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Magnetic Monopoles

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Abstract

Two of Maxwell's equations of electrodynamics are: $\nabla \cdot \boldsymbol{E} = 4\pi\rho_e$ and $\nabla \cdot \boldsymbol{B} = 0$, where $\boldsymbol{E}$, $\boldsymbol{B}$ and ρ_e are electric field, magnetic field, and electric charge density respectively. A fundamental question that the physics community is perplexed with since the 19C is this: Why the second of these equations is not $\nabla \cdot \boldsymbol{B} = 4\pi\rho_m$, where ρ_m is the magnetic charge density? Put in a slightly different way, it is an empirical fact of nature that magnets have two poles, namely, north and south poles. Why is it that objects with a single north or south pole do not appear to exist? No one has ever observed an isolated excess of one kind of magnetic charge—an isolated north pole, for example! Further, there does not exist any theoretical explanation why magnetic charges do not exist. The only conclusion that can be drawn from the more than one hundred and fifty years of fruitless search is that ordinary matter consists of electric charges (electric monopoles) and not magnetic charges (magnetic monopoles)! In this paper, we disprove this conclusion by showing that magnetic monopoles exist even though we cannot isolate them.

Keywords

Electromagnetism, Monopoles, Pseudo-Euclidean, Quantization

1. Introduction

It is an empirical fact that the magnetic field outside a magnetized rod looks like the electric field outside a rod that has an excess of positive charge at one end and negative charge at the other end of the rod. The implication of this analogy is that one end of the magnetized rod is the location of the excess of one kind of magnetic charge and the other end the location of the excess of the opposite kind. Continuing this analogy, we can call one end of the magnetized rod positive magnetic charge (north pole) and the other end negative magnetic charge (south pole), with the magnetic field directed from positive magnetic charge and terminating at the negative magnetic charge.

With magnetic charge as a possible source of the static magnetic field $\boldsymbol{B}$, one would have $\nabla\cdot\boldsymbol{B}=4\pi\rho_m$, where ρ_{m} stands for magnetic charge density, in complete analogy to the electric case where $\nabla\cdot\boldsymbol{E}=4\pi\rho_e$ where $\boldsymbol{E}$ and ρ_{e} are the electric field and electric charge density respectively. It is an empirical fact of nature also that isolated excess of magnetic charge does not exist; consequently it must follow that $\nabla\cdot\boldsymbol{B}=0$, everywhere!

As has been noted, magnets have two poles, north and south poles, why is it that objects with a single north or south pole, called magnetic monopoles, do not exist? That's a mystery that has occupied the attention of some of the great minds in physics since the 19C [1]. Numerous ingenious experiments have been designed to detect the magnetic monopole, sadly none of these experiments has produced acceptable result. For example, there was the speculation that pairs of magnetic monopoles might be created and fly apart in ultra-high energy accelerators. Several searches for such particles have detected none. On the theoretical platform, magnetic monopoles are predicted to exist in some gauge theories. Magnetic monopoles are also predicted to exist in some unified theories, Kaluza-Klein theories and superstring theory [2]. Whether monopoles cannot exist, and if so why not, remains an open question. In this paper, we prove that magnetic monopoles exist, but we cannot isolate them.

2. Classical Electrodynamics (CED)

Classical electrodynamics of J.C. Maxwell is the theory that describes the interaction of electromagnetism with charged particles in the framework of classical mechanics. We reproduce Maxwell's equations here for ease of reference [3],

$$\begin{aligned}&\nabla\cdot\boldsymbol{E}=4\pi\rho\,,\\&\nabla\times\boldsymbol{E}+\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}=0\,,\\&\nabla\cdot\boldsymbol{B}=0\,,\\&\nabla\times\boldsymbol{B}=\frac{4\pi}{c}\boldsymbol{J}+\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}\,.\end{aligned}\tag{1}$$

where,

$$\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\varphi\tag{2}$$

$\boldsymbol{B}=\nabla\times\boldsymbol{A}$.

In combination with the Lorentz force equation and Newton's second law of motion, these equations provide a complete description of the classical dynamics of interacting charged particles and electromagnetic fields.

It is usually claimed that Maxwell's equations, in the absence of sources, are symmetrical in $\boldsymbol{E}$ and $\boldsymbol{B}$, but asymmetrical when the sources are present. It must be noted, however, that Equation (1) have no physical meaning except when subject to the constraint Equation (2). When there are no sources both $\boldsymbol{E}$ and $\boldsymbol{B}$ vanish, consequently Maxwell's equations are totally asymmetrical in $\boldsymbol{E}$ and $\boldsymbol{B}$.

3. Quantum Electrodynamics (QED)

Quantum electrodynamics is the theory that describes the electromagnetic fields interaction with matter in the framework of quantum mechanics. It follows that CED is the classical analogue of QED. There is a straightforward procedure for obtaining classical dynamical equations from its analogous quantum equations and vice-versa:

$$\boldsymbol{p} = \frac{\hbar}{\mathrm{i}}\nabla, \quad p^0 = \mathrm{i}\hbar\frac{\partial}{\partial x^0} \tag{3}$$

It follows from Equations (1), (2), and (3) that in the quantum theory we must have,

$$\begin{aligned}
&\mathrm{i}\left(\boldsymbol{p}/\hbar\right)\cdot\boldsymbol{E} = 4\pi\rho,\\
&\mathrm{i}\left(\boldsymbol{p}/\hbar\right)\times\boldsymbol{E} - \mathrm{i}\left(p^o/\hbar\right)\boldsymbol{B} = 0,\\
&\mathrm{i}\left(\boldsymbol{p}/\hbar\right)\cdot\boldsymbol{B} = 0,\\
&\mathrm{i}\left(\boldsymbol{p}/\hbar\right)\times\boldsymbol{B} + \mathrm{i}\left(p^o/\hbar\right)\boldsymbol{E} = 4\pi\boldsymbol{J}
\end{aligned} \tag{1'}$$

and

$$\begin{aligned}
&\boldsymbol{E} = \mathrm{i}\left(p^o/\hbar\right)\boldsymbol{A} - \mathrm{i}\left(\boldsymbol{p}/\hbar\right)\varphi\\
&\boldsymbol{B} = \mathrm{i}\left(\boldsymbol{p}/\hbar\right)\times\boldsymbol{A}\,.
\end{aligned} \tag{2'}$$

Unlike equations (1) and (2), (1') and (2') are expressed entirely in terms of 4-operators whose components are observables; $p = \left(p^o, \boldsymbol{p}\right)$ and $A = \left(\varphi, \boldsymbol{A}\right)$, where $\varphi = A^0$. These operators satisfy the electromagnetic 4-operator constraints [4],

$$p\cdot p = 0,\quad p\cdot A = 0,\ A\cdot A = 0\,. \tag{4}$$

The second of Equation (4), $p\cdot A = 0$, gives the Lorentz (Coulomb) gauge condition of classical electrodynamics—a purely arbitrary condition there. Thus, the concept of gauge in classical electrodynamics is not fundamental; it is therefore not something to be exploited for the description of nature. Further, the geometry of the 4-operator background is pseudo-euclidean as it should be for a relativistic phenomenon in contradistinction to the Maxwell variables which are euclidean.

Equations (1') and (2') are the equations to be employed for the description of electromagnetic interactions with matter in the framework of quantum mechanics. These equations are completely new in the annals of the theories of the fundamental processes; they have nothing in common with what is usually referred to as quantum electrodynamics in conventional physics theories.

4. The Magnetic Charge

We now use the second of Equation (2') to solve the magnetic monopole problem. Just as the electric charge is a property of the electron, magnetic charge is also a property of the electron.

We focus on the free electrons of the bounded material medium, and let

$|e,\ a'\rangle$ denote the representative state ket of the medium, where e, the electronic charge, is the eigenvalue of each electronic operator (Q_e) and a' the totality of the eigenvalues of the operators of the system that commute with Q_e and amongst themselves. It is enough to consider one electron.

In the presence of the material medium the purely mathematical operator $(\boldsymbol{p}/\hbar)$ in the second equation of (2') is converted to a physical operator $(\boldsymbol{p}/\hbar)$ e, by the action of the operator Q_e on the ket $|e,\ a'\rangle$ where $\boldsymbol{p}$ stands for the momentum of the electron. It then follows from Equation (2') that

$$\boldsymbol{B} = \mathrm{i}(\boldsymbol{p}/\hbar)e \times \boldsymbol{A}. \tag{5}$$

In the non-relativistic regime we represent the operator $\boldsymbol{p}$ by $\boldsymbol{p} = m_e\boldsymbol{v}$, so that from (5) $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = (\mathrm{i}e/\hbar)m_e\boldsymbol{v} \times \boldsymbol{A}. \tag{6}$$

Because the material medium is bounded the electron moves in a closed path of arbitrary geometry, and hence acquires a magnetic moment $\boldsymbol{\mu}$ given by

$$\boldsymbol{\mu} = \left(\frac{e}{2m_e c}\right)\boldsymbol{L}, \tag{7}$$

where $\boldsymbol{L}$ is the orbital angular momentum of the electron. Then from equations (6) and (7) we obtain,

$$\boldsymbol{B} = (\mathrm{i}/\hbar)\left(\frac{e}{2m_e c}\right)\boldsymbol{L} \times \boldsymbol{A}. \tag{8}$$

Here m_e is the rest mass of the electron.

It follows from Equation (8) and our previous consideration that the system is characterized by three commuting operators, namely, Q_e, L_z and L^2, and hence by the state ket $|em\ell\rangle$, where m is the magnetic quantum number and ℓ azimuthal quantum number. Since $L_z|em\ell\rangle = m\hbar|em\ell\rangle$, it follows that $\boldsymbol{B}$ is quantized (discrete),

$$B = i\left(\frac{em}{2m_e c}\right)(\hat{z} \times A),\ \ m = -\ell, -\ell+1, \cdots, \ell. \tag{9}$$

In the electrostatic case, $\boldsymbol{E} = -\nabla\varphi$, where $\varphi = ef(r)$, $f(r)$ is a spherically symmetric function. It follows that $\boldsymbol{E} = -e\nabla f(r)$, where e is the electric charge of the electron. Analogously the quantity $\mathrm{i}(em/2m_e c)\hat{z}$, a pure imaginary vector quantity, appearing in Equation (9) is called the magnetic charge (or magnetic monopole) of the electron of strength $(|e||m|/2m_e c)$. On account of the possible values of m (see (9)), magnetic monopoles point only along the two directions of z since ℓ, appearing in (9), is non-negative and takes the values,

$$\ell = 0, 1,\ 2, \cdots, n-1, \tag{10}$$

i.e. n different values in all; n is called the principal quantum number. If $(em/2m_e c)$ is positive, we call it positive magnetic charge or north pole, and negative magnetic charge or south pole if negative—the same number of monopoles point in the z-direction and opposite to the z-direction. No monopoles are in the *x*- or *y*-directions since L_x and L_y do not commute with L_z. Of course the choice of L_z is not unique—we could have chosen L_x or L_y and the conclusions would be the same. We have here an unusual case of quantization: quantization in magnitude (strength) and direction.

Formally, we define magnetic monopole, denoted by $\boldsymbol{\mu}_q$ (*q*, for quantum), as a vector quantity,

$$\boldsymbol{\mu}_q = \mathrm{i}\left(\frac{em}{2m_e c}\right)\hat{z}, \tag{11}$$

which is not an observable, being a pure imaginary quantity. The magnetic monopole differs completely from electric monopole, a scalar quantity, which is an observable. By substituting the well known values of *e*, m_e, and c one obtains the strength of the magnetic monopole (in gaussian units) to be approximately 10^7.

It follows from Equations (9) and (11) that

$$\boldsymbol{B} = \boldsymbol{\mu}_q \times \boldsymbol{A}, \tag{12}$$

and

$$\nabla \cdot \boldsymbol{B} = 0 \text{, everywhere.} \tag{13}$$

5. Conclusions

The proof of the existence of magnetic monopoles in nature which we have presented here is indeed revolutionary. It is a discovery that opens up new vistas in the theories of the fundamental processes. We limit our discussion here to a small sample of its consequences.

We can now assert without an iota of equivocation that ordinary matter is made up of electric charges (electric monopoles) or magnetic charges (magnetic monopoles). This takes us back to Ampere's hypothesis, his idea that magnetism in matter is to be accounted for by a multitude of tiny rings of electric current distributed throughout the substance. Those tiny rings of current are the classical manifestation of a fundamental property of the electron called magnetic charge. There is a clear distinction between the electric and magnetic charges of the electron: The electric charge is real, an observable scalar quantity. Electrons that have this property may be called electrons of the first kind—they are scalar particles. On the other hand magnetic charge is a pure imaginary vector quantity, not observable-electrons that have this property, and may be called electrons of the second kind, which are pure imaginary vector particles.

The implication of the foregoing is that our conventional theory of the structure of matter is incomplete! Substances composed of electric charges are well known and have been extensively studied for centuries [5]. Substances of the second kind that is substances composed of magnetic charges, are not well known because their formal structure was still an open question before now, and the conventional theories of them had to be phenomenological. To obtain their structure we merely replace electric charges with magnetic charges in the old scheme.

An important question is this: why is it that we do not notice the magnetic fields of all the electrons constrained to move in closed paths of substances. The conventional answer is that there is mutual cancellation, because in an ordinary lump of matter there are electrons moving around in one direction as in the opposite direction. This is so because there is no special reason why a particular direction of rotation is intrinsically preferable, or otherwise to distinguish any unique axis of rotation. This explanation is palpably classical, and hence unsatisfactory. The fact is that electrons of the second kind are aligned in either direction of a particular axis of quantization, the z-axis, say. If the substance is of infinite extent the magnetic charges cancel out exactly; and we say, by analogy with the electrical case, that the substance is magnetically neutral. For substances of finite extent magnetic charges accumulate at the ends as positive magnetic charge (or north pole) at one end and negative magnetic charge (or south pole) at the other end because cancellation of the charges does not occur at the ends. Away from the ends the charges cancel out exactly as in the case of material of infinite extent, and the substance is said to be magnetically neutral there. Such a finite material is called a magnet. If the substance of finite length is cut into two pieces we obtain two magnets because of accumulation of positive magnetic charges at one new end and negative magnetic charge at the other. This way one can produce any number of magnets from a substance of finite length.

Geomagnetism is the science concerned with the earth's magnetic field. The field extends far above the surface of the earth, and functions as a guide to travelers, e.g. ships. The source of the field is, however, a mystery; but geophysicists believe that it is associated with dynamo action in the earth's liquid core! The sources of the field, from the foregoing paragraph, are magnetic monopoles. The earth is a finite body having two distinct uniform rotations, one about the geographic North-South axis and the other about the sun. The net rotation is about an axis (N' - S'), the z-axis say, that makes a small angle with the geographic North-South axis. Some of the free electrons in the earth are electrons of the second kind (magnetic monopoles). Monopoles with positive magnetic charge are aligned in the $S'N'$ direction and monopoles with negative magnetic charge point in the opposite direction. The earth is magnetically neutral except at the poles N' and S', because of the exact cancellation of the positive and negative magnetic charges within the earth. At N' the $S'N'$ monopoles are absent, and only the $N'S'$ monopoles survive. There is therefore excess accumulation of negative magnetic charges at N', which serves as the South Pole of a magnet. Similarly at S' the $N'S'$ monopoles are absent and only the $S'N'$ monopoles survive. Thus, excess positive magnetic charges accumulate at S' which serves as the North Pole of a magnet. The earth is thus a magnet with its North Pole (S') near the geographic South Pole and South Pole (N') near the geographic North Pole. This magnet is the source of earth's magnetic field.

By its very definition an electron is a monopole endowed with a certain fundamental property called charge. The nature of the charge depends on the frame of reference. If the electron resides in an inertial frame, we call it electric monopole because it is at rest there. Of course viewed from a different Lorentz frame the electron is an electric current, and is still electric monopole because of its electric charge. The magnetic field associated with the current arises because of wrong choice of Lorentz frame and hence is not fundamental. If the electron resides in a uniformly rotating frame, e.g. an electron moving in a closed path of any geometry, it becomes a magnetic monopole because of its magnetic charge which is a pure imaginary vector quantity. An electron that resides in a frame which executes combined uniform translation in a straight line and uniform rotation is called a complex monopole because its charge is complex. If the electron resides in an arbitrary frame the monopole description may not apply.

Given the foregoing remarks it becomes rather difficult to understand why anybody will look for an electron in an ultra-high energy machine like the LHC—recall the ATLAS and MoEDAL experiments. Theoretical physics should make a triumphant return to what it used to be in the 18C, 19C and early 20C—that is from machine-driven enquiry of today to an intellect-driven discipline its original domain.

Finally, as we have seen one can make any number of magnets from a given magnet. A fundamental question that must be answered is this: What is the smallest magnet that can be made from a given magnet?—that is the tiniest magnet that nature admits. It is an electron magnetic dipole consisting of a positive magnetic charge (North Pole) and a negative magnetic charge (South Pole) separated by the smallest energy gap, called Lamb Shift, measured by Willis Lamb and Robert Retherford around the middle of the 20C. The magnetic dipole moment of the electron has been measured and is well documented.

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