

Modelling the Dynamical State of the Projected Primary and Secondary Intra-Solution-Particle Movement System of Efavirenz *In-Vivo*

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Abstract

This work seeks to describe intra-solution particle movement system. It makes use of data obtained from simulations of patients on efavirenz. A system of ordinary differential equations is used to model movement state at some particular concentration. The movement states' description is found for the primary and secondary level. The primary system is found to be predominantly an unstable system while the secondary system is stable. This is derived from the state of dynamic eigenvalues associated with the system. The saturated solution-particle is projected to be stable both for the primary potential and secondary state. A volume conserving linear system has been suggested to describe the dynamical state of movement of a solution particle.

Keywords

Ordinary Differential Equations, Stability, Nonautonomous Linear System, Concentration-Varying Eigenvalues, Solution Particle

1. Introduction

The intra-solution-particle movement system is described in this work. A system which consists of the primary and secondary movement states that is proposed in Nemaura (2015) is investigated. Differential Equations are used to describe dynamical systems. Most systems are described relative to time [1] [2] [3]. In this work, they are used to describe a dynamical system of the state of a solution particle relative to concentration.

Multiple compartmental modelling finds its use in fields such as Physiologically Based Pharmacokinetics, Engineering and Mathematical Biology [1] [3] [4]. Other researchers work with constant parameters that are obtained for autonomous linear systems. In this

work, a simple system (nonautonomous linear system) whose parameters vary according to concentration is considered [5] [6]. The mixing problems have been widely studied by the use of ordinary differential equations [4] [7] [8]. Additionally, some researchers consider probable systems that arise from these mixing problems [7] [8]. This work proposes a system of linear ordinary differential equations that potentially govern the state of solution particle and shares a relation to the mixing problems. In addition, there is consideration of a volume conserving state of a solution particle. The dynamic eigenvalues from the corresponding matrix and stability of the concentration varying linear systems are proposed. The dynamic eigenvalues have found applications for linear time-varying systems [2].

This work highlights possible intra-particle movement potential/states inferred to be present in the solution particle, at primary and secondary level and attempts to give mathematical form as in Nemauro (2015) [9]. It proposes multiple-compartmental models, one for the primary level and the other for the secondary level. The resultant form gives a general representation of movement within a solution particle.

2. Methods

The primary and secondary movement system of projected simulated data from patients who had been on efavirenz is used [9]. The software used, were R and Mathematica.

2.1. The Primary System

The primary system is a sub-system of the secondary and describes potential. It is projected to consist of four main movement entities that is convection, saturation, passive and advection. The form entity being the advective component [9].

2.2. The Primary System of Solution Particle

A solution particle with concentration (x) is made up of four movement components (variables) at primary level C -convective, A -advective, P -Passive, and S -Saturation at primary level satisfying

$$C(x) = S(x) + A(x) - P(x), \quad (1)$$

$$= S(x) + A(x) + \bar{P}(x), \quad (2)$$

where $\bar{P} = -P$.

We analyse the following system of differential equations in order to infer on the overall process occurring in describing state of movement in a solution particle (See **Figure 1**).

$$\frac{dS}{dx} = \sigma^{\bar{A}_3}(x) - \mu_{SA}^{\bar{A}_2}(x)S + \mu_{AS}^{\bar{A}_2}(x)A = h(S, A, \bar{P}), \quad (3)$$

$$\frac{dA}{dx} = \mu_{SA}^{\bar{A}_2}(x)S - \left(\mu_{AS}^{\bar{A}_2}(x) + \tau_{A\bar{P}}^{\bar{A}_1}(x) + \sigma^{\bar{A}_3}(x) \right) A + \tau_{\bar{P}A}^{\bar{A}_1}(x)\bar{P} = q(S, A, \bar{P}), \quad (4)$$

$$\frac{d\bar{P}}{dx} = \tau_{A\bar{P}}^{\bar{A}_1}(x)A - \tau_{\bar{P}A}^{\bar{A}_1}(x)\bar{P} = w(S, A, \bar{P}), \quad (5)$$

Subject to,

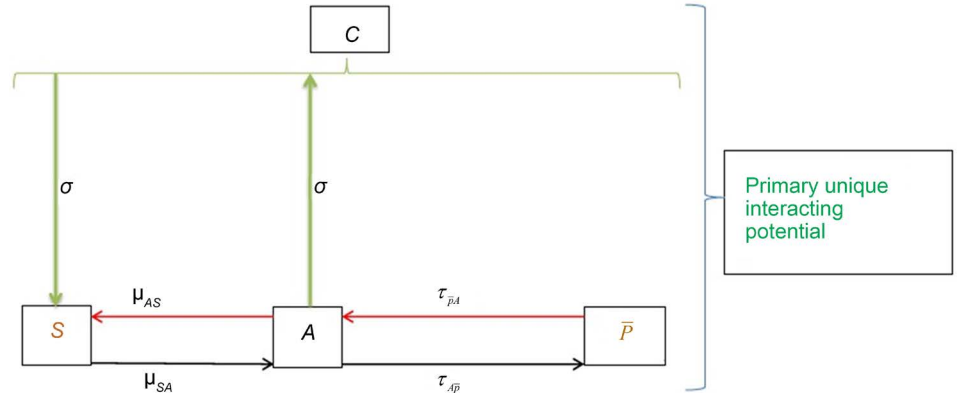


Figure 1. A compartmental representation of the model for solution particle movement state dynamics at primary level driven by the advective component. It is noted that this representation is not necessarily unique for all forms of advective component.

$$\sigma^{\dot{A}_3}(x) + \mu_{SA}^{\bar{A}_2}(x) + \mu_{AS}^{A_2}(x) = 0,$$

And

$$\tau_{AP}^{A_1}(x) + \tau_{PA}^{\bar{A}_1}(x) = 0,$$

With $\tau_{AP}^{A_1}, \tau_{PA}^{\bar{A}_1}, \mu_{SA}^{\bar{A}_2}, \mu_{AS}^{A_2}, \sigma^{\dot{A}_3} \in \mathbb{R}$ and are the variable parameters. These two conditions allows no net change in the volumetric consistencies of the four movement components, where

σ —single phase third generation solution sub-particle at primary level, form inducing movement interaction with four main movement entities,

μ —single phase second generation solution sub-particle at primary level with four main movement entities, and

τ —altering phase(s) first generation solution sub-particle at primary level with three main movement entities.

Solving the primary system above (1 - 5) in terms of advective components,

$$\begin{aligned} \sigma^{\dot{A}_3}(x) &= \frac{\frac{dC}{dx}}{1 - A}, \\ \mu_{SA}^{\bar{A}_2}(x) &= \frac{\sigma^{\dot{A}_3}(x) - \sigma^{\dot{A}_3}(x)A - \frac{dS}{dx}}{S + A}, \\ \mu_{AS}^{A_2}(x) &= \frac{\frac{dS}{dx} - \sigma^{\dot{A}_3}(x) - \sigma^{\dot{A}_3}(x)S}{S + A}, \\ \text{and } \tau_{AP}^{A_1}(x) &= \frac{\frac{d\bar{P}}{dx}}{A + \bar{P}} = -\tau_{PA}^{\bar{A}_1}(x). \end{aligned}$$

2.3. Stability Analysis of the Primary System

The equilibrium, steady state points are constant solutions, $S(x), A(x)$, and $\bar{P}(x)$

which satisfy the nonlinear system of equations $\frac{dS}{dx} = 0, \frac{dA}{dx} = 0$ and $\frac{d\bar{P}}{dx} = 0$. These points govern the behaviour of physical models. Thus we obtain $A^*(x) = 1$ (form potential), $S^*(x) = -1$ (space accessory potential), and $\bar{P}^*(x) = -1$ (negative binding-orientation potential). It is important to note that the convective potential at equilibrium (primary level) is negative (*nullifying/compensating accessory potential*). Furthermore, it has the value of -1 .

Local Analysis near Steady-State Points (S^*, A^*, \bar{P}^*)

Local analysis is studied near each steady state point (S^*, A^*, \bar{P}^*) . We begin by the following,

$$S(x) = S^*(x) + v(x), \tag{6}$$

$$A(x) = A^*(x) + \zeta(x), \tag{7}$$

$$\bar{P}(x) = \bar{P}^*(x) + \psi(x), \tag{8}$$

where $v(x)$, $\zeta(x)$ and $\psi(x)$ are small.

$$\frac{dv}{dx} = h(S^*(x) + v(x), A^*(x) + \zeta(x), \bar{P}^*(x) + \psi(x)) \approx \frac{\partial h}{\partial S} v + \frac{\partial h}{\partial A} \zeta + \frac{\partial h}{\partial \bar{P}} \psi, \tag{9}$$

$$\frac{d\zeta}{dx} = q(S^*(x) + v(x), A^*(x) + \zeta(x), \bar{P}^*(x) + \psi(x)) \approx \frac{\partial q}{\partial S} v + \frac{\partial q}{\partial A} \zeta + \frac{\partial q}{\partial \bar{P}} \psi, \tag{10}$$

$$\frac{d\psi}{dx} = w(S^*(x) + v(x), A^*(x) + \zeta(x), \bar{P}^*(x) + \psi(x)) \approx \frac{\partial w}{\partial S} v + \frac{\partial w}{\partial A} \zeta + \frac{\partial w}{\partial \bar{P}} \psi. \tag{11}$$

The partial derivatives are evaluated at the equilibrium point (S^*, A^*, \bar{P}^*) . The stability of the steady state point (S^*, A^*, \bar{P}^*) is investigated by studying the eigenvalues of the Jacobian matrix (A matrix of the partial derivatives). The Jacobian matrix for the primary system is given by,

$$J(S^*, A^*, \bar{P}^*) = \begin{bmatrix} \frac{\partial h}{\partial S} & \frac{\partial h}{\partial A} & \frac{\partial h}{\partial \bar{P}} \\ \frac{\partial q}{\partial S} & \frac{\partial q}{\partial A} & \frac{\partial q}{\partial \bar{P}} \\ \frac{\partial w}{\partial S} & \frac{\partial w}{\partial A} & \frac{\partial w}{\partial \bar{P}} \end{bmatrix}_{(S^*, A^*, \bar{P}^*)} = \begin{bmatrix} -\mu_{SA}^{\bar{A}_2} & \mu_{AS}^{\bar{A}_2} & 0 \\ \mu_{SA}^{\bar{A}_2} & -(\mu_{AS}^{\bar{A}_2} + \tau_{AP}^{\bar{A}_1} + \sigma^{\bar{A}_3}) & \tau_{PA}^{\bar{A}_1} \\ 0 & \tau_{AP}^{\bar{A}_1} & -\tau_{PA}^{\bar{A}_1} \end{bmatrix} \tag{12}$$

- (I) Undisturbed potential ($x = 0$), $(-\mu_{SA}^{\bar{A}_2} = \mu_{AS}^{\bar{A}_2} = 0, \tau_{AP}^{\bar{A}_1} = -\tau_{PA}^{\bar{A}_1})$ where $\tau_{AP}^{\bar{A}_1} = \infty$. The characteristic equation of J is given by,

$$\lambda^3 + \mu_{SA}^{\bar{A}_2} \tau_{AP}^{\bar{A}_1} \lambda = 0,$$

Eigenvalues of this system are given by, $(0, i\sqrt{\mu_{SA}^{\bar{A}_2} \tau_{AP}^{\bar{A}_1}}, -i\sqrt{\mu_{SA}^{\bar{A}_2} \tau_{AP}^{\bar{A}_1}})$. The equilibrium point has neutral stability. Thus the primary unique space interacting potential has a neutral equilibrium before “disturbances”.

- (II) Disturbed (dissolving) potential ($x > 0$), $(\sigma^{\bar{A}_3}, \mu_{SA}^{\bar{A}_2}, \mu_{AS}^{\bar{A}_2} \in \mathbb{R}, \tau_{AP}^{\bar{A}_1} = -\tau_{PA}^{\bar{A}_1})$
The dynamic eigenvalues follows from the characteristic equation of J given by,

$$\lambda^3 + \mu_{SA}^{\bar{A}_2} \left(\tau_{AP}^{A_1} - \mu_{SA}^{\bar{A}_2} - \mu_{AS}^{A_2} \right) \lambda + \mu_{SA}^{\bar{A}_2} \tau_{AP}^{A_1} \left(\mu_{SA}^{\bar{A}_2} + \mu_{AS}^{A_2} \right) = 0,$$

which is similar,

$$\lambda^3 + \mu_{SA}^{\bar{A}_2} \left(\tau_{AP}^{A_1} \right) + \mu_{SA}^{\bar{A}_2} \left(\sigma^{\bar{A}_3} \right) \lambda - \sigma^{\bar{A}_3} \mu_{SA}^{\bar{A}_2} \tau_{AP}^{A_1} = 0.$$

Thus the eigenvalues for this system are,

$$\lambda_1 = -\frac{M}{N} + \frac{N}{3},$$

$$\lambda_2 = \frac{zM}{2N} - \frac{z^*N}{6},$$

And

$$\lambda_3 = \frac{z^*M}{2N} - \frac{zN}{6},$$

where $z = 1 + i\sqrt{3}$, $z^* = 1 - i\sqrt{3}$, $M = \mu_{SA}^{\bar{A}_2} \left(\sigma^{\bar{A}_3} + \tau_{AP}^{A_1} \right)$, and

$$N = \left(\frac{27\sigma^{\bar{A}_3} \mu_{SA}^{\bar{A}_2} \tau_{AP}^{A_1} + \sqrt{729\sigma^{\bar{A}_3} \mu_{SA}^{\bar{A}_2} \tau_{AP}^{A_1 2} + 108\mu_{SA}^{\bar{A}_2 3} \left(\sigma^{\bar{A}_3} + \tau_{AP}^{A_1} \right)^3}}{2} \right)^{\frac{1}{3}}$$

- (III) Saturated potential ($\exists x_s \neq 0$: $\sigma^{\bar{A}_3}(x_s) = 0$, $-\mu_{SA}^{\bar{A}_2}(x_s) = \mu_{AS}^{A_2}(x_s) = 0$, $\tau_{AP}^{A_1}(x_s) = -\tau_{PA}^{A_1}(x_s) = 0$). The characteristic equation is given by,

$$\lambda^3 = 0.$$

2.4. The Secondary System of Solution Particle

The product of advective primary interaction potential with its unique space (the relative uptake) results in a secondary advective movement system [9]. A solution particle with concentration (x) is made up of three movement components (variables) at secondary level C_{ru} —convective, A_{ru} —advective, and S_{ru} —Saturation at secondary level satisfying

$$C_{ru}(x) = S_{ru}(x) + A_{ru}(x). \tag{13}$$

This is represented by the following constituent system (See **Figure 2**),

$$\frac{dS_{ru}}{dx} = \sigma^{(\bar{A}_{ru})_3}(x) - \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x) S_{ru} + \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}(x) A_{ru} = f(S_{ru}, A_{ru}), \tag{14}$$

$$\frac{dA_{ru}}{dx} = \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x) S_{ru} - \left(\sigma^{(\bar{A}_{ru})_3}(x) + \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}(x) \right) A_{ru} = g(S_{ru}, A_{ru}), \tag{15}$$

Subject to,

$$\sigma^{(\bar{A}_{ru})_3}(x) + \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x) + \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}(x) = 0,$$

With $\sigma^{(\bar{A}_{ru})_3}, \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}, \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2} \in \mathbb{R}$ (variable parameters). The condition allows no net change in the volumetric constituents of the movement components. Where,

σ_{ru} —single phase third generation solution sub-particle at secondary level, form

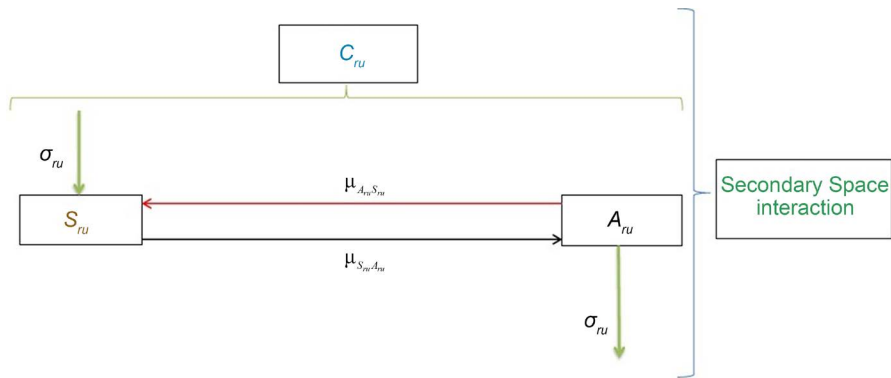


Figure 2. A compartmental representation of the model for solution particle movement state dynamics at secondary level driven by the advective component.

inducing movement interaction with four main movement entities,

μ_{ru} —single phase second generation solution sub-particle at secondary level with four main movement entities.

Solving the secondary system above (13) - (15),

$$\sigma^{(\hat{A}_{ru})_3}(x) = \frac{dC_{ru}}{dx} \cdot \frac{1}{1 - A_{ru}},$$

$$\mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x) = \frac{\sigma^{(\hat{A}_{ru})_3}(x) - \sigma^{(\hat{A}_{ru})_3}(x)A_{ru} - \frac{dS_{ru}}{dx}}{S_{ru} + A_{ru}},$$

and $\mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}(x) = \frac{\frac{dS_{ru}}{dx} - \sigma^{(\hat{A}_{ru})_3}(x) - \sigma^{(\hat{A}_{ru})_3}(x)S_{ru}}{S_{ru} + A_{ru}}.$

2.5. Stability Analysis of the Secondary System

The equilibrium, steady state points are constant solutions, $S_{ru}(x)$ and $A_{ru}(x)$, which satisfy the nonlinear system of equations $\frac{dS_{ru}}{dx} = 0$ and $\frac{dA_{ru}}{dx} = 0$. It is shown that this system is orbitally stable, which signifies that the solutions remain near the equilibrium point.

Local Analysis near Steady-State Points (S_{ru}^*, A_{ru}^*)

Considering,

$$\frac{dS_{ru}}{dx} = 0,$$

$$\frac{dA_{ru}}{dx} = 0.$$

Thus we obtain, $A_{ru}^*(x) = 1$ (form state), and $S_{ru}^*(x) = \frac{\sigma^{(\hat{A}_{ru})_3}(x) + \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}(x)}{\mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x)} = -1$

(space accessory state). It is important to note that the convective movement at secondary level is a stable state which is 0 (nullifying/stabilising accessory state). We study local analysis near each steady state point (S_{ru}^*, A_{ru}^*) for Equations (14) & (15). We begin by the following,

$$S_{ru}(x) = S_{ru}^*(x) + \epsilon(x), \tag{16}$$

$$A_{ru}(x) = A_{ru}^*(x) + \eta(x), \tag{17}$$

where $\epsilon(x)$ and $\eta(x)$ are small. Next we substitute Equations (16) & (17) into (18) & (19).

$$\frac{d\epsilon}{dx} = f(S_{ru}^*(x) + \epsilon(x), A_{ru}^*(x) + \eta(x)) \approx \frac{\partial f}{\partial S_{ru}} \epsilon + \frac{\partial f}{\partial A_{ru}} \eta, \tag{18}$$

$$\frac{d\eta}{dx} = g(S_{ru}^*(x) + \epsilon(x), A_{ru}^*(x) + \eta(x)) \approx \frac{\partial g}{\partial S_{ru}} \epsilon + \frac{\partial g}{\partial A_{ru}} \eta. \tag{19}$$

The Jacobian matrix for the secondary system is given by,

$$\tilde{J}(S_{ru}^*, A_{ru}^*) = \begin{bmatrix} \frac{\partial f}{\partial S_{ru}} & \frac{\partial f}{\partial A_{ru}} \\ \frac{\partial g}{\partial S_{ru}} & \frac{\partial g}{\partial A_{ru}} \end{bmatrix}_{(S_{ru}^*, A_{ru}^*)} = \begin{bmatrix} -\mu_{S_{ru}A_{ru}}^{(A_{ru})_2} & \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2} \\ \mu_{S_{ru}A_{ru}}^{(A_{ru})_2} & -(\sigma^{(\bar{A}_{ru})_3} + \mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}) \end{bmatrix}. \tag{20}$$

The eigenvalues λ_1 and λ_2 for this system are given by $-i\sqrt{\sigma^{(\bar{A}_{ru})_3} \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}}$ and $i\sqrt{\sigma^{(\bar{A}_{ru})_3} \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}}$ respectively. The secondary system gives a form of neutral stability.

3. Results

3.1. Numerical Projections of Sub-Particle and Particle of Solution for the Primary System

The graphical representation of the advective components τ, μ, σ (sub-systems) are shown (see **Figure 3**). The solution is developed from the primary system equation in Nemauro (2015) and the system (1 - 5).

The primary system and sub-system equations and estimated parameters are given (See Equations (21) - (27) and **Table 1**).

Primary system equation,

$$\overbrace{\Sigma^A(x)}^{A(x)} = \underbrace{\theta_1^\Sigma \left(\frac{\theta_2^\Sigma}{x} + 1 \right)}_{P(x)} + \frac{\theta_3^\Sigma x}{\theta_4^\Sigma + x} + \underbrace{\theta_5^\Sigma x e^{-\theta_6^\Sigma x}}_{C(x)}. \tag{21}$$

Primary σ —sub-particle equation given by,

$$\sigma^{\bar{A}_3}(x) = \theta_1^\sigma \left(e^{-\theta_2^\sigma x} - 1 \right) + \frac{\theta_3^\sigma x}{\theta_4^\sigma + x} + \theta_5^\sigma x e^{-\theta_6^\sigma x}. \tag{22}$$

Primary μ_{AS} —sub-particle equation given by,

$$\mu_{AS}^{\bar{A}_2}(x) = \theta_1^{\mu_{AS}} \left(e^{-\theta_2^{\mu_{AS}} x} - 1 \right) + \frac{\theta_3^{\mu_{AS}} x}{\theta_4^{\mu_{AS}} + x} + \theta_5^{\mu_{AS}} x e^{-\theta_6^{\mu_{AS}} x}. \tag{23}$$

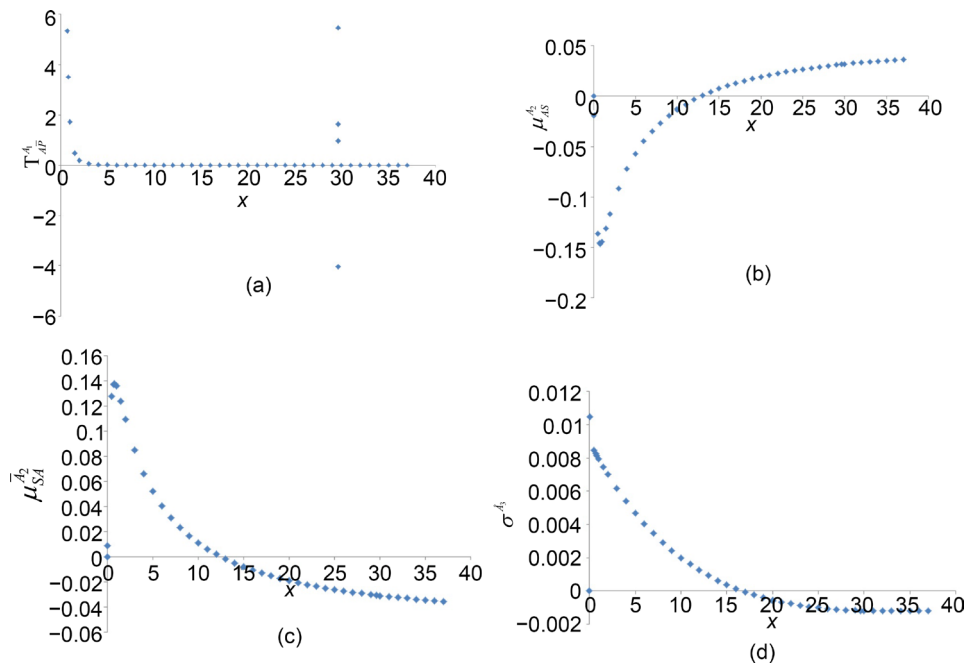


Figure 3. (a) The intra-advective movement potential of the primary sub-system from A to \bar{P} against concentration. Note the intra-advective movement potential of the primary sub-system from \bar{P} to A against concentration is equivalent to $-\tau_{A\bar{P}}^A$. (b) The intra-advective movement potential of the primary sub-system from A to S against concentration. (c) The intra-advective movement potential of the primary sub-system from S to A against concentration. (d) The intra-advective movement potential of the primary sub-system against concentration.

Table 1. Parameter estimates in modelling movement rates associated with the primary system.

Advective	Parameters	Estimate	Std Error	t value	Pr(> t)
Σ^A	θ_1^Σ	0.0034	0.0002	14.010	2.43×10^{-13}
	θ_2^Σ	0.8808	0.0874	10.076	2.75×10^{-10}
	θ_3^Σ	-0.0561	0.0057	-9.764	5.18×10^{-10}
	θ_4^Σ	7.4315	0.6922	10.737	7.49×10^{-11}
	θ_5^Σ	0.0089	0.0003	32.289	$< 2 \times 10^{-16}$
	θ_6^Σ	0.0598	0.0015	39.930	$< 2 \times 10^{-16}$
σ^A	θ_1^σ	0.0457	0.0145	3.146	0.0032
	θ_2^σ	1.4843	0.2279	6.512	1.01×10^{-7}
	θ_3^σ	0.0452	0.0147	3.073	0.0039
	θ_4^σ	0.2333(Fix)	-	-	-
	θ_5^σ	0.0083	0.0020	4.092	0.0002
	θ_6^σ	0.3241	0.0360	9.011	4.48×10^{-11}
μ_{AS}^A	$\theta_1^{\mu_{AS}}$	-0.1852	0.0047	-39.74	$< 2 \times 10^{-16}$
	$\theta_2^{\mu_{AS}}$	0.3691	0.0131	28.19	$< 2 \times 10^{-16}$
	$\theta_3^{\mu_{AS}}$	-0.2551	0.0064	-39.84	$< 2 \times 10^{-16}$
	$\theta_4^{\mu_{AS}}$	0.2333	0.0133	17.49	$< 2 \times 10^{-16}$
	$\theta_5^{\mu_{AS}}$	0.0072	0.0004	17.74	$< 2 \times 10^{-16}$
	$\theta_6^{\mu_{AS}}$	0.0254	0.0007	37.56	$< 2 \times 10^{-16}$

Continued

$\mu_{SA}^{\bar{A}_1}$	$\theta_1^{\mu_{SA}}$	0.1803	0.0047	38.489	$< 2 \times 10^{-16}$
	$\theta_2^{\mu_{SA}}$	0.3549	0.0170	20.869	$< 2 \times 10^{-16}$
	$\theta_3^{\mu_{SA}}$	0.2393	0.0028	84.021	$< 2 \times 10^{-16}$
	$\theta_4^{\mu_{SA}}$	0.2333(Fix)	–	–	–
	$\theta_5^{\mu_{SA}}$	–0.0061	0.0006	–9.459	1.21×10^{-11}
	$\theta_6^{\mu_{SA}}$	0.0242	0.0014	17.326	$< 2 \times 10^{-16}$
$\tau_{\alpha, A\bar{P}}^{A_1}$	$\theta_1^{(\alpha)}$	1.6961	0.0036	471.62	$< 2 \times 10^{-16}$
	$\theta_2^{(\alpha)}$	3.1504	0.002	1599.33	$< 2 \times 10^{-16}$
	$\theta_3^{(\alpha)}$	3.8206	0.0954	40.04	$< 2 \times 10^{-16}$
$\tau_{\beta, A\bar{P}}^{A_1}$	$\theta_1^{(\beta)}$	2.32×10^{-3}	1.313×10^{-11}	1.7664×10^8	$< 2 \times 10^{-16}$
	$\theta_2^{(\beta)}$	9.477×10^{-2}	4.514×10^{-4}	210	$< 2 \times 10^{-16}$
$\tau_{\gamma, A\bar{P}}^{A_1}$	$\theta_1^{(\gamma)}$	-2.33×10^{-3}	4.067×10^{-7}	–572.9	1.91×10^{-15}
	$\theta_2^{(\gamma)}$	8.529×10^{-2}	8.458×10^{-4}	100.8	6.41×10^{-11}

Primary $\tau_{A\bar{P}}$ —sub-particle equation given by,

$$\mu_{SA}^{\bar{A}_2}(x) = \theta_1^{\mu_{SA}} \left(e^{-\theta_2^{\mu_{SA}} x} - 1 \right) + \frac{\theta_3^{\mu_{SA}} x}{\theta_4^{\mu_{SA}} + x} + \theta_5^{\mu_{SA}} x e^{-\theta_6^{\mu_{SA}} x}. \tag{24}$$

Primary $\tau_{A\bar{P}}$ —sub-particle (α) equation given by,

$$\tau_{\alpha, A\bar{P}}^{A_1}(x) = \frac{\theta_1^{(\alpha)}}{x^{\theta_2^{(\alpha)}}} + \theta_1^{\tau_{A\bar{P}}^{(\alpha)}} e^{-\theta_3^{\tau_{A\bar{P}}^{(\alpha)}} x}, \quad 0 < x \leq x_n, \tag{25}$$

where $x_n : \left. \frac{d\tau}{dx} \right|_{x=x_n} = 0$.

Primary $\tau_{A\bar{P}}$ —sub-particle (β) equation given by,

$$\tau_{\beta, A\bar{P}}^{A_1}(x) = \frac{\theta_1^{(\beta)}}{x_m - x} + \theta_1^{\tau_{A\bar{P}}^{(\beta)}} e^{-\theta_2^{\tau_{A\bar{P}}^{(\beta)}} x}, \quad x_n \leq x < x_m. \tag{26}$$

Primary $\tau_{A\bar{P}}$ —sub-particle (γ) equation given by,

$$\tau_{\gamma, A\bar{P}}^{A_1}(x) = \frac{\theta_1^{(\gamma)}}{x - x_m} - \theta_1^{\tau_{A\bar{P}}^{(\gamma)}} e^{-\theta_2^{\tau_{A\bar{P}}^{(\gamma)}} x}, \quad x > x_m. \tag{27}$$

where $x_m = 29.6184242885$ is the point when the system is projected to show considerable loss of homogeneity-potential.

Furthermore, $\tau_{\bar{P}A} = -\tau_{A\bar{P}}$.

3.2. Numerical Projections of Sub-Particle and Particle of Solution Movement for the Secondary System

The graphical representation of the advective components μ, σ (sub-systems) are shown (see **Figure 4**). The solution is developed from the secondary system equation in Nemauro (2015) and the system (13 - 15).

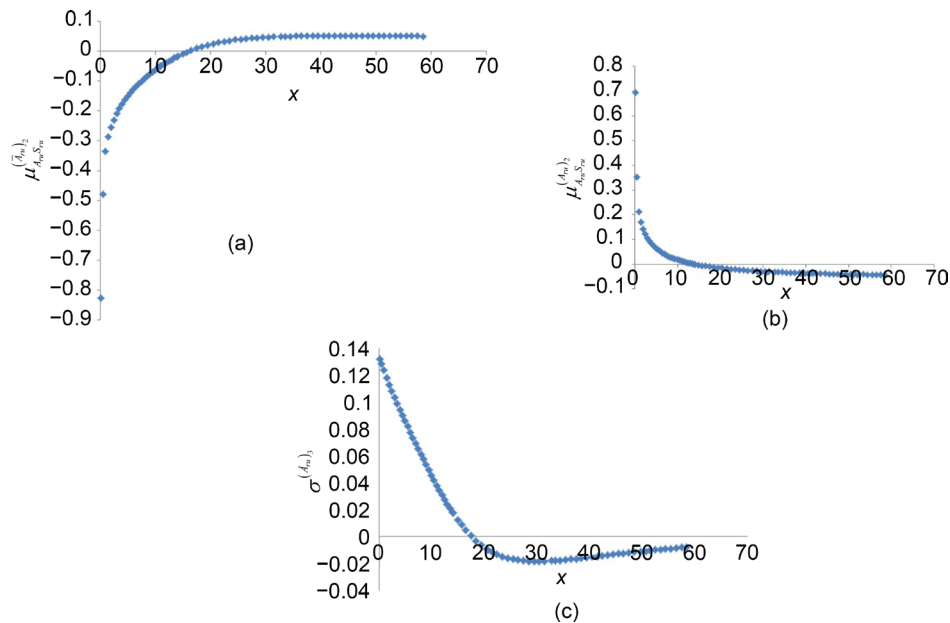


Figure 4. (a) The advective intra-secondary sub-system modelling the movement from A_{ru} to S_{ru} against concentration. (b) The advective intra-secondary sub-system modelling the movement from S_{ru} to A_{ru} against concentration. (c) The advective intra-secondary sub-system which initiates the process against concentration.

The secondary system and sub-system equations and estimated parameters are given (See Equations (28) - (31) and **Table 2**). Secondary system equation is given by,

$$\frac{A_{ru}(x)}{\Sigma^{A_{ru}}(x)} = \frac{\varphi_1^\sigma x}{\varphi_2^\sigma + x} + \frac{C_{ru}(x)}{\varphi_3^\sigma x e^{-\varphi_4^\sigma x}} \tag{28}$$

Secondary σ —sub-particle equation given by,

$$\sigma^{(\hat{A}_{ru})_3}(x) = \varphi_1^\sigma \left(\varphi_2^\sigma e^{-\varphi_3^\sigma x} - 1 \right) + \frac{\varphi_4^\sigma x}{\varphi_5^\sigma + x} + \varphi_6^\sigma x e^{-\varphi_7^\sigma x} \tag{29}$$

Secondary $\mu_{A_{ru}S_{ru}}$ —sub-particle equation given by,

$$\mu_{A_{ru}S_{ru}}^{(\bar{A}_{ru})_2}(x) = \varphi_1^{\mu_{A_{ru}S_{ru}}} \left(\frac{\varphi_2^{\mu_{A_{ru}S_{ru}}}}{x} + 1 \right) + \frac{\varphi_3^{\mu_{A_{ru}S_{ru}}} x}{\varphi_4^{\mu_{A_{ru}S_{ru}}} + x} + \varphi_5^{\mu_{A_{ru}S_{ru}}} x e^{-\varphi_6^{\mu_{A_{ru}S_{ru}}} x} \tag{30}$$

Secondary $\mu_{S_{ru}A_{ru}}$ —sub-particle equation given by,

$$\mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x) = \varphi_1^{\mu_{S_{ru}A_{ru}}} \left(\frac{\varphi_2^{\mu_{S_{ru}A_{ru}}}}{x} + 1 \right) + \frac{\varphi_3^{\mu_{S_{ru}A_{ru}}} x}{\varphi_4^{\mu_{S_{ru}A_{ru}}} + x} + \varphi_5^{\mu_{S_{ru}A_{ru}}} x e^{-\varphi_6^{\mu_{S_{ru}A_{ru}}} x} \tag{31}$$

3.3. Stability Numerical Analysis for Secondary System

Let $\pi = \sigma^{(\hat{A}_{ru})_3} \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}$ and the plot of π against x is given (See **Figure 5**). The results show that $\pi \geq 0$ for almost all values of concentration (x). The following is proposed $\sigma^{(\hat{A}_{ru})_3}(x) \mu_{S_{ru}A_{ru}}^{(A_{ru})_2}(x) = \pi(x) + \phi(x)$, where $\phi(x)$ is the error term in estimating

Table 2. Parameter estimates in modelling movement rates associated with the secondary system.

Advective	Parameters	Estimate	Std Error	t value	$Pr(> t)$
Σ^{A_m}	ϕ_1^Σ	-0.8281	0.0059	-141.37	$< 2 \times 10^{-16}$
	ϕ_2^Σ	6.9213	0.1615	42.87	$< 2 \times 10^{-16}$
	ϕ_3^Σ	0.1345	0.0007	186.53	$< 2 \times 10^{-16}$
	ϕ_4^Σ	0.0566	0.0002	233.96	$< 2 \times 10^{-16}$
$\sigma^{(\bar{A}_m)_3}$	ϕ_1^σ	-0.2354	0.0108	-21.89	$< 2 \times 10^{-16}$
	ϕ_2^σ	0.4450	0.0235	18.94	$< 2 \times 10^{-16}$
	ϕ_3^σ	0.2771	0.0221	12.57	$< 2 \times 10^{-16}$
	ϕ_4^σ	0.2354	0.0108	21.89	$< 2 \times 10^{-16}$
	ϕ_5^σ	9.199(Fix)	-	-	-
	ϕ_6^σ	-0.0100	0.0005	-19.57	$< 2 \times 10^{-16}$
	ϕ_7^σ	0.0470	0.0005	86.54	$< 2 \times 10^{-16}$
$\mu^{(\bar{A}_m)_2}_{A_m S_m}$	$\phi_1^{\mu_{A_m S_m}}$	-0.2665	0.0033	-81.76	$< 2 \times 10^{-16}$
	$\phi_2^{\mu_{A_m S_m}}$	0.4271	0.0084	51	$< 2 \times 10^{-16}$
	$\phi_3^{\mu_{A_m S_m}}$	0.2665	0.0033	81.76	$< 2 \times 10^{-16}$
	$\phi_4^{\mu_{A_m S_m}}$	9.1990	0.4912	18.73	$< 2 \times 10^{-16}$
	$\phi_5^{\mu_{A_m S_m}}$	0.0110	0.0003	33.19	$< 2 \times 10^{-16}$
	$\phi_6^{\mu_{A_m S_m}}$	0.0351	0.0004	87.31	$< 2 \times 10^{-16}$
$\mu^{(A_m)_2}_{S_m A_m}$	$\phi_1^{\mu_{S_m A_m}}$	0.1013	0.0026	38.40	$< 2 \times 10^{-16}$
	$\phi_2^{\mu_{S_m A_m}}$	1.1959	0.0400	29.90	$< 2 \times 10^{-16}$
	$\phi_3^{\mu_{S_m A_m}}$	-0.1013	0.0026	-38.4	$< 2 \times 10^{-16}$
	$\phi_4^{\mu_{S_m A_m}}$	9.199(Fix)	-	-	-
	$\phi_5^{\mu_{S_m A_m}}$	-0.0052	0.0002	-23.9	$< 2 \times 10^{-16}$
	$\phi_6^{\mu_{S_m A_m}}$	0.0304	0.0009	32.26	$< 2 \times 10^{-16}$

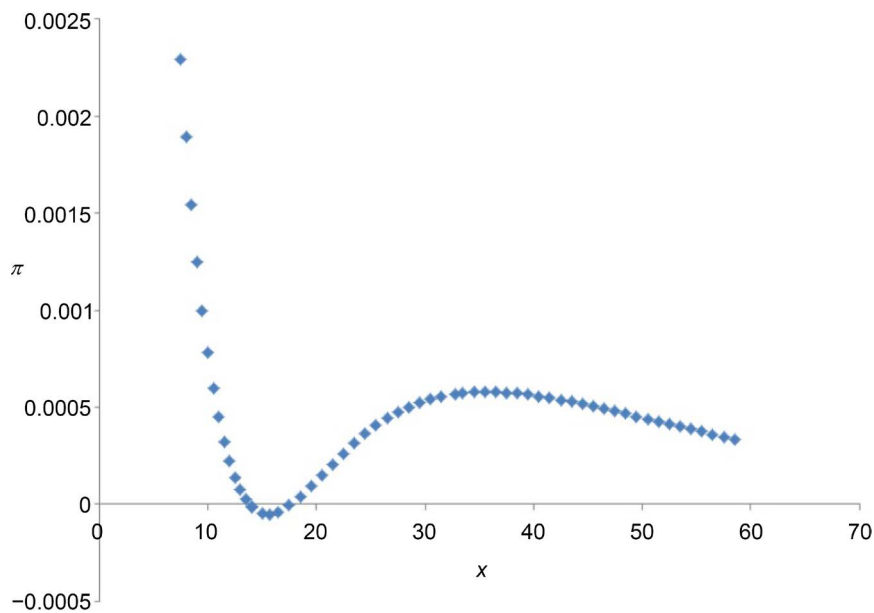


Figure 5. The value π of the characteristic equation against concentration x .

$\sigma^{(A_{ru})_3}(x)\mu^{(A_{ru})_2}(x)$. The system is inferred to be stable.

4. Discussion

A nonautonomous linear system is developed that enables characterisation of the projected movement in a solution-particle [5] [6]. There are four main movement potential components at primary level with respect to advection. They are held together by the sub-system also a potential which is inferred to have five distinct systems (that is the two μ 's, σ and two τ 's). While, the secondary system consists of two μ 's and σ . The variable parameters in the system of ordinary differential equations are inferred to describe (advective) form components of movement. The movement description gives the state of solution particle and there is suggestion of the constituent entities.

The primary system is generally projected to be an unstable system relative to concentration. The saturated system is inferred to have stable potential. The reaction allowing state (non-saturated concentration) has unstable equilibrium potential at primary level. However, the secondary system is stable.

This work has managed to describe the possible state of kinetics of a solution-particle of efavirenz. It has shown that a saturated state can be equated to a stable state both at primary and secondary level. The constructed matrix in this case is traceless that is it has zero trace. The volume conserving systems have been considered in modelling of the physical systems and the theoretical framework is also developed in Lie Theory [10] [11].

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