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1 + 1 = 3: Synergy Arithmetic in Economics

Mark Burgin[1], Gunter Meissner[2]

[1]UCLA Department of Mathematics, Marina Del Rey, USA
[2]University of Hawaii, Honolulu, USA
Email: mburgin@math.ucla.edu, meissner@hawaii.edu

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Abstract

Counting has always been one of the most important operations for human beings. Naturally, it is inherent in economics and business. We count with the unique arithmetic, which humans have used for millennia. However, over time, the most inquisitive thinkers have questioned the validity of standard arithmetic in certain settings. It started in ancient Greece with the famous philosopher Zeno of Elea, who elaborated a number of paradoxes questioning popular knowledge. Millennia later, the famous German researcher Herman Helmholtz (1821-1894) [1] expressed reservations about applicability of conventional arithmetic with respect to physical phenomena. In the 20th and 21st century, mathematicians such as Yesenin-Volpin (1960) [2], Van Bendegem (1994) [3], Rosinger (2008) [4] and others articulated similar concerns. In validation, in the 20th century expressions such as 1 + 1 = 3 or 1 + 1 = 1 occurred to reflect important characteristics of economic, business, and social processes. We call these expressions synergy arithmetic. It is common notion that synergy arithmetic has no meaning mathematically. However in this paper we mathematically ground and explicate synergy arithmetic.

Keywords

Synergy, Synergy Arithmetic, Non-Diophantine Arithmetics, Mergers and Acquisitions

1. Introduction

Everybody knows that 1 + 1 = 2. However, in the 21st century, expressions such as 1 + 1 = 3 occurred to reflect important characteristics of economic and business processes. It seems that this contradicts core mathematical axioms and is incorrect from a mathematical point of view.

We find similarities in the history of mathematics—what had been considered strange, ungrounded and inconsistent with the existing mathematics, was incorporated later in the main body of mathematical knowledge. Here are some examples:

In China and India, mathematicians used negative numbers for centuries before these numbers came to Europe. However, when the European mathematicians encountered negative numbers, critics dismissed their sensibility. Some of the notable European mathematicians, such as d'Alembert or Frend, did not want to accept negative numbers until the 18th century and referred to them as "absurd" or "meaningless" (Kline, 1980 [5], Mattessich, 1998 [6]). Even in the 19th century, it was common practice to ignore any negative results derived from equations, on the assumption that they were meaningless (Martinez, 2006 [7]). For instance, Lazare Carnot (1753-1823) affirmed that the idea of something being less than nothing is absurd (Mattessich, 1998 [6]). Outstanding mathematicians such as William Hamilton (1805-1865) and August De Morgan (1806-1871) had akin opinions. Similarly, irrational numbers and later imaginary numbers were firstly rejected. Today these concepts are accepted and applied in numerous scientific and practical fields, such as physics, chemistry, biology and finance.

The interaction between physics and mathematics gives us one more example. In the 20th century, physicists started using functions that took infinite values at some points. At first, mathematicians objected by saying that there are no such functions in mathematics (cf., for example, von Neumann, 1955 [8]). However, later they grounded utilization of these functions developing the theory of distributions and finding numerous new applications for this theory (Schwartz, 1950/1951 [4]).

2. Problems with the Conventional Arithmetic

Human beings have used conventional arithmetic for millennia before the most inquisitive thinkers started questioning its validity in certain settings. It dates back to ancient Greece, where mathematicians and philosophers already started to doubt the convenience of conventional arithmetic. The Sophists, who lived from the second half of the fifth century B.C to the first half of the fourth century B.C. asserted the relativity of human knowledge and suggested many paradoxes, explicating complexity and diversity in the real world. One of them, the famous philosopher Zeno of Elea (490 - 430 B.C.), who was said to be a self-taught boy from the country side, invented very notable paradoxes, in which he questioned the popular knowledge and intuition related to fundamental essences such as time, space, and numbers.

An example of this type of reasoning is the *paradox of the heap* (or the *Sorites paradox* where *σωρος* is the Greek word for "heap"). It is possible to formulate this paradox in the following way.

1) One million grains of sand make a heap.

2) If one grain of sand is added to this heap, the heap stays the same.

3) However, when we add 1 to any natural number, we always get a new number.

There are analogies of this paradox in our time. For instance, if you have ten million dollars (a "heap" of money) and somebody will give you a dollar, will you say that you have ten million and one dollar when asked about your assets? No, you will most likely say that you have the same ten million dollars (the same "heap").

As we know, Greek sages posed questions, but in many cases, including arithmetic, suggested no answers. As a result, for more than two thousand years, these problems were forgotten and everybody was satisfied with the conventional arithmetic. In spite of all problems and paradoxes, this arithmetic has remained very useful.

In recent times, scientists and mathematicians have returned to problems of arithmetic. The famous German researcher Herman Ludwig Ferdinand von Helmholtz (1821-1894) [1] was one the first scientists who questioned adequacy of the conventional arithmetic. In his "Counting and Measuring" (1887) [1], Helmholtz considered an important problem of the applicability of arithmetic to physical phenomena. This was a natural approach of a scientist, who judged mathematics by the main criterion of science—observation and experiment.

The scientist's first observation was that as the concept of a *number* is derived from some practice, usual arithmetic has to be applicable in all practical settings. However, it is easy to find many situations when this is not true. To mention but a few described by Helmholtz, one raindrop added to another raindrop does not make two raindrops. It is possible to describe this situation by the formula $1 + 1 = 1$.

In a similar way, when one mixes two equal volumes of water, one at 40˚ Fahrenheit and the other at 50˚, one does not get two volumes at 90˚. Alike, the conventional arithmetic fails to describe correctly the result of combining gases or liquids by volumes. For example (Kline, 1980), one quart of alcohol and one quart of water yield about 1.8 quarts of vodka.

Later the famous French mathematician Henri Lebesgue facetiously pointed out (cf. Kline, 1980 [5]) that if one puts a lion and a rabbit in a cage, one will not find two animals in the cage later on. In this case, we will also have $1 + 1 = 1$.

However, since very few paid attention to the work of Helmholtz on arithmetic, and as still no alternative to the conventional arithmetic has been suggested, these problems were mostly forgotten. Only years later, in the second part of the 20th century, mathematicians began to doubt once more the absolute character of the ordinary arithmetic, where $2 + 2 = 4$ and $2 \times 2 = 4$. Scientists and mathematicians again started to draw attention of the scientific community to the foundational problems of natural numbers and the conventional arithmetic. The most extreme assertion that there is only a finite quantity of natural numbers was suggested by Yesenin-Volpin (1960) [2], who developed a mathematical direction called ultraintuitionism and took this assertion as one of the central postulates of ultraintuitionism. Other authors also considered arithmetics with a finite number of numbers, claiming that these arithmetics are inconsistent (cf., for example Van Bendegem, 1994 [3] and Rosinger, 2008 [4]).

Van Danzig had similar ideas but expressed them in a different way. In his article (1956), he argued that only some of natural numbers may be considered finite. Consequently, all other mathematical entities that are called traditionally natural numbers are only some expressions but not numbers. These arguments are supported and extended by Blehman, *et al.* (1983) [9].

Other authors are more moderate in their criticism of the conventional arithmetic. They write that not all natural numbers are similar in contrast to the presupposition of the conventional arithmetic that the set of natural numbers is uniform (Kolmogorov, 1961 [10]; Littlewood, 1953 [11]; Birkhoff and Bartee, 1967 [12]; Dummett 1975 [13]; Knuth, 1976 [14]). Different types of natural numbers have been introduced, but without changing the conventional arithmetic. For example, Kolmogorov (1961) [10] suggested that in solving practical problems it is worth to separate *small, medium*, *large*, and *super-large* numbers.

Regarding geometry, it was discovered that there was not one but a variety of arithmetics, which were different in many ways from the conventional arithmetic. It is natural to call the conventional arithmetic by the name *Diophantine arithmetic* because the Greek mathematician Diophantus, who lived between 150 C.E. and 350 C.E., and who extensively contributed to the development of conventional arithmetic. Consequently, new arithmetics acquired the name *non-Diophantine arithmetics*.

Burgin built first Non-Diophantine arithmetics of whole and natural numbers (Burgin, 1977 [12]; 1997 [15]; 2007 [16]; 2010 [17] [18]) and Czachor extended this construction developing Non-Diophantine arithmetics of the real and complex numbers (Czachor, 2015 [19]).

In the following section, we will show that non-Diophantine arithmetics occur in economics, starting with mergers and acquisitions.

3. Examples of Non-Diophantine Arithmetic

In the following section, we will show that non-Diophantine arithmetics occur in economics, business and social settings, starting with mergers and acquisitions.

Mergers and acquisitions (M & A) are processes in which the operating units of two companies are combined. Whereas in a merger, two approximately equally sized companies consolidate into one entity; in an acquisition, a larger company takes over the smaller one.

There are different motivations for M & As, the most typical being the following:

1) *Synergies* or *economies of scale* are achieved when the larger company may be able to lower the per unit purchasing cost due to higher bulk orders, or lower fixed cost by removing or combing duplicate departments such as research, accounting, or compliance.

2) *Increased market share* usually leads to the greater than before market power.

3) *Technology driven* M & As are aimed at gaining access to new technologies.

4) *Tax reduction* in M & A is the situation when a company may acquire a loss generating, yet presumably long term profitable company to reduce taxes.

5) In the case of *CEO activism*, a CEO may want to amplify his standing by increasing company size and seemingly showing leadership in M & A.

The success of M & As varies strongly. With respect to the cost saving due to synergy and economies of scale, many M & As do achieve their objective, as we can see in **Figure 1**.

However, with respect to expected revenue, only few companies achieve the desired objective, as we can see in **Figure 2**.

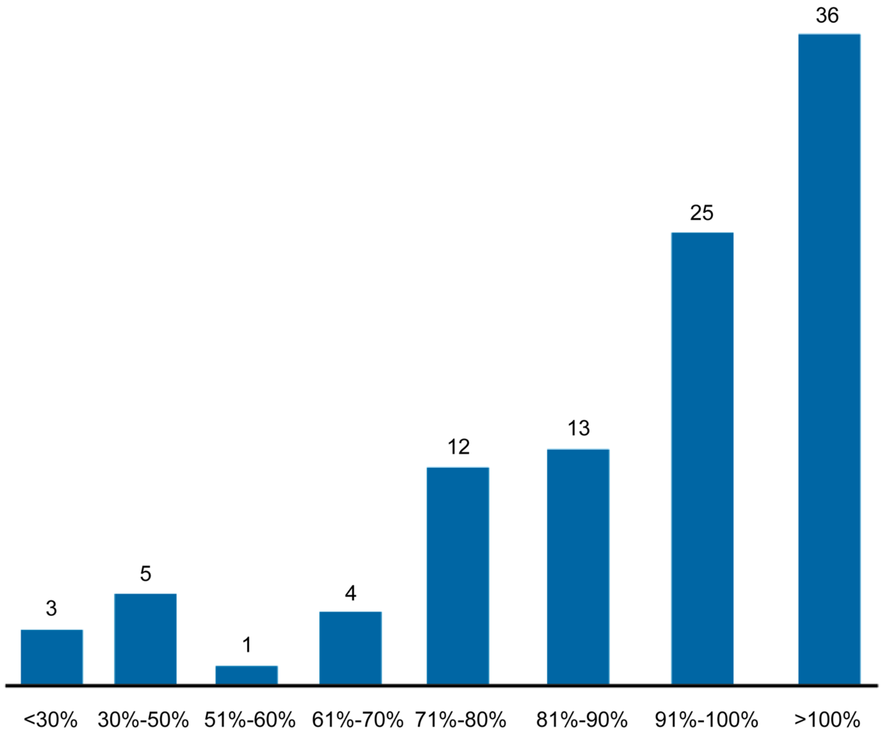

Figure 1. Mergers and Acquisitions achieving the expected cost savings (Source: McKinsey 2004 [20]).

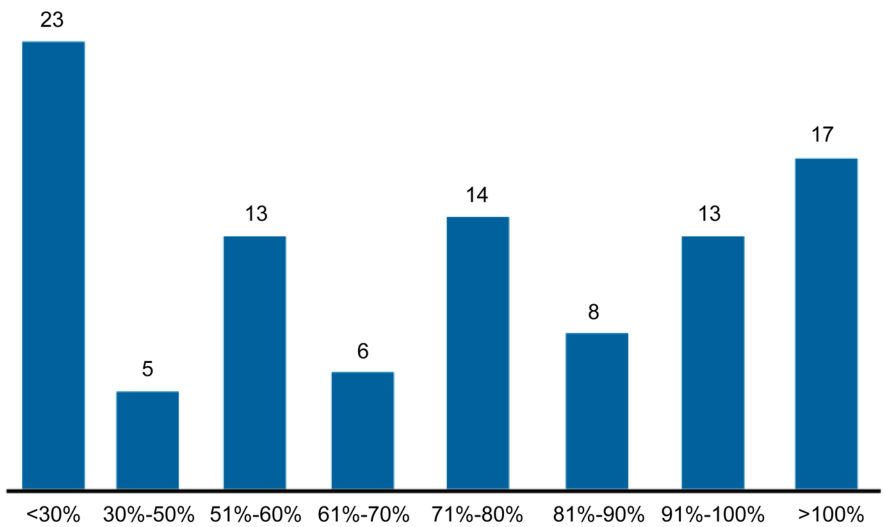

Figure 2. Mergers and Acquisitions meeting the percentage of expected revenue (Source: McKinsey 2004 [20]).

Successful M & As, those that are in the right part of **Figure 2**, are sometimes described by the expression 1 + 1 = 3 (see, for example, Beechler, 2003 [21]) because this expression reflects the fact that after a M & A the sum of the two combined entities, 3, is bigger than the two parts considered separately, 1 and 1.

In a similar way, the Cambridge Business English Dictionary (2011) [22] defines that when two companies or organizations join together, they achieve more and are more successful than if they work separately, or in other words, the merger results in 2 + 2 = 5. Therefore, s*ynergy* emerges when the cooperation of two systems gives a result greater than the sum of their individual components.

Most studies of M & As (see for example (Cartwright and Schoenburg, 2006 [23]; Krug and Aguilera, 2005 [24]; Agraval and Jaffe, 2000 [25]) find that the majority of M & As do not meet their expectations, see the left part of **Figure 2**. The main reasons for the failure of many M & A are

1) Poor cultural fit or lack of cultural compatibility,

2) Overestimation of synergy effects,

3) Underestimation of cost involved in the M & A process,

4) Employee turnover.

When M & As do not meet their expectations, it is possible to argue that these situations are correctly reflected by expressions 1 + 1 = 1, or even 1 + 1 = 0. This state of affairs reflects *negative synergy* or *system friction*.

Expression 1 + 1 = 3 may also describe synergies in other areas. Michael Angier (2005) [26] defines synergy as the phenomenon of two or more people getting along and benefiting one another, *i.e.*, the combination of energies, resources, talents and efforts equal more than the sum of the parts. It is possible to describe this phenomenon by the expression 1 + 1 = 3.

In addition, the expression 1 + 1 = 3 also emerges in other areas, for example in the exploration of the features of visual information (Tufte, 1990 [27]). For example in an expression of two words, e.g., "every thing", the white space between the words provides additional information. Its absence changes the meaning, e.g., without the white space, we have "everything". Adding two words (symbols), *i.e.*, 1 + 1, we obtain 3 meaningful symbols. It is possible to call these expressions 1 + 1 = 3 and 2 + 2 = 5, synergy arithmetic.

Furthermore, there are examples of non-Diophantine arithmetic in everyday life. Imagine that you come to a supermarket and you can see an advertisement "Buy one, get one free". It actually means that you can buy two items for the price of one. Such advertisement may refer to almost any product: bread, milk, juice etc. For example, if one bottle of orange juice costs $2, and you get two for one, then we have the equality 2 + 2 = 2. This is incorrect in the conventional arithmetic but is true for some non-Diophantine arithmetic, as we will show below.

Another example: when a cup of milk is added to a cup of popcorn then only one cup of mixture will result because the cup of popcorn will very nearly absorb a whole cup of milk without spillage. So we have 1 + 1 = 1. This is impossible to replicate with conventional arithmetic but it is true for some non-Diophantine

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arithmetics.

One more example is when you want to buy a car, which according to the newspaper advertisement, costs $19,990. Coming to the dealership, you find that the price is five dollars more. Do you think that the new price is different from the initial one or you consider it practically one and the same price? It is natural to suppose that any sound person has the second opinion. Consequently, we come to the same "paradox": if k is the price of the car, then in the Diophantine arithmetic $k + 5$ is not equal to k, while with respect to you, they are basically identical.

Critics may object that we use Non-Diophantine arithmetics to explain these phenomena. They may say that we use the conventional arithmetic but only transform its operations according to some formulas. This objection is similar to the 18th century Europe claim that people do not use negative numbers but only employ positive numbers with additional symbols.

Now let us look whether the laws of non-Diophantine arithmetic can reflect the economic and psychological phenomena considered above.

4. Laws of Non-Diophantine Arithmetics

Here we consider only arithmetics in the set $\boldsymbol{W}$ of whole numbers. We start with a more general concept of a prearithmetic (Burgin, 2010 [23]).

If X is a subset of the set $\boldsymbol{R}$ of all real numbers, then the *arithmetical completion* of X consists of all sums and products of elements from X. For instance, if we take the set {1} that has only one element 1, then its arithmetical completion is the set $\boldsymbol{N} = \{1, 2, 3, \cdots\}$ of all natural numbers because in $\boldsymbol{R}$, any natural number is the sum of some quantity of the number 1.

Let us consider two functions $f: \boldsymbol{W} \to \boldsymbol{R}$ and $g: \boldsymbol{R} \to \boldsymbol{W}$. Functions f and g allow defining two new operations in the set $\boldsymbol{W}$:

$$a \oplus b = g\left(f(a) + f(b)\right)$$

$$a \circ b = g\left(f(a) \cdot f(b)\right).$$

Here a and b are whole numbers, + is addition and · is multiplication of real numbers, while $\oplus$ is addition and $\circ$ is multiplication of numbers in prearithmetic defined by functions f and g. Let us take the set A which is the domain of f, *i.e.*, the subset of $\boldsymbol{W}$ where f is defined.

The structure $\boldsymbol{A} = \langle A, \oplus, \circ \rangle$ is called a *whole-number prearithmetic*. In other words, a whole number prearithmetic is a set of whole number with operations $\oplus$ and $\circ$.

Naturally the conventional arithmetic $\boldsymbol{W}$ is a whole-number prearithmetics. Another example of whole-number prearithmetics is a modular arithmetic, which is studied in mathematics and used in physics and computing. In modular arithmetic, operations of addition and multiplication are defined but in contrast to the conventional arithmetic, its numbers "wrap around" upon reaching a certain value, which called the modulus. For instance, when the modulus is equal to 10, the modular arithmetic $\boldsymbol{Z}_{10}$ contains only ten numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and when the result of the operation the conventional arithmetic is larger than 10, then it is reduced to these numbers in the modular arithmetic. Readers can find information about modular arithmetics in many books and on the Internet. Here we only give some examples for the modular arithmetic $\boldsymbol{Z}_{10}$:

$$2 + 2 = 4$$

but

$$5 + 5 = 0$$

and

$$7 + 8 = 5$$

In addition,

$$3 \cdot 3 = 9$$

but

$$5 \cdot 5 = 5$$

and

$$4 \cdot 4 = 6$$

Note that in a general case, operations $\oplus$ and $\circ$ are partial, *i.e.*, they are not defined for all numbers from $\boldsymbol{W}$.

When a whole-number prearithmetic satisfies the following conditions, it becomes a *general Non-Diophantine arithmetic*.

Condition A1. f: $\boldsymbol{W} \to \boldsymbol{R}$ is a total function.

Condition A2. The arithmetical completion of the image of f is a subset of the domain of g.

Condition A3. The composition $g \circ f : \boldsymbol{W} \to \boldsymbol{W}$ is a projection, *i.e.*, its image coincides with the whole $\boldsymbol{W}$.

For instance, condition A2 is true when g is also a total function, e.g., $g(x) = \lceil x \rceil$, or when f maps a whole number into a whole number and g is defined for all whole numbers, e.g., $f(x) = 2x$, and $g(x) = 3x$.

An important class of Non-Diophantine arithmetics is formed by projective arithmetics. To build a projective arithmetic, we take a non-decreasing function h: $\boldsymbol{R} \to \boldsymbol{R}$ and its inverse relation h^{-1} defining the following two functions h_{T} and h^{T}:

$$h_{\mathrm{T}}(x) = \lceil h(x) \rceil$$

$$h^{\mathrm{T}}(x) = \lfloor h^{-1}(x) \rfloor$$

Here $\lceil a \rceil$ is the ceiling of a natural number a, which is the least whole number that is larger than a, and $\lfloor a \rfloor$ is the floor of a natural number a, which is the largest whole number that is less than a.

For instance, taking the number 2.75, we have $\lceil 2.75 \rceil = 3$ and $\lfloor 2.75 \rfloor = 2$.

Taking $f = h_{\mathrm{T}}$ and $g = h^{\mathrm{T}}$, we build the whole-number prearithmetic $\boldsymbol{AW} = \langle \boldsymbol{W}, \oplus, \circ \rangle$ with

$$a \oplus b = h^{\mathrm{T}}\left(h_{\mathrm{T}}(a) + h_{\mathrm{T}}(b)\right)$$

$$a \circ b = h^{\mathrm{T}}\left(h_{\mathrm{T}}(a) \cdot h_{\mathrm{T}}(b)\right)$$

This prearithmetic is a *projective whole-number arithmetic* if the following conditions are satisfied:

1) $h_{\mathrm{T}}(0_{\mu}) = 0$;

2) $h(x)$ is a strictly increasing function;

3) for any elements *a* and *b* from *U* from $a \le b$, we have $h_{\mathrm{T}}(Sa) - h_{\mathrm{T}}(a) \le h_{\mathrm{T}}(Sb) - h_{\mathrm{T}}(b)$.

Here *Sa* is the number that follows *a* in the Diophantine arithmetic, e.g., *S*2 = 3 and *S*7 = 8.

It is possible to find a theory of this and other Non-Diophantine arithmetics in Burgin, 1977; 1997; 2007; and 2010. Here we consider only simple examples and some properties of Non-Diophantine arithmetics because the goal of this work is a demonstration of a possibility of the rigorous mathematics to correctly and consistently interpret seemingly paradoxical statements, which describe situations in various spheres of real life.

Example 1. Let us take the function $f(x) = 2x$ and its inverse function $f^{-1}(x) = (1/2)x$. In this case, $f_{\mathrm{T}}(a) = f(a)$ for all whole numbers *a* and if *c* = 2*d*, then $f^{\mathrm{T}}(c) = f^{-1}(c)$. This allows us to build the whole-number arithmetic $\boldsymbol{A} = \langle \boldsymbol{W}, \oplus, \circ \rangle$ and perform summation and multiplication in it finding some sums and products:

$$1 \oplus 1 = (1/2)(2 \cdot 1 + 2 \cdot 1) = (1/2)(2+2) = (1/2)(4) = 2$$

$$2 \oplus 2 = (1/2)(2 \cdot 2 + 2 \cdot 2) = (1/2)(4+4) = (1/2)(8) = 4$$

However,

$$1 \circ 1 = (1/2)((2 \cdot 1) \cdot (2 \cdot 1)) = (1/2)(2 \cdot 2) = (1/2)(4) = 2$$

$$2 \circ 2 = (1/2)((2 \cdot 2) \cdot (2 \cdot 2)) = (1/2)(4 \cdot 4) = (1/2)(16) = 8$$ [1]

Example 2. Let us take the function $f(x) = x + 1$ and its inverse function $f^{-1}(x) = x - 1$. In this case, $f_{\mathrm{T}}(a) = f(a)$ for all whole numbers *a* and *f* $f^{\mathrm{T}}(c) = f^{-1}(c)$. This allows us to build the whole-number arithmetic $\boldsymbol{A} = \langle \boldsymbol{W}, \oplus, \circ \rangle$ and perform summation and multiplication in it finding some sums and products:

$$1 \oplus 1 = ((1+1)+(1+1)) - 1 = (2+2) - 1 = 4 - 1 = 3$$

$$2 \oplus 2 = ((2+1)+(2+1)) - 1 = (3+3) - 1 = 6 - 1 = 5$$

and

$$1 \circ 1 = ((1+1) \cdot (1+1)) - 1 = (2 \cdot 2) - 1 = 4 - 1 = 3$$

$$2 \circ 2 = ((2+1) \cdot (2+1)) - 1 = (3 \cdot 3) - 1 = 9 - 1 = 8$$

Example 3. Let us take the function $f(x) = \log_2 x$ and its inverse function $f^{-1}(x) = 2^x$. This allows us to build the whole-number arithmetic $\boldsymbol{A} = \langle \boldsymbol{W}, \oplus, \circ \rangle$ and perform summation and multiplication, in it finding some sums and prod-

[1]For example, if the function $f(x) = x + 5$, then $f^{-1}(x) = x - 5$.

ucts:

$$1 \oplus 1 = 2^{(\log_2 1 + \log_2 1)} = 2^{(0+0)} = 2^0 = 1$$

$$2 \oplus 2 = 2^{(\log_2 2 + \log_2 2)} = 2^{(1+1)} = 2^2 = 4$$

while

$$1 \circ 1 = 2^{(\log_2 1 \cdot \log_2 1)} = 2^{(0 \cdot 0)} = 2^0 = 1$$

$$2 \circ 2 = 2^{(\log_2 2 \cdot \log_2 2)} = 2^{(1 \cdot 1)} = 2^1 = 2$$

Example 4. Let us take the function $f(x) = x - 1$ and its inverse function $f^{-1}(x) = x + 1$. In this case, $f_{\mathrm{T}}(a) = f(a)$ for all whole numbers *a* and $f^{\mathrm{T}}(c) = f^{-1}(c)$. This allows us to build the whole-number arithmetic $\boldsymbol{A} = \langle \boldsymbol{W}, \oplus, \circ \rangle$ and perform summation and multiplication in it finding some sums and products:

$$1 \oplus 1 = ((1-1)+(1-1))+1 = (0+0)+1 = 0+1 = 1$$

$$2 \oplus 2 = ((2-1)+(2-1))+1 = (1+1)+1 = 2+1 = 3$$

$$2 \oplus 1 = ((2-1)+(1-1))+1 = (1+0)+1 = 1+1 = 2$$

and

$$1 \circ 1 = ((1-1) \cdot (1-1))+1 = (0 \cdot 0)+1 = 0+1 = 1$$

$$2 \circ 2 = ((2-1) \cdot (2-1))+1 = (1 \cdot 1)+1 = 1+1 = 2$$

One more unusual property of Non-Diophantine arithmetics is related to physics. Physicists often use relations $a \ll b$, which means that *a* is much smaller than *b*, and $b \gg a$, which means that *b* is much smaller than *a*. However, these relations do not have an exact mathematical meaning and are used informally. In contrast, Non-Diophantine arithmetics provide rigorous interpretation and formalization for these relations. Namely Burgin, 1997 [28],

$$a \ll b \quad \text{if and only if} \quad b \oplus a = b^{\,2}$$

Note that this is impossible in conventional mathematics because for any number *a* > 0, the sum *b* + *b* is larger than *b*. At the same time, there are Non-Diophantine arithmetics, in which $b \oplus a = b$ is true for different *a*, *b* > 0. One of this type of arithmetic is considered in Example 4.

5. Conclusions

Expressions such as 1 + 1 = 3, 2 + 2 = 5, 2 + 2 = 3, 1 + 1 = 1 and 1 + 1 = 0 are symbolically used in economics and other realms of human activity. In addition, books exist which use these expressions as metaphors (Archibald, 2014 [29]; Trott, 2015 [30]; The Business Book, 2014 [31]). Although for a long time these expressions were considered mathematically meaningless, we show that they are mathematically correct in some Non-Diophantine arithmetics or in prearithmetics. Therefore these expressions are approved and authorized by mathematical laws and are able to reflect phenomena in economics and physics.

[2]The operator ⊕ follows the standard rules of arithmetic and has no restrictions.

At the same time, rules of Non-Diophantine arithmetics are implicit in many realms of everyday life. Therefore, Non-Diophantine arithmetics are also becoming an integral part of sciences such as psychology, sociology and education, where they can explain and extend the known laws and principles and discover new ones.

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