pmatrix}0.0189 & 0.0362 & 0.0255\\ 0.0362 & -0.0049 & -0.0118\\ 0.0255 & -0.0118 & -0.0142\end{pmatrix}. \tag{22}$$

where,

$$\Delta M_\nu=\begin{pmatrix}0.0032 & 0.0039 & -0.0068\\ 0.0039 & 0.0034 & -0.0035\\ -0.0068 & -0.0035 & -0.0059\end{pmatrix}$$

$m_1=0.0485, m_2=-0.0493, m_3=0.0005$ in eV.

5. Summary

We have started with a parameter ϵ equating this to $\sin\theta_{13}$ and construct the PMNS matrix, U_{PMNS}. Then we represent the neutrino masses $(m_{i=1,2,3})$ in terms of the same parameter $\sin\theta_{13}$, *i.e.* ϵ. We verify our hypothesis by comparing the ranges of the mass squared differences as a result of our ansatz with the 1σ range, experimentally obtained. We take the range of ϵ as the experimental 1σ range of $\sin\theta_{13}$ [6]. We obtain the range of Δm^2_{21} and Δm^2_{23} as $(7.46 - 7.58)\times10^{-5}\ \text{eV}^2$ and $(2.42 - 2.44)\times10^{-3}\ \text{eV}^2$ respectively. The respective ranges obtained, lie within the experimental 1σ boundary [6]. This provides a support to our hypothesis m_i as $m_i(\epsilon)$. This is to be emphasised that the U_{PMNS} matrix as proposed in Equation (7) satisfies the unitary condition and is not dependent on the choice of the order of ϵ. The introduction of δ_{cp} does not affect $\tan^2\theta_{12}$ and $\tan^2\theta_{23}$ in our calculation. The maximum J_{cp} obtained is 0.034 (with respect to $\epsilon=\sin\theta_{13}=0.156$). Finally we concentrate on the construction of M_ν, the neutrino mass matrix. The present investigation though phenomenological, gives a complete picture of the texture of the neutrino mass matrix which can be employed in other applications regarding baryon asymmetry of the universe [18]. Although we have constructed the mass matrix for inverted hierarchical model, yet we can extend our technique to Normal as well as Quasidegenerate mass models.

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